Rosenbrock-type methods for geothermal reservoirs simulation

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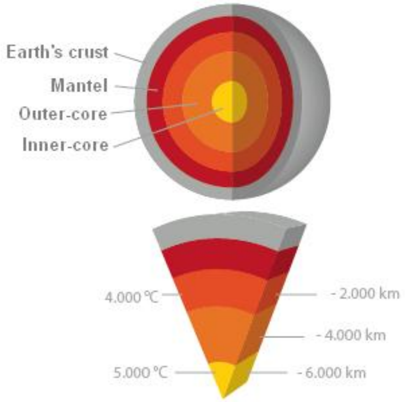
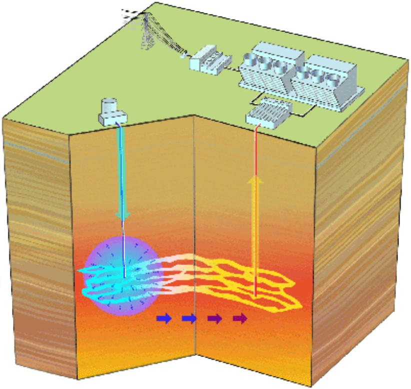
Outline

What is geothermal energy?





Geothermal reservoir simulation: AIMS, Challenge and research strategies

1. AIMS
   - Predict reservoir production
   - Optimal production strategies
   - Understand physical processes
2. Challenge
   - Coupled highly nonlinear physical processes
   - Coupled processes on multiple scales
   - Heterogeneous environments
   - Working in fixed-grid with phase change
3. Our goal

   Propose an alternative efficient, stable and accurate time stepping methods where Newton iterations are no required at every time step as in standard implicit methods mostly used currently in reservoir simulation.

Geothermal with one phase flow

1. Energy Equation

$$\begin{cases} (1-\phi)\rho_s c_{ps}\dfrac{\partial T_s}{\partial t} = (1-\phi)\nabla\cdot(\mathbf{k}_s\nabla T_s) + (1-\phi)q_s + he(T_f - T_s) \\ \phi\rho_f c_{pf}\dfrac{\partial T_f}{\partial t} = \phi\nabla\cdot(\mathbf{k}_f\nabla T_f) - \nabla\cdot(\rho_f c_{pf}\mathbf{v}T_f) + \phi q_f + he(T_s - T_f) \end{cases}$$

2. Darcy's Law

$$\mathbf{v} = -\frac{\mathbf{K}}{\mu}\left(\nabla p - \rho_f \mathbf{g}\right), \qquad (2)$$

3. Mass balance equation

$$\frac{\partial \phi\rho_f}{\partial t} = -\nabla\cdot(\mathbf{v}\rho_f) + Q_f, \qquad (3)$$

Geothermal with one phase flow

1. State functions $\mu, \rho_f, C_pf, \alpha_f, \beta_f$
2. Slightly compressible rock and compressible fluid

$$\begin{cases} \phi = \phi_0 \left(1 + \alpha_b(p - p_0)\right) \\ \alpha_f = -\dfrac{1}{\rho_f}\dfrac{\partial \rho_f}{\partial T_f}, \qquad \beta_f = \dfrac{1}{\rho_f}\dfrac{\partial \rho_f}{\partial p}. \end{cases} \tag{4}$$

3. Model equations

$$\begin{cases} (1-\phi)\rho_s c_{ps}\dfrac{\partial T_s}{\partial t} = (1-\phi)\nabla\cdot(\mathbf{k}_s \nabla T_s) + (1-\phi)q_s + he(T_f - T_s) \\ \phi\rho_f c_{pf}\dfrac{\partial T_f}{\partial t} = \phi\nabla\cdot(\mathbf{k}_f \nabla T_f) - \nabla\cdot(\rho_f c_{pf}\mathbf{v}T_f) + \phi q_f + he(T_s - T_f) \\ -\phi\rho_f\alpha_f\dfrac{\partial T_f}{\partial t} + \rho_f\left(\phi\beta_f + \phi_0\alpha_b\right)\dfrac{\partial p}{\partial t} = \nabla\cdot\left(\dfrac{\rho_f\mathbf{K}}{\mu}\left(\nabla p - \rho_f\mathbf{g}\right)\right) + Q_f \end{cases}$$

Finite volume for space discrete

- **Keys features of the method**
  1. Integrate each equations over each control volume Ω_i.
  2. Use the divergence theorem to convert the volume integral into the surface integral in all divergence terms.
  3. Use two-point flux approximations for diffusion heat and flow fluxes
- **Semi-discrete system after space discretization**

$$\begin{cases} \dfrac{dT_h}{dt} = G(T^h, p_h, t), \\ \\ \dfrac{dp_h}{dt} = G_3(p_h, T_f^h, t) + \dfrac{(\phi\alpha_f)(T_f^h, p_h)}{(\phi\beta_f + \phi_0\alpha_b)(T_f^h, p_h)} \cdot G_2(T_s^h, T_f^h, p_h, t), \\ \\ G(T^h, p_h, t) = (G_1(T_s^h, T_f^h, t), G_2(T_s^h, T_f^h, p_h, t))^T, \\ T_h = (T_s^h, T_f^h)^T \approx (T_s, T_f)^T. \end{cases} \tag{6}$$

Rosenbrock-Type methods: Construction

- Motivation

  When the equations are non-linear, implicit equations can in general be solved only by iteration. This is a severe drawback, as it adds to the problem of stability, that of convergence of the iterative process. An alternative, which avoids this difficulty, is ...., (H.H. Rosenbrock 1962/63

- Consider the following ODEs

$$y' = f(y)$$

- The corresponding diagonally implicit Runge-Kutta method is given by

$$k_i = hf(y_n + \sum_{j=1}^{i-1} a_{i,j}k_j + a_{i,i}k_i), \quad y_{n+1} = y_n + \sum_{i=1}^{s} b_i k_i \tag{7}$$

Rosenbrock-Type methods: Construction

- Linearization

$$k_i = hf(g_i) + f'(g_i)a_{i,i}k_i, \quad g_i = y_n + \sum_{j=1}^{i-1} a_{i,j}k_j + a_{i,i}k_i. \qquad (8)$$

- The equation (8) can be interpreted as the application of one Newton iteration to each stage of previous RK method.
- No continuation of iterating until convergence, a new class of methods are deduced with judicious choice of coefficients $a_{i,j}$ to ensure their convergence, their stability and the accuracy.
- The s-stage Rosenborck methods is given by

$$k_i = hf(y_n + \sum_{j=1}^{i-1} a_{i,j}k_j) + hf'(y_n)\sum_{j=1}^{i} \gamma_{i,j}\, k_j, \quad y_{n+1} = y_n + \sum_{i=1}^{s} b_i k_i. \qquad (9)$$

- Difference with RK, extra coefficients $\gamma_{i,j}$ are needed.

Rosenbrock-Type methods: Embedded approximations

- To control the local errors and adaptivity purposes, cheaper and stable scheme is needed, the corresponding embedded approximation associated to Rosenbrock-Type methods is given by

$$y_{n+1}^{1} = y_n + \sum_{i=1}^{s} \hat{b}_i k_i. \tag{10}$$

- For Rosenbrock -type method of order p, the coefficients $\hat{b}_i$ are determined using the consistency conditions such that the embedded approximation is order $p-1$.
- The the embedded approximation is always more stable that the associated scheme and the local error is estimated as $err = norm(y_n - y_n^1)$.

Application to geothemal model

- The second order scheme ROS2(1) and the third order scheme denoted ROS3p are used.
- We solve sequentially the following systems

$$\begin{cases} \dfrac{dT_h}{dt} = G(T^h, p_h, t) \\ T_h(0),\ p_h(0) \quad \textbf{given}, \end{cases} \tag{11}$$

and

$$\begin{cases} \dfrac{dp_h}{dt} = G_3(p_h, T_f^h, t) + \dfrac{(\phi\alpha_f)(T_f^h, p_h)}{(\phi\beta_f + \phi_0\alpha_b)(T_f^h, p_h)} \cdot G_2(T_s^h, T_f^h, p_h, t) \\ = G_4(T_h^h, p_h, t), \\ T_h(0),\ p_h(0) \quad \textbf{given}. \end{cases} \tag{12}$$

Two-phase mixture model problems (C.Y. Wang, 2007)

1. The mass conservation of the two phase is given

$$\frac{\partial \phi \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_f. \tag{13}$$

   Here Q_f is the source of liquid and vapor.

2. The momentum conservation and is given by

$$\mathbf{u} = -\frac{\mathbf{K}}{\mu} \left[\nabla p - \rho_k(s)\mathbf{g} \right], \tag{14}$$

3. The model is obtained by adding the equations of mass conservation of liquid phase and vapor phase, $\rho \mathbf{u} = \rho_l \mathbf{u}_l + \rho_v \mathbf{u}_v$, $\rho = \rho_l s + (1-s)\rho_v$, $\rho_k = \rho_l \lambda_l + \rho_v \lambda_v$, $\mu = \rho \upsilon$, $\upsilon = 1/(kr_l/\upsilon_l + kr_v/\upsilon_v)$ with

$$\mathbf{u}_i = -\mathbf{K}\frac{kr_i}{\mu_i} \left[\nabla p - \rho_i \mathbf{g} \right], \quad i = \{l, v\}. \tag{15}$$

Two-phase mixture model problems

1. Monotone transformation of the thermodynamic state variables
$H = \rho(h - 2h_{vsat})$, $\rho h = \rho_l s h_l + \rho_v (1-s) h_v$

$$\Omega \frac{\partial H}{\partial t} + \nabla \cdot (\gamma_h \mathbf{u} H) = \nabla \cdot (\Gamma_h \nabla H) + \nabla \cdot \left[f(s) \frac{K \Delta \rho h_{fg}}{\nu_v} \mathbf{g} \right] \quad (16)$$

2. The temperature T and liquid saturation s can be calculated as

$$T = \begin{cases} \dfrac{H + 2\rho_l h_{vsat}}{\rho_l c_{pl}} & H \le -\rho_l (2h_{vsat} - h_{lsat}) \\ T_{sat} & -\rho_l (2h_{vsat} - h_{lsat}) < H \le -\rho_v h_{vsat} \\ T_{sat} + \dfrac{H + \rho_v h_{vsat}}{\rho_v c_{pv}} & -\rho_v h_{vsat} < H \end{cases}$$

$$s = \begin{cases} 1 & H \le -\rho_l (2h_{vsat} - h_{lsat}) \\ -\dfrac{H + \rho_v h_{vsat}}{\rho_l h_{fg} + (\rho_l - \rho_v) h_{vsat}} & -\rho_l (2h_{vsat} - h_{lsat}) < H \le -\rho_v h_{vsat} \\ 0 & -\rho_v h_{vsat} < H. \end{cases}$$

Model problem (C.Y. Wang and al.)

1. Two-phase mixture model problem (C.Y. Wang and al.)

$$\begin{cases} \frac{\partial \phi \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_f \\ \Omega \frac{\partial H}{\partial t} + \nabla \cdot (\gamma_h \mathbf{u} H) = \nabla \cdot (\Gamma_h \nabla H) + \nabla \cdot \left[f(s) \frac{K \Delta \rho h_{fg}}{\nu_v} \mathbf{g} \right] \\ \Omega = \phi + \rho_s c_{ps} (1 - \phi) \dfrac{dT}{dH} \end{cases}$$

2. Wang model were recently tested with great success for steady state mass conservation by different authors
3. For geothemal, steady state mass conservation is less realistic

Our adapted model problem

- Decomposition

$$\begin{aligned} \frac{\partial(\phi\rho)}{\partial t} &= \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} \\ \phi\frac{\partial\rho}{\partial t} &= \phi\frac{\partial\rho}{\partial p}|_H\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial H}|_p\frac{\partial H}{\partial t} = \phi\rho\beta_H\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial H}|_p\frac{\partial H}{\partial t}. \end{aligned} \tag{17}$$

- Here, β_H is called the pseudo fluid compressibility at constant mixture pseudo enthalpy H

$$\beta_H = \frac{1}{\rho}\frac{\partial\rho}{\partial p}|_H = -\frac{1}{V}\frac{\partial V}{\partial p}|_H \tag{18}$$

- We assume that the rock is weakly compressibility

$$\phi = \phi_0(1 + \alpha_b(p - p_0)) \qquad \frac{\partial\phi}{\partial t} = \phi_0\alpha_b\frac{\partial p}{\partial t}. \tag{19}$$

Our adapted model problem

- Note that in one phase region, by simplication we have:

$$\beta_H = \frac{\beta \rho c_p + \alpha(1 - \alpha T)}{\rho\left(c_p - \alpha(h - 2h_{vsat})\right)}, \chi := \left(\frac{\partial \rho}{\partial H}\right)_p = \frac{\alpha}{\alpha(h - 2h_{vsat}) - c_p} \quad (20)$$

- As we are dealing with two phase flow with phase change we compute the coefficients by

$$\chi = -\frac{1}{v^2}\left(\frac{\partial v}{\partial H}\right)|_p = -\frac{1}{v^2}\frac{\frac{\partial v}{\partial h}|_p}{\frac{\partial H}{\partial h}|_p} \quad (21)$$

$$\beta_H = -\rho\frac{\frac{\partial v}{\partial p}|_h\frac{\partial H}{\partial h}|_p - \frac{\partial v}{\partial h}|_p\frac{\partial H}{\partial p}|_h}{\frac{\partial H}{\partial h}|_p} \quad (22)$$

Our adapted model problem

1. Our adapted geothermal model problem

$$\begin{cases} \rho\left(\phi\beta+\phi_0\alpha_b\right)\dfrac{\partial p}{\partial t}=\nabla\cdot\left(\frac{\rho\mathbf{K}}{\mu}\left[\nabla p-\rho_k\mathbf{g}\right]\right)+Q_f-\phi\chi\dfrac{\partial H}{\partial t} \\ \Omega\frac{\partial H}{\partial t}+\nabla\cdot\left(\gamma_h\mathbf{u}H\right)=\nabla\cdot\left(\Gamma_h\nabla H\right)+\nabla\cdot\left[f(s)\frac{K\Delta\rho h_{fg}}{\nu_v}\mathbf{g}\right] \end{cases} \tag{23}$$

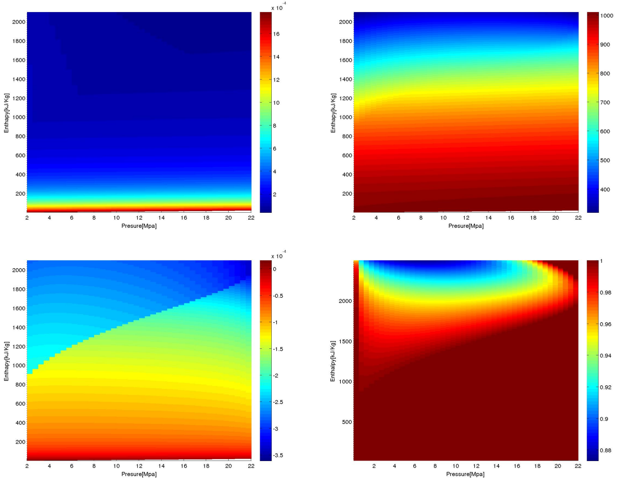
2. Expression of some coefficients

$$\gamma_h = \frac{\left[\frac{\rho_v}{\rho_l}(1-s)+s\right]\left[h_{vsat}(1+\lambda_l)-h_{lsat}\lambda_l\right]}{\left(2h_{vsat}-h_{lsat}\right)s+\left(\rho_v\frac{h_{vsat}}{\rho_l}\right)(1-s)} \tag{24}$$

$$\Gamma_h = k_{eff}\frac{dT}{dH} \tag{25}$$

$$f(s) = \frac{\frac{kr_v kr_l}{\nu_l}}{\frac{kr_l}{\nu_l}+\frac{kr_v}{\nu_v}}, \tag{26}$$

Graphs of some coefficients with $kr_l = s, kr_v = 1 - s$ (for μ). In order μ, ρ, χ, γ_h



Finite volume methods for space discretization

- Semi discrete system

$$\begin{cases} \rho\left((\phi\beta)_\delta + \phi_0\alpha_b\right)\dfrac{dp_\delta}{dt} = G_1(p_\delta, H_\delta) - (\chi\phi)(p_\delta, H_\delta)\dfrac{d(H_\delta)}{dt}, \\ \Omega_\delta \dfrac{dH_\delta}{dt} = G_2(H_\delta, p_\delta) \end{cases} \quad (27)$$

- We solve sequentially the following systems

$$\begin{cases} \Omega_\delta \dfrac{dH_\delta}{dt} = G_2(H_\delta, p_\delta) \\ H_\delta(0),\ p_\delta(0) \quad \textbf{given}, \end{cases} \quad (28)$$

$$\begin{cases} \psi(H_\delta, p_\delta)\dfrac{dp_\delta}{dt} = G_1(p_\delta, H_\delta) - (\chi\phi)(p_\delta, H_\delta)\dfrac{d(H_\delta)}{dt}, \\ \\ \psi(H_\delta, p_\delta) := \rho\left((\phi\beta_H)_\delta + \phi_0\alpha_b\right) \\ H_\delta(0),\ p_\delta(0) \quad \textbf{given}. \end{cases} \quad (29)$$

Rosenbrock-type scheme for differential algebraic equations

1. Consider the following differential algebraic equation in implicit form as it appears in our model problem

$$\begin{cases} C(\mathbf{y},t)\dfrac{d\mathbf{y}}{dt} = \mathbf{f}(\mathbf{y},t), \qquad t \in [0,\tau] \\ \mathbf{y}(0) = \mathbf{y}_0, \end{cases} \tag{30}$$

2. The following transformation is needed $z = \dfrac{d\mathbf{y}}{dt}$, we therefore have

$$\frac{d\mathbf{y}}{dt} = z, \qquad C(\mathbf{y},t)z - \mathbf{f}(\mathbf{y},t) = 0 \tag{31}$$

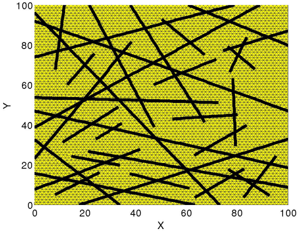
3. Applying the RM and set $\epsilon = 0$ to

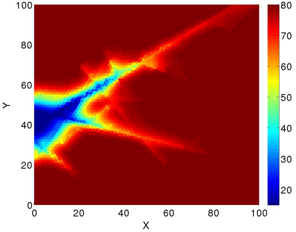
$$\frac{d\mathbf{y}}{dt} = z, \qquad \epsilon\frac{d\mathbf{z}}{dt} = C(\mathbf{y},t)z - \mathbf{f}(\mathbf{y},t). \tag{32}$$

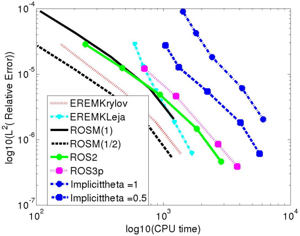
Rosenbrock-type scheme for DAE for energy equation

$$\begin{cases}
\left(\frac{1}{\tau_n\gamma}\Omega_\delta(H_\delta^n,p_\delta^n)-\mathbf{A}_n\right)\mathbf{k}_{ni}=G_2(H_\delta^n+\sum_{j=1}^{i-1}a_{ij}\mathbf{k}_{nj},p_\delta^n) \\
-\Omega_\delta((H_\delta^n,p_\delta^n)\sum_{j=1}^{i-1}\frac{c_{ij}}{\tau_n}\mathbf{k}_{nj}+\left((\Omega_\delta(H_\delta^n,p_\delta^n)-\Omega_\delta(H_\delta^n+\sum_{j=1}^{i-1}a_{ij}\mathbf{k}_{nj},p_\delta^n)\right) \\
\left((1-\sigma_i)z_n+\sum_{j=1}^{i-1}\frac{s_{ij}}{\tau_n}\mathbf{k}_{nj}\right), \\
H_\delta^{n+1}=H_\delta^n+\sum_{i=1}^{s}b_i\mathbf{k}_{ni}, \\
H_1^{n+1}=H_\delta^n+\sum_{i=1}^{s}\hat{b}_i\mathbf{k}_{ni} \\
z^{n+1}=z_n+\sum_{i=1}^{s}b_i\left(\frac{1}{\tau}\sum_{j=1}^{i}(c_{i,j}-s_{i,j})\mathbf{k}_{ni}+(\sigma_i-1)z_n\right) \\
z_1^{n+1}=z_n+\sum_{i=1}^{s}\hat{b}_i\left(\frac{1}{\tau}\sum_{j=1}^{i}(c_{i,j}-s_{i,j})\mathbf{k}_{ni}+(\sigma_i-1)z_n\right)
\end{cases}$$

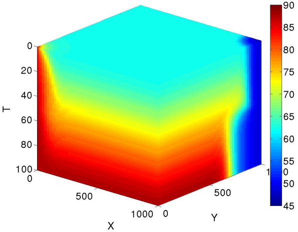
Geothermal simulation in 2 D fractured reservoir

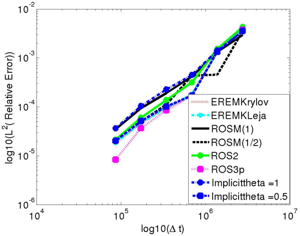


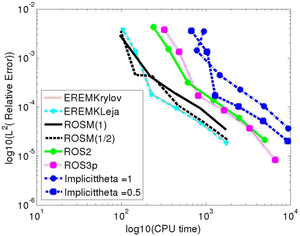




Geothermal simulation in 3 D







End

Thank You!!!!!!!!!!
Merci Beaucoup!!!!!!!
Tusen Takk!!!!!!!!!!!
muito obrigado!!!!!!