Earthquake induced oscillations of high rise buildings and other vertical structures

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World's largest earthquake test. Japan, 2009.

NEES (Network for Earthquake Engineering Simulation),
Simpson Strong-Tie and Colorado State University

“ *Recent earthquakes have shown that damage in non-structural components and in building contents can have large economic consequences as well as safety and egress concerns. ... (2) typically more than 75% of the construction cost is associated with non-structural components; and (3) localized damage in certain non-structural systems can affect the functionality of large portions of the building.*” - **Reinoso and Miranda, 2005.**

- Need models to simulate effect of oscillations.
- Tall buildings are often modelled as vertical beams.
- [RM05] - 14 articles use beam models for buildings.
- [RM05] - Building Seismic Safety commission and American Society of Civil Engineers use analytical studies and recovered data for safety specifications of new buildings.

Timoshenko model

- Rigorous derivation from three-dimensional linear elasticity presented in **Cowper, 1966**. Inspires confidence in the model.
- **Stephen and Puchegger, 2006**; **Labuschagne, Van Rensburg and Van der Merwe, 2009** - Timoshenko theory compared to multi-dimensional model. Timoshenko theory is an excellent approximation in the case of beam applications, i.e. for transverse loads.
- **Van Rensburg and Van der Merwe, 2006**; [LVV09] - Timoshenko model compared to Rayleigh and Euler-Bernoulli models. These models can be useful when β is large.
- Rayleigh and Euler-Bernoulli models are special cases of Timoshenko model.

Timoshenko model

Equations of motion:

$$\rho A \partial_t^2 w = \partial_x V + Q, \tag{1}$$
$$\rho I \partial_t^2 \phi = V + \partial_x M, \tag{2}$$

The **constitutive equations** for the moment M and the shear force V are

$$M = EI \partial_x \phi, \tag{3}$$
$$V = AG\kappa^2 \big(\partial_x w - \phi\big). \tag{4}$$

Dimensionless form of the **Timoshenko model**

$$\partial_t^2 w = \partial_x V + Q, \tag{5}$$

$$\frac{1}{\alpha}\,\partial_t^2 \phi = V + \partial_x M, \tag{6}$$

$$M = \frac{1}{\beta}\,\partial_x \phi, \tag{7}$$

$$V = \partial_x w - \phi. \tag{8}$$

The boundary conditions for a cantilever beam are

$$w(0,t) = \phi(0,t) = 0$$

at the clamped end and

$$M(1,t) = 0 \;\text{ and }\; V(1,t) = 0$$

at the free end.

Rayleigh model

Assume that the cross section remains perpendicular to the neutral plane. This implies that $\partial_x w = \phi$.

$$\partial_t^2 w = \frac{1}{\alpha}\,\partial_t^2\partial_x^2 w - \partial_x^2 M + Q, \tag{9}$$

$$M = \frac{1}{\beta}\,\partial_x^2 w. \tag{10}$$

The boundary conditions are the same as for the Timoshenko beam except that $\partial_x w(0,t) = 0$ replaces $\phi(0,t) = 0$.

Shear-T model

Han, Benaroya and Wei, 1999 consider four beam theories where in one shear is taken into account but not rotary inertia.

Shear-T model

$$\begin{aligned} \partial_t^2 w &= \partial_x V, & (11) \\ 0 &= V + \partial_x M. & (12) \end{aligned}$$

The constitutive equations and boundary conditions are the same as for the Timoshenko model.

Stiffness parameter $\frac{1}{\beta}$

$$\beta = \frac{AG\kappa^2\ell^2}{EI} \left(\alpha = \frac{A\ell^2}{I} \text{ and } \gamma = \frac{\beta}{\alpha} \right)$$

[VV06]; [LVV09] - Timoshenko model compared to Rayleigh and Euler-Bernoulli models. These models can be useful when β is large.

- Depending on initial data / manner of excitation, value of β between 300 and 1200 may be sufficient.
- For $\beta \approx 300$ fundamental frequency for these models is acceptable but not the higher frequencies.
- For $\beta < 100$ they should not be considered.

Modes of vibration

Natural frequencies of vibration is used to compare beam models. This approach was also used in

- [SP06] and [LVV09] - Timoshenko v.s. multi-dimensional model;
- [VV06] and [LVV09] - Timoshenko v.s. Rayleigh and Euler-Bernoulli.

For the modal analysis we follow [VV06].

Eigenvalue problem Timoshenko

Consider Equations (5) and (6) of Timoshenko model, do separation of variables to obtain eigenvalue problem

$$\begin{aligned} -u'' + \psi' &= \lambda u, && (13) \\ -\frac{1}{\beta}\psi'' - u' + \psi &= \frac{\lambda}{\alpha}\psi, && (14) \end{aligned}$$

with the boundary conditions given by

$$u(0) = \psi(0) = u'(1) - \psi(1) = \psi'(1) = 0. \quad (15)$$

To calculate eigenvalues and eigenfunctions use method in [VV06].

To calculate eigenvalues for **Shear-T model**, use eigenvalue problem for Timoshenko with $\lambda = 0$ in equation (14).

To justify this, replace $\dfrac{1}{\alpha}$ by $\dfrac{\gamma}{\beta}$ and let $\gamma = 0$. (λ depends continuously on γ.)

Frequency equation:

$$\left(\frac{\lambda+\mu^2}{\lambda-\omega^2}+\frac{\lambda-\omega^2}{\lambda+\mu^2}\right)\cosh\mu\cos\omega+\left(\frac{\omega}{\mu}-\frac{\mu}{\omega}\right)\sinh\mu\sin\omega = 2,$$

but with

$$\omega^2 = \frac{\lambda}{2}\left(\sqrt{1+\frac{4\beta}{\lambda}}+1\right) \text{ and } \mu^2 = \frac{\lambda}{2}\left(\sqrt{1+\frac{4\beta}{\lambda}}-1\right).$$

Comparison of Shear-T and Timoshenko eigenvalues

$\beta_{LA52} = 50.$
For Timoshenko model $\gamma = 0.25$ and $\gamma = 0$ for Shear-T model.

LA-52: North-South oscillation		
	Timoshenko model	Shear-T model
k	λ_k	λ_k
1	0.2190	0.2232
2	5.3522	5.8336
3	27.3517	30.4359
4	69.5214	78.4895
5	132.8139	150.5247
6	201.4049	244.7589

Beam models for high-rise structures

Adapted Timoshenko model

$$
\begin{aligned}
\rho^* \partial_t^2 u &= \partial_x S + P, && (16)\\
\rho^* \partial_t^2 w &= \partial_x V + Q, && (17)\\
\frac{\rho^*}{\alpha}\, \partial_t^2 \phi &= V + \partial_x M + S \partial_x w, && (18)\\
M &= \frac{1}{\beta}\, \partial_x \phi, && (19)\\
V &= \partial_x w - \phi, && (20)\\
S &= \frac{1}{\gamma} \partial_x u. && (21)
\end{aligned}
$$

Parameter ρ^*

- Entire structure cannot be considered as a beam.
- Seems reasonable that part of building may be modelled as beam. (Reinforced concrete frames, steel frames and shear walls are mentioned in [RM05].)
- Additional mass that does not contribute to stiffness of the structure is present.
- Let ρ_{RM} denote mass per unit length used in [RM05], then $\rho_{RM} > \rho A$, where ρA is mass per unit length of the "beam".
- Let $\rho^* = \dfrac{\rho_{RM}}{\rho A}$, then $\rho^* > 1$.

- Only consider transverse vibration.
- $S = \mu(1 - x), \;\; \mu = \dfrac{\rho g \ell}{G\kappa^2} << 0.1.$
- A force density considered in **Wang, Fung and Huang, 2001** but not in [RM05].
- Effect of S is hardly noticable.

Adapted Timoshenko model

$$\rho^* \partial_t^2 w = \partial_x V, \tag{22}$$

$$\frac{\gamma \rho^*}{\beta} \partial_t^2 \phi = V + \partial_x M + S \partial_x w. \tag{23}$$

Note that $\dfrac{1}{\alpha}$ was replaced by $\dfrac{\gamma}{\beta}$.
$w(0,t) = w_E(t), \;\; u(0,t) = \phi(0,t) = 0.$
$M(1,t) = 0 \text{ and } V(1,t) = 0.$

Earthquake induced oscillations

- The force density $Q = 0$.
- In general $u(0,t) \neq 0$.

Equivalent problem

The earthquake model problem is equivalent to an artificial "wind problem" for a cantilever beam.

The boundary condition $w(0,t) = w_E(t)$ can be homogenized: Let $\tilde{w}(x,t) = w(x,t) - w_E(t)y(x)$ and $\tilde{V} = \partial_x \tilde{w} - \phi$.

Equations (22) and (23) are transformed as follows

$$\rho^* \partial_t^2 \tilde{w} = \partial_x \tilde{V} - \rho^* w_E - \rho^* \ddot{w}_E y, \quad (24)$$

$$\frac{\gamma \rho^*}{\beta} \partial_t^2 \phi = \tilde{V} + w_E y' + \partial_x M - \partial_x w S, \quad (25)$$

where $y(x) = 1 + x - \frac{1}{2}x^2$.

Boundary conditions:

$y(0) = 1$ implies

$$\tilde{w}(0,t) = w_E(t) - w_E(t)y(0) = 0.$$

At the top

$$\tilde{V}(1,t) = V(1,t) - w_E(t)y'(1) = V(1,t) = 0.$$

The other boundary conditions remain unchanged, i.e.

$$M(1,t) = 0 \text{ and } \phi(0,t) = 0.$$

We now have a model problem for a **cantilever** beam.

Shear-M model

It is derived from a model in **Miranda, 1999** for a building in equilibrium subjected to a distributed load Q (equivalent problem). A shear beam is combined with an Euler-Bernoulli (flexural) beam.

$$\rho^* \partial_t^2 w - \sigma \partial_x^2 w + \frac{1}{\beta} \partial_x^4 w = Q, \quad \text{where} \quad \sigma = \frac{G_s A_s}{G A \kappa^2}. \qquad (26)$$

In [RM05] the boundary conditions are not discussed. At $x = 0$ may use the boundary conditions for Rayleigh and at the top

$$\partial_x^2 w(1, t) = 0 \quad \text{and} \quad \partial_x w(1, t) - \frac{1}{\beta\sigma} \partial_x^3 w(1, t) = 0.$$

Note that gravity is neglected in this model.

Stiffness ratio parameter in [RM05]: $\alpha_M = \beta\sigma$.

Eigenvalue problem

$$\begin{aligned}
u^{(4)} - \alpha_M u'' - \lambda \alpha_M u &= 0, \quad \text{with} \\
u(0) = u'(0) &= 0, \\
\frac{1}{\alpha_M} u'''(1) - u'(1) &= 0, \\
u''(1) &= 0.
\end{aligned}$$

- Authors make use of their model to obtain the values of the parameters.
- Values of β and σ are not given separately in article - only α_M is given.

From the boundary conditions we also obtain the following frequency equation

$$\left(2\frac{\mu^2\omega^2}{\beta} - \omega^2 + \mu^2\right)\cosh\mu\cos\omega + \left(2\mu\omega - \frac{\mu^3\omega}{\beta} + \frac{\mu\omega^3}{\beta}\right)\sinh\mu\sin\omega + \frac{\mu^4 + \omega^4}{\beta} - \mu^2 + \omega^2 = 0, \quad \text{with}$$

$$\mu^2 = \frac{\beta}{2}\left(1 + \sqrt{1 + \frac{4\lambda}{\beta}}\right) \text{ and } \omega^2 = \frac{\beta}{2}\left(-1 + \sqrt{1 + \frac{4\lambda}{\beta}}\right).$$

Comparison of two buildings using data from [RM05].

	LA-52	LA-54
Height	$\pm 200m$	$\pm 200m$
Floor dimensions	$48m \times 48m$	$60m \times 37m$
α_M	$\alpha_{M,\mathrm{NS}} = 7.8^2$ $\alpha_{M,\mathrm{EW}} = 6.6^2$	$\alpha_{M,\mathrm{NS}} = 27.5^2$ $\alpha_{M,\mathrm{EW}} = 30^2$
Fundamental period	$T_{\mathrm{NS}} = 5.8$ $T_{\mathrm{EW}} = 6$	$T_{\mathrm{NS}} = 6.2$ $T_{\mathrm{EW}} = 5.2$
Peak ground acceleration	$\mathrm{PGA}_{\mathrm{NS}} = 165$ $\mathrm{PGA}_{\mathrm{EW}} = 109$	$\mathrm{PGA}_{\mathrm{NS}} = 165$ $\mathrm{PGA}_{\mathrm{EW}} = 98$
Peak roof acceleration	$\mathrm{PRA}_{\mathrm{NS}} = 389$ $\mathrm{PRA}_{\mathrm{EW}} = 220$	$\mathrm{PRA}_{\mathrm{NS}} = 177$ $\mathrm{PRA}_{\mathrm{EW}} = 139$

Simulation

- Nature of the disturbance should be taken into account - will determine number of modes involved. (If manner of excitation is such that only first mode is considered, then Euler-Bernoulli beam may still be fine.)
- Earthquake models: don't know how many modes are involved - simulation is necessary.
- To investigate effect of disturbance our preliminary experiment was to simulate each model separately to observe the transient response of the structure.

Transient response of a building due to earthquake using Timoshenko model. Full period of the ground disturbance $\tau_g = 8$, $w(0,t) = w_E = D\sin(Ct)$.

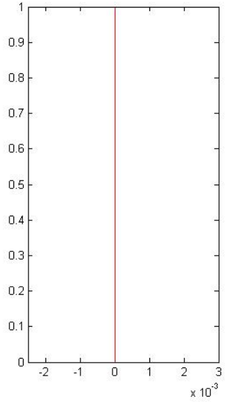
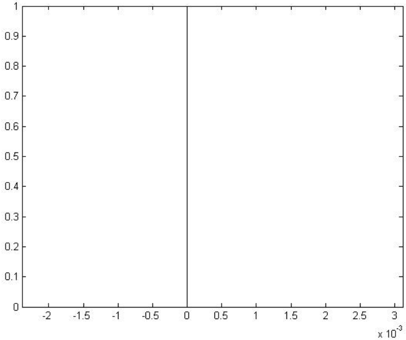


Illustration of effect of β using Timoshenko model

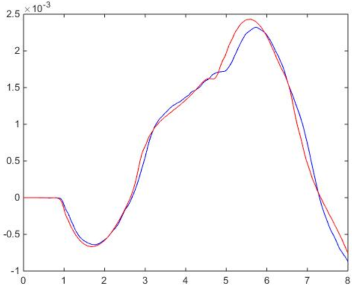
$\beta = 50$ (in red) v.s. $\beta = 800$ (in blue).



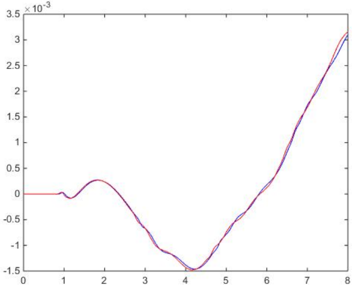
Comparison of models

Consider the motion of top of building for full period of ground motion.

$\beta = 50$

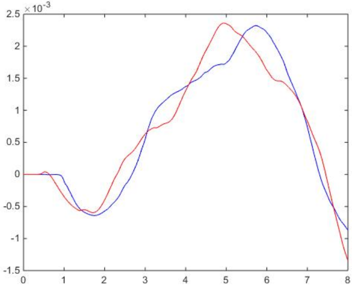


$\beta = 800$

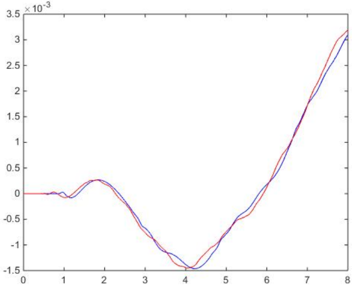


Timoshenko (blue) v.s. Shear-T (red)

$\beta = 50$



$\beta = 800$



Timoshenko (blue) v.s. Rayleigh (red)

Conclusion

- Rayleigh and Euler-Bernoulli only for $300 < \beta < 1200$.
- Shear-T compares well to Timoshenko - but difficulty in programming and no gain.
- Shear-M cannot be compared to Timoshenko using [RM05] data. Solution: Artificial building or data from another artical.

END

Thank you