South African Mathematical Society

SAMS

65th SAMS CONGRESS

06-08 DECEMBER 2022

STELLENBOSCH UNIVERSITY

PROGRAMME AND BOOK OF ABSTRACTS

Stellenbosch UNIVERSITY IYUNIVESITHI UNIVERSITEIT

MaSS DSI-NRF CENTRE OF EXCELLENCE IN MATHEMATICAL & STATISTICAL SCIENCES

65th SAMS CONGRESS

06-08 DECEMBER 2022

STELLENBOSCH UNIVERSITY

Session key:

CATL	Finite Groups	FAOT	General	Graph Theory
Lie Symmetries	Math Education	Plenaries	SANUM/DEs	WiM

Day 1 schedule: (Tues 06 Dec)

Time\ Room	1003	1032	1033	1041	2118	2121
08:00-08:30						Opening
08:30-09:20	Topological Data Analysis - a new tool from old ideas. (Chair: N Bishop)					J Murugan
09:25-09:50	B Rodrigues	G Janelidze	A Ran	R Durandt	I Chokoe	J Nordstrom
09:50-10:15	A Prins	M Hoefnagel	D J van Rensburg	L Tendela	K Tshivhi	J Nordstrom
10:15-10:45	COFFEE					
10:45-11:10	S Madanha	S Marques	M van Straaten	T Khemane	B Moutsinga	C Harley
11:10-11:35	B Razakarinoro	S Chili	A van der Merwe	E Kikianty	L Galane	G Marewo
11:35-12:00	M Sias	E Inyangala	J Jaftha	A Campbell	F Mhlanga	J Munyakazi
12:00-13:30	LUNCH					
13:30-14:20	Pedagogical practices in integration of IKS in teaching and learning geometry. (Chair: R Durandt)					Z Jojo
14:25-14:50	V Ranaivomanana	Z Janelidze	B Zinsou	R Durandt	B Bah	R Adetona
14:50-15:15	N Bishop	S Parle	S ter Horst	C Msipha	S Josias	L Rundora
15:15-15:45	COFFEE					
15:45-16:10		J Gray	P de Jager	Panel Discussion	J King	H Jafari
16:10-16:35		G Joubert	P Zurlo	Panel Discussion	P Steenkamp	H Magau
16:35-17:00		G Feierabend	V Makhoshi	Panel Discussion	V Venter	N Ndou
17:00-19:00	Welcome function / Poster session (main foyer)					

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Day 2 schedule: (Weds 07 Dec)

Time\ Room	1003	1032	1033	1004	2118	2121
08:30-09:20	Blow-up in nonlinear heat equations: before, at, and after. (Chair: N Hale)					JAC Weideman
09:25-09:50	I Allie	J Madden	C Budde	C Khalique	A Flemming	S Childs
09:50-10:15	E Mphako-Banda	A Razafindrakoto	T Olabiyi	B Sebogodi	R Kleinhans	G Madito
10:15-10:45	COFFEE					
10:45-11:10	P Dankelmann	O Otafudu	T Nazir	C Majola	B van Zyl	A Maphiri
11:10-11:35	S Mukwembi	K Dayaram	M Khumalo	G Pai	R Ganzevoort	S Mohammadi
11:35-12:00	S Mafunda	F Assfaw	A Bokodisa	J Bodibe	H Zietsman	N Tibane
12:00-13:30	LUNCH					
13:30-14:20	Congruences in non-algebraic settings. (Chair: L Van Wyk)					S Veldsman
14:25-14:50	A Alochukwu	C Rathilal	A Swartz	T Motsepa	K Hohls	A Ramanantoanina
14:50-15:15	B Du Preez	S Mthethwa	T Lukoto	O Adeyemo	N van Rensburg	F Nyabadza
15:15-15:45	COFFEE					
15:45-16:10	M Kassaye	M Nxumalo	D Rabearivony	S Mbusi	S du Toit	S Ali
16:10-16:35	C Kriel	B Iragi	M Robdera	B Bartlett	M Labuschagne	M Rudziva
16:35-17:00	R Roux	B Tlharesakgosi	J Conradie	Y Gaxela	B Stapelberg	J Du Plessis

Day 3 schedule: (Thurs 08 Dec)

Time\ Room	1003	1032	1033	1004	2118	2121
08:30-09:20	Packing colorings of graphs. (Chair: S Mukwembi)					B Jonck
09:25-09:50	O Oyewumi	J Picado	Howell & Rabie	K Plaatjie	A de Villiers	
09:50-10:15	N Ralaivaosaona	S Moshokoa	M Hoenselaar	L Moleleki	J Bidie	N Hale
10:15-10:45	COFFEE					
10:45-11:10	Z Shozi	T Ncongwane	K Marais	M Mafora	AWIM Workshop	E Nel
11:10-11:35	S Xhanti	T Phawe	F Van Niekerk	M Lephoko	AWIM Workshop	M Pheko
11:35-12:00	B Mugwangwavari	M Zweni	A Craig	M Isaac	AWIM Workshop	M Mkhatshwa
12:00-13:00	LUNCH					
13:00-13:25	R Maartens	T Meyer	C Robinson	M Kekana	M Roberts	E Thomann
13:25-13:50	E Joubert	C Msipha	Y Hardy	T Itumeleng	M Roberts	E Thomann
14:00-16:00						Closing & AGM

Abstracts and talk titles are available online at https://appliedmaths.sun.ac.za/samscongress2022/programme/

Presenting authors are requested read the author instructions at https://appliedmaths.sun.ac.za/samscongress2022/programme/instructions.shtml

PLENARY TALKS

Pedagogical practices in integration of IKS in teaching and learning geometry

Prof Zingiswa Jojo, *University of South Africa*

SAMS; Subject Classification Number: 16

For geometry proficiency, visualization for the heuristic exploration of complex situations, modelling, and reasoning, which are the processes that lead to proof and explanation, are necessary for cognitive understanding in mathematics. The knowledge of basic concepts in geometry is crucial for the visualisation and comprehension of properties of geometrical figures. This paper reports on how learners in disadvantaged communities improved their understanding of geometric concepts when they used indigenous materials. From a variation and van Hiele's theoretical perspective, the paper responds to the question: What are the pedagogical practices in integration of IKS in the teaching and learning of mathematics? Results indicated that in cooperative social environments the use of indigenous knowledge and tools assist the learners to develop a particular type of thought to understand, describe, and represent, objects found in their proximity. Furthermore, as they observe, and imitate each other, they subsequently develop higher mental skills.

Bio: Zingiswa Jojo is a full professor in the department of Mathematics Education at UNISA. She is a scholar who serves as the local organizing committee of AMESA, the Commission for African Women in Mathematics (AMU-CAWM), an academic board member in the Athens Institute for Education and Research, and a leader of several projects aimed at promoting Mathematics teaching and learning. Zingiswa holds a PhD in Mathematics Education from the University of KwaZulu Natal.
She has published and presented numerous papers, hosted in some leading universities located in countries like Umea in Sweden, Rio de Janeiro in Brazil, Athens in Greece, Dublin in Ireland, e-Learning in Kigali, Rwanda and PACOM in Congo Brazzaville.

Packing colorings of graphs

Prof Betsie Jonck, *University of the Witwatersrand*

SAMS; Subject Classification Number: 6

Consider a graph $G(V, E)$. A function $\pi : V \to \{1, \ldots, k\}$ is a packing coloring of order k if $\pi(u) = \pi(v)$ implies the distance between u and v is more than $\pi(u)$. The minimum number of colors with which the vertices of a graph G can be packing colored is called the packing chromatic number of G, denoted by $\chi_\rho(G)$. We will discuss the packing chromatic number of various graphs with emphasis on the upper bound and the lower bound of the packing chromatic number of the 2- dimensional infinite square lattice/grid. A discussion of the packing chromatic number of the torus will follow. A comparison will be made between the packing chromatic numbers of the grid and the torus.

References

[1] A survey on packing colorings, B Bresar, J Ferme, S Klavzar, D F Rall, Discussiones Mathematicae Graph Theory 40, 2020, 923-970

Bio: Betsie did her undergraduate studies at UP; her graduate Studies at UJ (RAU). She taught at High School Die Fakkel (1980 – 1992) and was a lecturer/senior lecturer and associate professor at UJ (RAU) (1993 – 2016). From 2016 until now, she is employed at Wits. Betsie was HoS/Deputy HoS of Mathematics at UJ and then at Wits (18 years in total). She became full professor in March 2019.

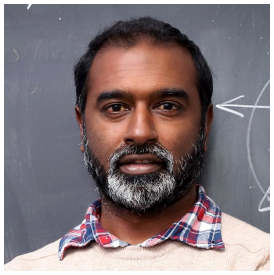
Betsie published 23 ISI articles, supervised / co-supervised to completion 7 doctoral and 15 masters students. She presented at 32 conferences, here and overseas. She externals for universities and evaluated 7 times other schools / departments of mathematics at universities in the country.

Topological Data Analysis - a new tool from old ideas

Jeff Murugan, *University of Cape Town*

SAMS; Subject Classification Number: 23

Topological Data Analysis (TDA) is a novel, and relatively new approach to analysing high-dimensional data sets. It does this by fo- cussing on global properties like the shape and connectivity of the data giving it a significant advantage over more conventional tools based on cluster analysis, a localised property of the data. However, some of its mathematical foundations, like algebraic topology and discrete Morse theory, are perceived as an intimidatingly steep upramp into the subject. Consequently, it has enjoyed much less popularity as a data-analysis tool than less abstract methods. This talk will give an introduction to this fascinating subject by focusing on a small set of simple examples, chosen primarily for their pedagogical value. I will illustrate the universality of the method by discussing two applications: 1. to the intriguing data set of fast radio burst (FRB) observations in astrophysics and 2. to the study of quantum phase transitions in condensed matter physics.



Bio: Jeff Murugan is Professor of Mathematical Physics at the University of Cape Town (UCT). Prof Murugan was a postdoctoral fellow in Brown University's High Energy Theory group, a member of the School of Natural Sciences of the Institute for Advanced Study in Princeton, a Research Associate in the Division of Physical Sciences at the American Museum of Natural History in New York, and a Simons Associate at the International Center for Theoretical Physics in Trieste.

A String Theorist by training, his work currently lies at the nexus of quantum information and quantum matter where he is one of the co-discoverers of the 3-dimensional web of dualities and, together with Douglas Stanford and Edward Witten, is in the MSW class of disordered conformal field theories. He is also the recipient of UCT's Distinguished Teacher Award for 2018.

Congruences in non-algebraic settings

Stefan Veldsman, *Nelson Mandela University and La Trobe University*

SAMS; Subject Classification Number: 29

A congruence is a fundamental and essential tool in algebra. It will be shown that a congruence can also be defined for non-algebraic mathematical objects, leading to the natural counterparts of the algebraic isomorphism theorems, subdirect products and Birkhoff's Theorem. In particular, this theory of non-algebraic congruences provides the last missing piece of the puzzle that completes the full correspondence between the radical theory of algebraic structures and the theory of connectednesses and disconnectednesses of topological spaces and graphs. However, as we shall see, it also brought some unexpected differences between the algebraic and the non- algebraic radical theories to the fore.

Bio: Stefan Veldsman started his academic career in 1977 as a junior lecturer at the University of Port Elizabeth (now called Nelson Mandela University). With a life in academia, he held teaching and research positions at the University of Port Elizabeth, the Rand Afrikaans University (now the University of Johannesburg), Sultan Qaboos University (in Muscat, Oman) and a short term visiting professor position at the Technical University in Vienna.

In September 2019, he retired from the Nelson Mandela University where he is now Emeritus Professor. Since January 2020 he is also an Honorary Research Fellow at the School of Engineering and Mathematical Sciences, La Trobe University (Melbourne).

Blow-up in nonlinear heat equations: before, at, and after

Prof André Weideman, *Stellenbosch University*

SAMS; Subject Classification Number: 23

The blow-up phenomenon in nonlinear evolution equations has been an active area of research for decades. In this talk we focus on a nonlinear diffusion equation, one which exhibits a point blow-up in finite time. One can distinguish the three stages of the evolution as before blow-up, at blow-up, and post blow-up. Much is known about the first, less about the second, and very little about the third. We discuss the various methods that can be brought to bear on each of these stages, including analytical appproximation methods such as perturbation theory as well as numerical approximation methods. (Joint work with Marco Fasondini and John King.)

Bio: Professor Weideman graduated from the University of the Free state in 1986, and occupied positions at that university and at Oregon State University, as well as visiting positions at MIT and the University of Utah. He is currently Distinguished Professor of Applied Mathematics at Stellenbosch University.

CONTRIBUTED TALKS

Self-Branding and perfecting your elevator pitch – an interactive workshop

Maambele Ambie Khosa, *Stellenbosch University*

SAMS; Subject Classification Number: 12

Ever wondered what could be the best way to communicate your research in academic and non-academic events? This session will explore the famous elevator pitch, self-branding, and strategic use of social media. It is a networking event open to everyone, facilitated by Maambele Ambie Khosa [1] and hosted by African Women in Mathematics (AWiM) [2]. Everyone is welcome.

References

[1] https://za.linkedin.com/in/maambelekhosa

[2] https://math.sun.ac.za/awmc/

Efficient solution of Burgers equation using B-spline approximation functions

Rasheed A. Adetona*, **Nabendra Parumasur**, and **Pravin Singh**
University of KwaZulu-Natal

SAMS; Subject Classification Number: 23

This paper discussed the application of orthogonal collocation finite element method using a third order B-spline basis functions to ordinary and partial differential equations. Collocation was done at Gaussian points to get optimal solution, hence the name orthogonal. The method is used to solve various particular solutions of Burgers' equation and the results compared favourably with existing results. The stability and convergence of the method are also given consideration. Our results show that it is unconditionally stable, second order accurate in time and space and performed better than existing results based on other quadratic splines in literature.

Solitary wave solutions and classical symmetry reductions of a generalized bi-dimensional nonlinear wave equation in engineering physics.

Oke Davies Adeyemo*, **Chaudry Masood Khalique**,
International Institute for Symmetry Analysis and Mathematical Modelling,
Department of Mathematical Sciences, North-West University,

SAMS; Subject Classification Number: 3

We perform classical symmetry reductions of a bi-dimensional nonlinear wave equation that is very important in modelling diverse occurrences with applications in engineering physics, fluid dynamics, cosmology, plasma physics and so on [1–6]. This leads to various invariant and solitary wave solutions of interests. In addition, a steady-state series solution of the model is found. Besides, the associated conserved vectors [4–6] of the understudy equation are also investigated, thus yielding some vectors of note in physical sciences.

References

[1] N. Dimakis, A. Giacomini, S. Jamal, G. Leon, Noether symmetries and stability of ideal gas solutions in Galileon cosmology, Physical Review D, 95 (2017) 064031.

[2] W.F. Ames, Nonlinear Partial Differential Equations in Engineering, Academic Press, New York, 1965.

[3] S.J. Chen, Y.H. Yin, W.X. Ma, X, Lü, Abundant exact solutions and interaction phenomena of the (2+1)-dimensional YTSF equation, Analysis and Mathematical Physics, 9 (2019) 2329–2344.

[4] O.D. Adeyemo, C.M. Khalique, Analytic solutions and conservation laws of a (2+1)-dimensional generalized Yu-Toda-Sasa-Fukuyama equation, Chinese Journal of Physics, 77 (2021) 927–744.

[5] O.D. Adeyemo, L. Zhang, C.M. Khalique, Bifurcation theory, Lie group-invariant solutions and conservation laws of a generalized (2+1)-dimensional BK equation Type II in plasma physics and fluid mechanics, Mathematics, 10 (2022) 2391.

[6] A.A. Zaidi, M.D. Khan,I. Naeem, Conservation Laws and Exact Solutions of Generalized Nonlinear System and Nizhink-Novikov-Veselov Equation, Mathematical Problems in Engineering, 2018 (2018).

A numerical study of double-diffusive convection in the anisotropic porous layer under rotational modulation with internal heat generation

Samah A. Ali*, **Precious Sibanda**, **Munyaradzi Rudziva**, **Osman A.I. Noreldin**,
University of KwaZulu-Natal.

SAMS; Subject Classification Number: 23

Double-diffusive convection in a non-uniformly rotating anisotropic fluid layer with internal heating is investigated. The normal mode technique is used to obtain the critical stationary and oscillatory Rayleigh numbers. The analysis for the nonlinear case is based on the minimal truncated double Fourier series which gives rise to the nonlinear Lorenz type equations. A local quasilinearization block hybrid method (LQBHM) is used to solve the coupled nonlinear Lorenz type equations. The solution obtained using this method is compared with solutions obtained using the ode45 solver. The numerical results indicate that the LQBHM is accurate, efficient, and flexible. A weakly nonlinear analysis is used to investigate the rate of heat and mass transfer in the fluid system.

Resistance and flow-resistance in snarks

Imran Allie*, *University of Cape Town*
Edita Macajova, *Comenius University Bratislava*
Martin Skoviera, *Comenius University Bratislava*

SAMS; Subject Classification Number: 06

Vizing's theorem categorises graphs into two classes: those which are $\Delta(G)$-edge-colourable (class one, colourable), and those which are not (class two, uncolourable). By $\Delta(G)$-*edge-colourable* we mean graphs whose edges can be coloured with $\Delta(G)$ colours such that no two adjacent edges have the same colour. Cubic class two graphs are of particular interest, for the reason that many major open conjectures in graph theory are reducible to just to the consideration of these graphs. In contemporary times, so-called measures of uncolourability have been studied in order to develop insights into cubic class two graphs. Let G be cubic. Resistance, $r(G)$, is defined as the minimum number of edges which can be removed from G in order to render a class one graph, an intuitive measure of uncolourability. Flow resistance, $r_f(G)$, defined as the minimum number of zero edges in a 4-flow of G, is another measure of uncolourability, although less intuitive. ($r(G) = r_f(G) = 0$ if and only if G is colourable). It has been conjectured, with good reason, that $r(G) \geq r_f(G)$ for all cubic G [1]. In this talk, we explain the motivation for this conjecture, and prove a surprising result which disproves it. We prove that the ratio of flow-resistance to resistance can in fact be arbitrarily large.

References

[1] M. A. Fiol, G. Mazzuoccolo and E. Steffen, Measures of edge-uncolourability of cubic graphs, *Electron. J. Combin.* **25** (2018), #P4.54.

Bounds on the fault-(edge)-diameter of graphs

Alex Alochukwu*, *University of the Witwatersrand*
Peter Dankelmann, *University of Johannesburg*

SAMS; Subject Classification Number: 06

The distance between two vertices u and v in a connected graph G is the length of a shortest $u-v$ path in G. The diameter of G is the largest of the distances between all pairs of vertices of G. If the removal of not more than k vertices (edges) never disconnects the graph G, we say that G is $(k+1)$-connected ($(k+1)$-edge-connected). The k-fault diameter and k-edge-fault diameter of a $(k+1)$-connected or $(k+1)$-edge connected graph G is the largest diameter of the subgraphs obtained from G by removing up to k vertices and edges respectively.

Few bounds on the fault-(edge)-diameter are known. Recently, the second author [Bounds on the fault-diameter of graphs, Networks 70(2) (2017), 132-140] observed that the k-fault diameter of a $(k+1)$-connected graph G with n vertices is bounded from above by $n-k+1$ and showed that this bound can be improved to approximately $\frac{4n}{k+2}$ if G is triangle-free and $\frac{5n}{(k-1)^2}$ if G does not contain 4-cycles. He also gave similar bounds on the k-edge-fault-diameter.

In this talk, we present these results and show that the above bound for C_4-free graphs can be improved to $\frac{3n}{k^2-k+1} + 3k^2 - 3k + 5$ if, in addition, the graph is also bipartite. We also discuss results on the k-edge-fault diameter for $(k+1)$-edge connected C_4-free and bipartite C_4-free graphs. We construct graphs to show that our bounds on the k-fault-(edge)-diameter of bipartite C_4-free graphs are best possible.

Interior operators and the category of (pre)sheaves

Fikreyohans Solomon Assfaw, *University of the Western Cape*

SAMS; Subject Classification Number: 4

Motivated by the theory of categorical closure operators, the categorical notion of interior operators was introduced by [3]. These operators have received more recent attention and a few papers are published on the subject (see [1]). Working in an arbitrary category in which each pullback functor commutes with the join of subobjects, we further study interior operators. In particular, we define and investigate interior operators on the category of pre(sheaves). Furthermore, we examine the notions of heredity, openness, closedness, initiality, and finality with respect to the defined interior operators.

References

[1] F. S. Assfaw and D. Holgate, Codenseness and openness with respect to an interior operator, *Appl. Categ. Structures.* **29(2)** (2021), 235–248.

[2] S. Mac Lane and I. Moerdijk, A first introduction to topos theory, *Springer-Verlag*, 1992.

[3] Vorster, S., Interior operators in general categories, *Quaest. Math.* **23(4)** (2000), 405–416.

Efficient and robust optimization methods for training binarized deep neural networks

Bubacarr Bah*, *African Institute for Mathematical Sciences (AIMS) South Africa & Stellenbosch University*

Jannis Kurtz, *University of Siegen, Germany*

SAMS; Subject Classification Number: 23

Compared to classical deep neural networks its binarized versions are, among other things, useful for applications on resource-limited devices due to their reduction in memory consumption and computational demands. In this work we study deep neural networks with binary activation functions and continuous or integer weights (BDNN). We show that the BDNN can be reformulated as a mixed-integer linear program with bounded weight space which can be solved to global optimality by classical mixed-integer programming solvers. Additionally, a local search heuristic is presented to calculate locally optimal networks. Furthermore to improve efficiency we present an iterative data-splitting heuristic which iteratively splits the training set into smaller subsets by using the k-mean method. Afterwards all data points in a given subset are forced to follow the same activation pattern, which leads to a much smaller number of integer variables in the mixed-integer programming formulation and therefore to computational improvements. Finally for the first time a robust model is presented which enforces robustness of the BDNN during training. All methods are tested on random and real datasets and our results indicate that all models can often compete with or even outperform classical DNNs on small network architectures confirming the viability for applications having restricted memory or computing power, details in [1].

References

[1] J. Kurtz and B. Bah, Efficient and Robust Mixed-Integer Optimization Methods for Training Binarized Deep Neural Networks. *arXiv preprint arXiv:2110.11382*, 2021.

Coherent states in complex geometry with application to modular forms and representation theory

Bruce Bartlett, *Stellenbosch University*

SAMS; Subject Classification Number: 9, 20

A Hermitian holomorphic line bundle over a Hermitian complex manifold carries a fundamental geometric object called the Bergman kernel. This kernel can be thought of as the ?correlation function? for pairs of points on the manifold and leads to the concept of coherent states. I will give an overview of the usefulness of the coherent states approach to well-known constructions such as the Riemann mapping theorem. I will introduce coherent loop states, obtained by integrating coherent states around Bohr-Sommerfeld loops, and give an overview of their applications to the study of inner products of modular forms and to the representation theory of SU(2) (this latter is work of my PhD student Nzaganya Nzaganya).

Numerical solution for vibrating overhead transmission lines

Judith Bidie, *Tshwane University of Technology*

SAMS; Subject Classification Number: 23

Overhead power lines are structures used to transmit and distribute electric power, as well as transmit electrical energy across large distances. A thin uniform rod with fixed ends is considered to represent a power line of unit length with minimal sag. A high voltage transmission line equation of motion will be derived using Lagrange Equation, with and without damping.

A change of variables is effected and then the Garlekin-Kantorovich method used to solved the resulting partial differential equation. It assumes that the solution to the partial differential equation is in trigonometric series form, like the one that can be found using the method of separation of variables. Thereafter, orthogonality of eigenfunctions is used, and the resulting system of coupled ordinary differential equations of initial value type is solved for both free and forced vibrations. For this study, MATHEMATICA is used in all computations and graphics because of its data handling and graphics capabilities.

Furthermore, this solution is approximated by using the numerical method of lines, which uses finite difference equations. This will be useful to check whether numerical method of lines can be applied to more complicated partial differential equations, extending to Rayleigh-Love, Rayleigh-Bishop and Mindlin-Hermann models.

The propagation of gravitational waves through matter

Nigel T. Bishop*, **Monos Naidoo**, **Petrus J. van der Walt**
Rhodes University

SAMS; Subject Classification Number: 20

Using linearized perturbations within the Bondi-Sachs formalism in general relativity, we consider the problem of a gravitational wave (GW) source surrounded by a spherical dust shell. It is shown that the shell causes the GWs to be modified both in magnitude and phase; and that if the shell is viscous, then the shear induced in the velocity field results in an energy transfer so damping the magnitude of the GWs. Both effects can be significant if the shell radius r is much smaller than the GW wavelength λ, and there are astrophysical scenarios for which the modification to the GW signal is large enough to be measurable. These scenarios include core collapse supernovae (CCSNe), quasinormal mode emission from the remnant of a binary neutron star merger, and primordial GWs. Importantly, there are feasible values of the CCSNe parameters for which viscous damping of the GW signal would be almost complete, implying that the detection of GWs from a CCSNe is unlikely.

A numerical evolution of the full Einstein and matter field equations with GW extraction far from the source will properly include all effects described above. However, in situations such as CCSNe shear viscosity is neglected and approximate methods are used to estimate the GW extraction. Results obtained in these cases need to be corrected.

Symmetry analysis and conservation laws of the Alice Bob-KP equation

Bodibe Jonathan Lebogang*, **Chaudry Masood Khalique**,
International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University

SAMS; Subject Classification Number: 3

In this talk, we study the nonlinear partial differential equation (NLPDE), namely, the Alice Bob-KP equation [1] which has many applications in fields such as dispersive media [2,3] and multicomponent plasmas [4]. Using the technique of Lie symmetry analysis [5], Firstly, we compute the Lie point symmetries of the Alice Bob-KP equation and then perform symmetry reductions. Exact solutions are obtained. Furthermore, conservation laws for this equation are derived using the multiplier method [6].

References

[1] H.Y. Wu, J.X. Fei, Z.Y. Ma, J.C. Chen, W.X. Ma, Symmetry Breaking Soliton, Breather, and Lump Solutions of a Nonlocal Kadomtsev-Petviashvili System, Complexity (2020) 6423205, 13 pages

[2] M. J. Ablowitz and H. Segur, Solitons and the Inverse Scattering Transform, SIAM, Philadelphia, PA, USA, 1981.

[3] V. I. Petviashvili and O. V. Pokhotelov, Solitary Waves in Plasmas and in the Atmosphere, Energoatomizdat, Moscow, Russia, 1989.

[4] G. C. Das and J. Sarma, "Evolution of solitary waves in multicomponent plasmas," Chaos, Solitons and Fractals, vol. 9, no. 6, pp. 901–911, 1998.

[5] N.H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley and Sons, Chichester, NY,USA,1999.

[6] P.J. Olver, Applications of Lie Groups to Differential Equations, Springer-Verlag, New York, 1993.

A parallel hybrid bregman subgradient extragradient method for a system of pseudomonotone equilibrium and fixed point problems

Anele Bokodisa* and **Lateef Jaloaso**
Sefako Makgatho Health Sciences University

SAMS; Subject Classification Number: 25

In this monograph, we introduce a new parallel hybrid subgradient extragradient method for solving the system of the pseudomonotone equilibrium problem and common fixed point problem in real reflexive Banach spaces. The algorithm is designed such that its convergence does not require prior estimation of the Lipschitz-like constants of the finite bifunctions underlying the equilibrium problems. Moreover, a strong convergence result is proven without imposing strong conditions on the control sequences. We further provide some numerical experiments to illustrate the performance of the proposed algorithm and compare with some existing methods.

A Lumer–Phillips type generation theorem for bi-continuous semigroups

Christian Budde*, *University of the Free State*
Sven-Ake Wegner, *University of Hamburg*

SAMS; Subject Classification Number: 3, 10, 11

The famous 1960s Lumer–Phillips theorem states that a closed and densely defined operator $A\colon \mathrm{D}(A) \subseteq X \to X$ on a Banach space X generates a strongly continuous contraction semigroup if and only if $(A, \mathrm{D}(A))$ is dissipative and the range of $\lambda - A$ is surjective in X for some $\lambda > 0$. We will investigate a version of this result for bi-continuous semigroups and apply the latter amongst other examples to the transport equation as well as to flows on infinite networks. This is joint work with S.-A. Wegner (Hamburg, Germany).

Examining exam questions

Anita L. Campbell, *Academic Support Programme for Engineering and Centre for Research in Engineering Education; University of Cape Town*

SAMS; Subject Classification Number: 16

A hazzard of teaching a topic multiple times is that our growing expertise may lead to us making examination questions more difficult. To counteract potential 'difficulty creep', we explored lecturers' and engineering students' impressions of the difficulty levels of first year mathematics examination papers. A comparison of difficulty impressions by lecturers and students will be presented, along with an analysis of interview data showing what students found most difficult.

Quantitative, epidemiological prediction and the original SARS-CoV-2 pandemic

S. J. Childs, *Department of Mathematics and Applied Mathematics, University of Fort Hare*

SAMS; Subject Classification Number: 12

One of the foremost casualties of the first SARS-CoV-2 pandemic might well have been the field of quantitative epidemiology itself. Yet this presentation will demonstrate that the original pandemic in South Africa conformed to the standard model of an infectious disease; to the extent that an exact prediction of its epidemic threshold (around 3.5 million reported cases) was made as early as the 2nd of July, 2020. To this end, an exposition of a very simple formula for the basic reproduction number, r_0, over a chosen time increment, is provided. This formula is both far more accurate and efficient than other methods which have recently been in vogue.

In August of 2020, a purported, early threshold was touted and the questions of undetected, T-cell-mediated immunity and whether SARS-CoV-2 was actually endemic to South Africa needed to be settled. Two data defficiencies were formulated and experimented with. It was quickly discovered that, if either did, indeed, play a role in an unforseen threshold, then the more compelling of the two phenomena might be that the true level of infection had been underestimated by a multiplicative factor; something which was already a recognized phenomenon. The existance of a significant, imperceptible, immune fraction of the population was demonstrated to drive r_0 up, and, in so doing, move thresholds to even lower values. In contrast, there was very little difference in the r_0's, should infections have been underestimated by a factor. Although this phenomenon could, to a limited extent, reconcile the observed with the predicted, it remained an inescapable fact that, in July, r_0 did change fairly abruptly in all the contemplated scenarios of data-deficiency. This was not something suggestive of a threshold and the strong, seasonal dependence of the South African pandemic was then correctly diagnosed, as early as in August of 2020.

On graded algebras

Simon Thulani Chili, *University of Cape Town*

SAMS; Subject Classification Number: 4, 29

Given a semigroup S and a unital commutative ring K, by an S-graded K-algebra we mean an S-indexed family $A = (A_s)_{s\in S}$ of K-modules equipped with a family of K-bilinear multiplications

$$(A_s \times A_t \to A_{st})_{s,t\in S}$$

all written as $(x, y) \mapsto xy$ and satisfying the associative condition $x(yz) = (xy)z$ for all $s, t, u \in S$ and $x \in A_s$, $y \in A_t$, $z \in A_u$. A morphism $f : A \to B$ of S-graded K-algebras is a family $f = (f_s)_{s\in S}$ of K-module homomorphisms $f_s : A_s \to B_s$ with $f_s(x)f_t(y) = f_{st}(xy)$ for all $s, t \in S$ and $x \in A_s$, $y \in A_t$. We denote the category of S-graded K-algebras by $\mathbf{Alg}(K, S)$.

In this talk we study various topics on the semi-abelian (in the sense of [1]) category of S-graded K-algebras, that is, Smith and Huq commutators, theory of split extensions, internal categories and crossed modules in the context of $\mathbf{Alg}(K, S)$. Smith commutator was first introduced for congruences of algebras in Mal'cev varieties in [2] and Huq commutator of pair of morphisms with same codomain in [3]. The notion of crossed modules was introduced in [4] in study of groups with the algebraic properties of relative homotopy groups.

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Second law analysis of dissipative nanofluid flow past a moving heated cylindrical surface

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SAMS; Subject Classification Number: 10, 20, 23

Inherent irreversibility in nanofluid convection over a heated moving surface plays a crucial role in the efficient operation of different engineering processes and quality of industrial products. The interminable list of its engineering applications includes production of composite substances, rocket launching, torpedo, missile, fired bullet, the processing of porous materials, submarine, thermal insulation, fuel production, glass processing, thermal solar panels, and so forth. In this paper, the combined effects of viscous dissipation and nanofluid suction/injection on unsteady Cu-Water nanofluid convection past a heated moving permeable cylindrical surface is theoretically examined. Using appropriate similarity transformation, the governing equations are rendered dimensionless and solved numerically by via shooting method coupled with the Runge-Kutta Fehlberg integration scheme. The expression for entropy production is computed by employing the solutions obtained from the model momentum and energy equations. Effects of physical flow parameters on velocity, temperature, skin friction, Nusselt number, entropy generation rate and Bejan number are displayed graphically and discussed.

When are mixed convergences topological?

Jurie Conradie, *University of Cape Town*

SAMS; Subject Classification Number: 11Number

Generalized inductive limit topologies (sometimes called mixed topologies) occur surprisingly often in the theory of topolgical vector spaces. In its most general form, such a topology is the finest vector topology coinciding with a given vector topology on all the sets of a family of "bounded" sets (a bornology). In the context of vector lattices, many Mackey topologies are mixed topologies.

In the theory of convergence spaces, there is a similar notion of an inductive limit, or mixed convergence, often referred to as specified sets convergence. There are convergences which are, in general, not derived from a topology that are of this type; an example is order convergence in a vector lattice. We show that a mixed convergence derived from a topological convergence need not itself be topological, and investigate the necessary conditions for a mixed convergence to be topological.

Convergences are usually defined in terms of filters, but there is a recently developed parallel theory in terms of nets which can sometimes be used to simplify arguments.

This is joint work with Michael O'Brien, Vladimir Troitsky and Jan Harm van der Walt.

Dual representations of ortholattices

Andrew Craig*, *University of Johannesburg*
Gerhard Dorfer, *Technical University of Vienna, Austria*
Miroslav Haviar, *Matej Bel University, Slovakia & University of Johannesburg*
Klarise Marais, *University of Johannesburg*

SAMS; Subject Classification Number: 34

An ortholattice is a bounded lattice with a complement operation (i.e. a unary map $'$ satisfying $x \vee x' = 1$ and $x \wedge x' = 0$) that is both involutive and order-reversing. This operation is known as the orthocomplement. We use digraphs with topology, due to Ploščica [3], to provide a representation of the underlying lattice of an ortholattice. We then equip these digraphs with an additional unary map to represent the orthocomplement. We will contrast our dual representation with the approaches of Goldblatt [2] and Dzik et al. [1].

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Distances and a generalisation of cages

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SAMS; Subject Classification Number: 6

Let G be a connected graph. The diameter and Wiener index of G are defined as the largest and the sum, respectively, of the distances between all pairs of vertices of G. The average eccentricity of G is the arithmetic mean of the eccentricities of the vertices of G, where the eccentricity of a vertex v is defined as the distance between v and a vertex farthest from v.

In this talk we present bounds on these distance measures for graphs of girth 6 in terms of order, minimum degree and maximum degree. We further show that, in a sense, our bounds are best possible. In this context a natural generalisation of cages arises, which is interesting in its own right. The cage problem considers the minimum order of regular graphs of prescribed degree and girth. In our generalisation we consider not regular graphs, but prescribe minimum and maximum degree.

Revisiting Isbell's formulation of the Zassenhaus lemma

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SAMS; Subject Classification Number: 4, 15, 18

Isbell's paper [1] shows that the construction in the proof of the Jordan-Hölder-Schreier theorem by Zassenhaus [4] is stronger than what is stated. Moreover Isbell's formulation shows that the Jordan-Hölder-Schreier theorem is a consequence of the Zassenhaus lemma. We shall show that Isbell's formulation and proofs of the Zassenhaus lemma and the Jordan-Hölder-Schreier theorem may be extended to the context of the noetherian form. This context, introduced in [3] and further developed in [2], is a self-dual context which covers all group-like structures.

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Non-local sectional curvature as applied to random geometric graphs, the earth and fractals

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SAMS; Subject Classification Number: 21

Using the spherical cosine rule we propose an algorithm for calculating or estimating the sectional curvature of path metric spaces (of which any graph, hypergraph and manifold is a very relevant special case) at finite length scales. Being defined at a finite length scale allows for ready application to geometric graphs and fractals, where local definitions of curvature often fails to give an accurate picture of mesoscopic geometry. In order to compare the efficacy of the proposed algorithm against existing methods, a new form of metric distortion is introduced, which numerically captures how well a graph represents a given metric space. This is used to numerically motivate that as this distortion vanishes, so does the error of the estimator. Compelling comparisons on random geometric graphs motivate this geometric definition over existing methods mostly developed for complex network theory. It is also shown that in a continuum such as an oblate spheroid representing the earth, expected behaviour is maintained as long as a small enough length scale is chosen. In scale invariant fractals such as the Sierpinski triangle, curvature can only be formally 0 or infinite. Introducing this finite length scale curvature then acts as a regularizer, allowing a much richer story of exactly how the curvature diverges as the considered length scale vanishes. The proposed algorithm is expected to have applications and implications in discrete models of quantum gravity, discrete geometry, computational geometry and data science.

The degree diameter problem for plane graphs with large faces

Brandon Du Preez, *University of Cape Town*

SAMS; Subject Classification Number: 6

The face-degree of a face in a plane graph is the length of the walk bounding it. A plane graph is k-face-degree-regular if every face has face-degree k. The well-known degree-diameter problem asks for the maximum order $n(\Delta, D)$ of a graph with maximum degree Δ and diameter D (for an excellent and detailed survey, see [1]). In this talk, we discuss the degree-diameter problem for (face-degree-regular) plane graphs. In particular, we focus on the results of [2] — in which the degree diameter problem for $2D$-face-degree-regular and $(2D + 1)$-face-degree-regular plane graphs is solved.

On the path to solving this case of the degree diameter problem, we also explore the relationship between girth, face-degree and diameter in plane graphs with small diameter and large face-degrees.

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How does the teaching design influence tertiary students' learning of mathematical modelling?

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SAMS; Subject Classification Number: 16

The transition from school to university is a substantial hurdle in students' learning of mathematics, also because many students lack basic skills and abilities and have a constricted disposition towards mathematics. Accordingly, the learning of mathematical modelling is particularly challenging for students. Mathematical modelling is the process of solving real-life problems by translating between reality and mathematics. Modelling requires certain competencies (e.g., structuring, mathematising, interpreting, validating) and is thus cognitively demanding. According to empirical findings at the secondary school level, students ought to acquire as much experience of independent work in modelling as possible, whenever necessary supported by the teacher as modestly as possible. So, the question follows: What are appropriate ways of teaching mathematical modelling at the tertiary level?

In our project CoSTAMM (Comparative Studies into Teaching Approaches for Mathematical Modelling), we implemented a mathematical modelling unit (diagnostic test, pre-test, five lessons with ten tasks, post-test) for tertiary students and investigated the effects of two different teaching styles (an independence-oriented style and a more traditional teacher-guided style) in 2019 and 2022. The overarching purpose of this project is to enhance the teaching and learning of modelling, with an emphasis on the tertiary level, guided by the principle of finding an appropriate balance between students' independence and teacher's (lecturer's) guidance. In 2019, three groups of first-year engineering students participated in the project, and in 2022 two groups of first-year engineering students and one group of second-year analytical chemistry students. Both the 2019 and 2022 results show significant learning progress for all students, with the biggest growth for the independence-oriented style. In our talk we will present the modelling unit, the test instruments and the evaluation methods, and we will report on the results from the 2019 and 2022 implementations, including comparisons between the different groups.

Students' preparedness for, and response to learning first-year mathematics fully online

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SAMS; Subject Classification Number: 16

In this talk we will present empirical results from a study that investigated first-year mathematics students' preparedness for, and response to, online learning. In 2020, due to the COVID-19 pandemic, lecturers and students were faced with a 'sudden' shift from face-to-face to online teaching and learning, and the focus was to save the academic year with no student left behind. Students (from both advantaged and disadvantaged communities) were also expected to adapt to these changes instantaneously. In 2021 the pandemic situation was similar and the first-semester offering of first-year mathematics modules continued fully online at most universities in South Africa. A blended learning environment is not new to first-year mathematics students and has been promoted and encouraged in the years leading up to the pandemic. However, the complete and sudden change in environments, particularly to fully online, was new to most lecturers and students. Several challenges were reported that gave rise to this investigation. We expected students to learn productively when they have (1) regular and reliable access to technological resources, and (2) self-regulated study habits. Quantitative data were collected through a self-designed questionnaire during the first-semester of 2021. The large sample was from a public university in South Africa, where students were registered for one of seven first-year mathematics modules. Main results confirm students are partially prepared for learning first-year mathematics fully online and they have unequal scenarios related to access to technological resources. Further, first-year students' approach to solving mathematics problems while studying online has a significant effect on their performance.

How to include policies in disease models

Farai Nyabadza, *University of Johannesburg*

SAMS; Subject Classification Number: 10

The management of HIV/AIDS has evolved ever since advent of the disease in the past three decades. Many countries have had to revise their policies as new information on the virus, and its transmission dynamics emerged. In this paper, we track the changes in Botswana's HIV/AIDS response and treatment policies using a piece-wise system of differential equations. The policy changes are easily tracked in three epochs. Models for each era are formulated from a **grand model** that can be linked to all the epochs. The grand model's steady states are determined and analysed in terms of the model reproduction number, R_0. The model exhibits a backward bifurcation, where a stable disease-free equilibrium coexists with a stable endemic equilibrium when $R_0 < 1$. The stability of the models for the other epochs can be derived from that of the grand model by setting some parameters to zero. The models are fitted to HIV/AIDS prevalence data from Botswana for the past three decades. The changes in the populations in each compartment are tracked as the response to the disease and treatment policy changed over time. Finally, projections are made to determine the possible trajectory of HIV/AIDS in Botswana. The implications of the policy changes are easily seen, and a discussion on how these changes impacted the epidemic are articulated. The results presented have crucial impact on how policy changes affected and continue to influence the trajectory of the HIV/AIDS epidemic in Botswana.

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A functorial presentation of parsers

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SAMS; Subject Classification Number: 4

In computer science a parser is a program that translates strings to a different kind of data structure that can often be represented as a tree. This talk discusses how (certain) parsers can be described as functors from a category of strings to a category of trees. The topic of this work evolved from a Haskell implementation [1] of a proof assistant software, whose syntax is entirely based on bracketed expressions. An example parser, that is considered, operates on such bracketed expressions, which are discussed in detail by Laing [2].

The presentation is based on my Honours project under the supervision of Professor Zurab Janelidze.

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A framework for modelling spatio-temporal competition and spread of invasive plant species in South Africa

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SAMS; Subject Classification Number: 10, 25

The study of artificially introduced plant and animal species is important in South Africa, a country rich in indigenous biodiversity and home to three biodiversity hotspots recognised by Conservation International. An introduced species is one living outside of its native distribution range, and has been introduced to a new environment either by accidental or deliberate human activity. Invasive alien species are a sub-category of introduced species that negatively impact the natural species of a particular area. These invasions cause natural functioning ecosystems to break down, leading to further invasions and can ultimately lead to the extinction of the indigenous species of the area.

In this presentation, we consider the application of automated discretised processes aimed at conducting spatial analyses of ecosystems in South Africa that contain invasive species. In particular, our study is aimed at adopting a machine learning algorithmic approach towards determining which topographical, spatial and climatic factors are to be attributed to the occurrence and density of invasive species in an ecosystem. This approach allows for predicting regions requiring investigation due to likely unmapped occurrences or predicted future spread, based on the area's suitability in terms of sustaining these species. Next, a spatio-temporal modelling approach is applied to simulate the competition between, and spread of, invasive species within a suitable study region identified in South Africa. Finally, the development and testing of dynamic adaptive management strategies are discussed in order to control current infestations and prevent future invasions.

Stochastic differential game formulation on the reinsurance and investment problem

Dr L C Galane, *University of Limpopo*

SAMS; Subject Classification Number: 25

This talk presents a stochastic differential game between two insurance companies, a big one and a small one. The distinctions of these companies is that the big company has sufficient asset to invest in a risk-free asset and a risky asset and is allowed to purchase proportional reinsurance or acquire new business, and the small company can transfer part of the risk to a reinsurer via proportional reinsurance. The game studied here is zero-sum, where the big company is trying to maximize the expected exponential utility of the difference between two insurance companies' surpluses at the terminal time to keep its advantage on surplus, while simultaneously the small company is trying to minimize the same quantity to reduce its disadvantage. The relationships between the surplus processes and the price process of the risky asset are considered. The Nash equilibrium strategy is obtained through verification theorem which rest on the stochastic control theory.

A decision support framework for the selection of appropriate retail sales time series forecasting methods

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SAMS; Subject Classification Number: 10, 26

Within a high-turnover retail environment, there is a significant trade-off between limiting in-store inventory levels and mitigating the risk of stock-outs. More specifically, retail organisations aim to minimise the capital tied up in inventory without a loss in service level. Therefore, in order to better manage inventory, retailers often consider the prediction of customer behaviour as a main priority. In practice, however, forecasting processes are automated and practitioners have limited knowledge pertaining to the selection of appropriate forecasting methods.

The primary objective in this presentation is to describe the design of a generic framework capable of assisting a retail forecasting practitioner in the selection of an appropriate method for sales forecasting based on the time series data available. The framework takes as input time series sales data and facilitates the configuration, transformation and extraction of valuable information from these sales data in order to classify/categorise the data set according to a taxonomy of time series data with similar attributes. Moreover, the framework is employed to evaluate, based on well-established benchmark data, the difference in forecast quality that results from the application of various forecasting methods classified as traditional statistical techniques, machine learning techniques and ensemble techniques. The framework provides as output a selection of appropriate forecasting methods for predicting unit sales on different levels of product aggregation (*e.g.* SKUs, classes, departments) associated with the input data set.

The framework is verified by applying it to the well-known M5 forecasting competition data which are publicly available. Furthermore, the best results obtained during the competition are employed as a benchmark to compare with the forecasting results generated by applying the framework to the data. To demonstrate its practical value, the framework is finally validated in the form of a case study involving real-life retail sales data.

Solutions and conservation laws of the new extended KP equation

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SAMS; Subject Classification Number: 3

In this talk, the 2D new extended Kadomtsev–Petviashvili equation [1] is considered and analysed using the Lie symmetry analysis technique [2]. Firstly, Lie point symmetries are obtained and used to derive some exact solutions of this model. Moreover symmetry reductions result in several nonlinear ordinary differential equations, which we solve with the aid of Kudryashov's and the simplest equation method. Furthermore we compute the conserved vectors of the underlying equation using the direct method [3].

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Split extension cores for internal semi-abelian algebras in a cartesian closed category

J. R. A. Gray, *Stellenbosch University*

SAMS; Subject Classification Number: 04

It is well known that a split extension of a group B with kernel X is determined up to isomorphism by a group action of B on X. It turns out that an action of B on X can be presented as an algebra structure for X with respect to the monad on the category of groups, obtained from the functor sending each split extension to its kernel. This fact, discovered by D. Bourn and G. Janelidze [3], is, as explained there, a general categorical phenomenon which holds in every semi-abelian category (in the sense of G. Janelidze, L. Márki and W. Tholen [4]).

If X is a B-group (that is, X is a group equipped with an action of B on X) and S is a subgroup of X, one can show that there is a largest sub-B-group of X contained in S, which is called the action core of S with respect to X [2]. This fact can be stated in terms of internal object actions (whose name derives from [1] and are algebras over the above mentioned monad) and leads to the categorical definition of an action core. We call the corresponding notion for split extensions, split extension cores.

The main aim of this talk is to show that if $\mathcal{V}$ is a semi-abelian variety of universal algebras admitting split extension cores and $\mathbb{C}$ is a cartesian closed category with small limits, then internal $\mathcal{V}$ algebras in $\mathbb{C}$ admit split extension cores.

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A spectral collocation method for functional and delay differential equations

Nick Hale, *Stellenbosch University*

SAMS; Subject Classification Number: 23

A simple but effective method for the numerical solution of functional and delay differential equations (DDEs) via spectral collocation is described. The method is demonstrated by means of several examples of linear and nonlinear DDEs with various delay types, including discrete, proportional, continuous, and state-dependent delay. The approach is a natural extension of standard spectral methods based on polynomial interpolation and can be readily incorporated in existing spectral discretisations, for example, the Chebfun/Chebop framework for the automated solution of differential equations.

Some block and partial operations on matrices

Yorick Hardy, *University of the Witwatersrand*

SAMS; Subject Classification Number: 29

We consider maps of the form $A \otimes B \to A$ (respectively $A \otimes B \to B$), induced by a map $B \to 1$ (respectively $A \to 1$). In the case where A and B are spaces of square matrices, these maps are called block (respectively partial) operations. This talk will characterise some classes of block and partial operations, with the main focus on their role in the theory of Kronecker quotients and tensor decompositions in terms of quotients. In this talk we will primarily consider two types of Kronecker quotients, namely linear quotients which are described in terms of the block and partial trace, and multiplicative quotients which are described in terms of determinant-like operations.

A provably stable and high order accurate solution of the Frank-Kamenetskii partial differential equation

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SAMS; Subject Classification Number: 23, 20

The Frank-Kamenetskii partial differential equation (FKPDE) [1],

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{k}{x}\frac{\partial u}{\partial x} + \exp(u), \qquad x \in [0,1], \tag{1}$$

models a thermal explosion in a vessel. The shape factor, k, describes the shape of the vessel where $k = 0$ for a slab, $k = 1$ for a cylinder and $k = 2$ for a sphere. The variable, u, is the dimensionless temperature, x is the dimensionless spatial coordinate, and t is the dimensionless time. The boundary conditions are given by

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \qquad u(1,t) = 0. \tag{2}$$

An investigation of asymptotic solutions at the boundary $x = 0$ proves that the solution $u(x,t)$ is defined at the boundary in spite of the singularity that arises. We employ a multi-domain numerical procedure, where we utilise the Galerkin method on the domain $[0,\epsilon]$ and the finite difference method on the domain $[\epsilon,1]$. The SBP-SAT (summation by parts - simultaneous approximation term) [2,3,4] methodology assists us in coupling these two numerical schemes at $x = \epsilon$, so that we end up deriving provably stable and convergent numerical schemes for solving equation (1). Results obtained in this paper can be applied to the Navier-Stokes equations in a cylindrical coordinate system.

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On the commutativity of binary products and coequalisers

Michael Hoefnagel, *Stellenbosch University*

SAMS; Subject Classification Number: 4

Many algebraic categories satisfy the property: given two coequaliser diagrams $C_1 \rightrightarrows X_1 \to Y_1$ and $C_2 \rightrightarrows X_2 \to Y_2$, their product $C_1 \times C_2 \rightrightarrows X_1 \times X_2 \to Y_1 \times Y_2$ is again a coequaliser diagram, i.e., *binary products commute with coequalisers*. The aim of this talk is to present an algebraic analysis of this property, showing how it may be characterised for general varieties of algebras, as well as showing how it provides a natural setting in which to study Huq-centrality of morphisms. Much of the behaviour of Huq-centrality for unital categories [1] is retained in our setting, including categories which are (weakly) unital, but also categories outside of the unital setting.

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Facets of $\mathrm{Spec}(R)$

Micheala Hoenselaar, *University of Johannesburg*

SAMS; Subject Classification Number: 7, 29

Thanks to Grothendieck's algebraic geometry, the Zariski topology on the prime spectrum of a (commutative) ring (with identity) has been studied a considerable amount. On the other hand, although topologies like the patch, inverse, Alexandroff, coarse upper, coarse lower, interval, Scott, Lawson, *etc,* have mostly been studied in the general context of topology, lattice theory, and commutative algebra, not much has been done on the spectrum of a ring. The purpose of this talk is to give an overview on some of the existing results about these topologies on spectra of rings and then discuss some new ones.

Properties of a semi-linear Timoshenko rod model with axial force and spectral theory of the related linear model

K.A. Hohls*, *University of Pretoria*
N.F.J. van Rensburg, *University of Pretoria*

SAMS; Subject Classification Number: 3, 30

In this presentation, numerical experiments resulting from finite element calculations will be used to investigate the properties of a semi-linear Timoshenko rod model with axial force suggested by M.H. Sapir and E.L. Reiss [Dynamic Buckling of a nonlinear Timoshenko beam, *SIAM J. Appl. Math.* **37** (1979), 290-301]. These properties include the possibility of a non-trivial equilibrium as well as dynamic buckling. In order to aid in the investigations, results from the spectral theory of the related linear Timoshenko rod model will also be considered.

The nonlinear system of partial differential equations considered in the model is

$$\partial_t^2 w = \partial_x \left(\partial_x w - \phi\right) + \left(\frac{D}{\gamma} + \frac{1}{2\gamma}\int_0^1 (\partial_x w)^2\right)\partial_x^2 w, \tag{3}$$

$$\frac{1}{\alpha}\partial_t^2 \phi = \partial_x w - \phi + \partial_x\left(\frac{1}{\beta}\partial_x \phi\right). \tag{4}$$

This mechanical system is a special case of a problem of the form

$$u'' = Au + f(u) \tag{5}$$

on some Hilbert space, where A is a linear operator and f a nonlinear mapping.

Hyper near-vector spaces

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SAMS; Subject Classification Number: 15

Near-vector spaces are a generalisation of vector spaces, as near-rings are a generalisation of rings. A number of authors have attempted to define the notion of a near-vector space, we will focus on André's near-vector spaces [1]. Hyper nearrings and hyper vector spaces have been defined and studied (see [2] and [3], for example). It is thus natural to progress to the notion of a hyper near-vector space. In this talk we discuss the construction of a hyper near-vector space generalised from the hyper vector spaces defined in [2]. We obtain a space having similar properties to Johannes André's near-vector space. We define important concepts including independence, the notion of a basis, regularity, subhyperspaces and as a highlight prove that there is a Decomposition Theorem for these spaces.

This work is joint work with Prof B. Davvaz, Department of Mathematics, Yazd University, Iran.

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Star-multiplication and crossed modules in right Ω-loops

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SAMS; Subject Classification Number: 4

Internal categorical structures play an important role in categorical algebra. In semi-abelian categories, internal categories form a variety, namely the variety of crossed modules [1]. The notion of star multiplication was introduced by G. Janelidze in [1] where it was applied to the description of crossed modules in a semi-abelian category. In the same paper, the question of describing semi-abelian categories with the property that every star-multiplicative graph uniquely extends to an internal category structure was asked.

N. Martins-Ferreira [3] introduced conditions that provide a simple description of internal groupoids as crossed modules in the semi-abelian categories of groups and rings. The aim of this talk is to describe star-multiplication in varieties of right Ω-loops in the sense of [2].

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Quasi-uniformities and syntopogenous structures on frames

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SAMS; Subject Classification Number: 13

In early *1960*, Császár [2] introduced the notion of syntopogenous structures for spaces with the intention of studying, in the same setting, topological, uniform and proximity spaces. Recently, in [1], these orders were extended to a general category. In particular, topogenous orders were shown to play vital roles among which the unification of neighborhood, closure and interior operators.

In [4], quasi-uniformities and syntopogenous structures were successfully studied. Motivated by the fact that many researchers have focused their attention on the observation that the important aspect of a topological space is not its set of points but its lattice of open subsets, in this talk, we present the point-free counterpart of the results in [1] and [4].

We define topogenous orders on the category of frames. We show that they embrace nuclei and interior operators and many orders on frames including the Császár order introduced in [3].

Following [3] and [5], for syntopogenous and quasi-uniformities on frames respectively, we study entourage quasi-uniformities using syntopogenous structures on frames. We establish a Galois connection between syntopogenous structures and quasi-uniformities on a frame. This Galois connection permits us to establish which syntopogenous structures are isomorphic to the en tourage quasi-uniformities. Furthermore, this isomorphism lies at the center of the observation that a quasi-uniformity on a frame is a family of nuclei. In concluding we show that our results can be extended to the weil quasi-uniformities.

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Conservation laws and solutions of the generalised modified Camassa-Holm equation

Mohapi Isaac Keoagile* and **Chaudry Masood Khalique**,
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SAMS; Subject Classification Number: 3

In this talk, we investigate the general modified Camassa-Holm equation equation [1] which has many applications in fields such as physics, medicine, finance and other related science fields. Using the technique of Lie symmetry analysis [2], we first compute its Lie point symmetries. Thereafter, group-invariant solutions are determined under each symmetry. Finally, conservation laws for this equation are derived using multiplier approach [2].

References

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Exact solutions of flow and pressure variation inside a horizontal filter chamber using Lie symmetry analysis.

T.J. Itumeleng*, **G. Magalakwe**, **L.D. Moleleki**, *North West University*

SAMS; Subject Classification Number: 20

Physical model exact solutions improve crucial industrial production processes by giving operators and designers a greater grasp of how systems operate in practice. The current case study aims to find exact momentum and pressure variation solutions during the unsteady-state filtration process to advance fluid purification. Lie symmetry analysis is used to transform a system of PDEs representing the flow and pressure variation into solvable ODEs without changing the dynamics of the case study. The velocity (momentum) and pressure solutions are then determined by integrating the obtained solvable ODEs. The effects of physical parameters resulting from the process dynamics are examined using the exact solutions acquired to identify the parameter combination that results in the maximum permeates outflow. Graphical representations of the flow velocity and pressure are presented and analysed as well as the skin friction tabular. The results indicate that as time evolves, permeate outflow decreases because internal momentum, work done, and pressure diminish over time due to the clustering of particles. In addition, low wave speed, small porosity, more chamber space, high injection rate, and stronger magnetic effects are ideal for minimising the effects of no-slip condition at the bottom wall during operation.

A comparison on new integral transforms

H. Jafari, *University of South Africa*

SAMS; Subject Classification Number: 12, 16, 23

Integral transforms are important to solve real problems. Appropriate choice of integral transforms helps to convert differential equations as well as integral equations into terms of an algebraic equation that can be solved easily [1,2].

During last two decades many integral transforms in the class of Laplace transform are introduced such as Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, G_transform, Sawi and Kamal transforms.

In this work, we compare some integral transforms in the class of Laplace transform which are introduced during last few decades. After that we propose a general integral transforms which is covered all of those integral transforms [1].

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A Toeplitz-like operator with rational symbol having poles on the unit circle

Jacob Jaftha*, *University of Cape Town*
Gilbert Groenewald, *North West University*
Sanne ter Horst, *North West University*
Andre Ran, *North West University & Vrije Universiteit*

SAMS; Subject Classification Number: 11

Let $\Omega \in \mathrm{Rat}^{m\times m}$ with possibly poles on $\mathbb{T}$, where $\mathrm{Rat}^{m\times n}$ the space of $m \times n$ rational matrix functions and define the Toeplitz-like operator $T_\Omega\,(H^p_m \to H^p_m)$ as follows:

$$\mathrm{Dom}\,(T_\Omega) = \left\{ \begin{array}{ll} f \in H^p_m : & \Omega f = h + r \text{ where } h \in L^p_m(\mathbb{T}), \\ & \text{and} \quad r \in \mathrm{Rat}^m_0(\mathbb{T}) \end{array} \right\}$$

$$T_\Omega f = \mathbb{P}h \text{ where } \mathbb{P} \text{ is the Riesz projection of } L^p_m(\mathbb{T}) \text{ onto } H^p_m.$$

This talk will discuss some basic properties of the operator T_Ω such as

1. T_Ω is a densely defined closed operator and its domain contains the polynomials, and
2. T_Ω satisfies a Brown-Halmos type condition on its domain.

as well as Fredholm properties. The construction of a Wiener-Hopf-type factorization of Ω, where Ω has poles on $\mathbb{T}$, is shown to allow one to determine the Fredholm properties of the operator T_Ω. In addition, we will contrast some properties with the scalar case as well as T_Ω with classical block Toeplitz operators.

Satellites in semi-abelian algebra

George Janelidze, *University of Cape Town*

SAMS; Subject Classification Number: 4

There are various known notions of satellites of functors at various levels of generality, one of which, originally introduced in [1], is simply a modified version of the notion of Kan extension. The purpose of this talk is to show how a very special case of it is related to the weak form of action representability introduced in [2].

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Optimal noetherian forms

Zurab Janelidze, *Department of Mathematical Sciences, Stellenbosch Universityand & National Institute for Theoretical and Computational Sciences*

SAMS; Subject Classification Number: 04

Noetherian forms provide a symmetric (or 'self-dual') theory for establishing homomorphism theorems for (non-abelian) group-like structures. Even though existence of such structure was conjectured more than 70 years ago by one of the co-founders of category theory, Saunders Mac Lane (see [3]), their discovery is recent (see [1,2,4] and the references there). It turns out, however, that even those realms of mathematical structures which appear to be far from being 'group-like' can be *given* a noetherian form, and moreover, there may be many (in fact, infinitely many) noetherian forms in each case. In this talk we introduce a notion of an 'optimal' noetherian form: it is a noetherian form which can be isomorphically embedded in any other noetherian form (over the same context of mathematical structures, or more generally, over the same category) that, roughly speaking, describes the same homomorphism theorems as the first one. The question is, when does an optimal noetherian form exist, and if it exists, how does it look like? There is much mystery surrounding this question. We can only give a (positive) answer in various special scenarios. In particular, in a recent joint work in progress with Francois van Niekerk (on which the present talk is based), we showed that every topos has an optimal noetherian form. The background theory that led to this result has another interesting application: the unit interval gives rise to an increasing chain of optimal noetherian forms, which is unbounded on both sides. We will discuss these results and conclude with some open questions pertaining to optimal noetherian forms.

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Efficient and Robust Optimization Methods for Training Binarized Deep Neural Networks

G.J. Groenewald, *North-West University*
D.B. Janse van Rensburg*, *North-West University*
A.C.M. Ran, *orth-West University & Vrije Universiteit Amsterdam*

SAMS; Subject Classification Number: 11

In this talk we will discuss small rank perturbations of H-expansive and H- unitary matrices paying particular attention to the location of eigenvalues of these matrices with respect to the unit circle.

The talk is based on joint work with Gilbert Groenewald and Andre Ran.

Jacobian norm regularisation and conditioning in neural ODEs

Shane Josias*, *Applied Mathematics & School for Data Science and Computational Thinking, Stellenbosch University*
Willie Brink, *Applied Mathematics, Stellenbosch University*

SAMS; Subject Classification Number: 23

A recent line of work regularises the dynamics of neural ordinary differential equations (neural ODEs), in order to reduce the number of function evaluations needed by a numerical ODE solver during training. For instance, in the context of continuous normalising flows, the Frobenius norm of Jacobian matrices are regularised under the hypothesis that complex dynamics relate to an ill-conditioned ODE and require more function evaluations from the solver. Regularising the Jacobian norm also relates to sensitivity analysis in the broader neural network literature, where it is believed that regularised models should be more robust to Gaussian and adversarial perturbations in their input. We investigate the conditioning of neural ODEs under different Jacobian regularisation strategies, in a binary classification setting. Regularising the Jacobian norm indeed reduces the number of function evaluations required, but at a cost to generalisation. Moreover, naively regularising the Jacobian norm can make the ODE system more ill-conditioned, contrary to what is believed in the literature. As an alternative, we regularise the condition number of the Jacobian and observe a lower number of function evaluations without a significant decrease in generalisation performance. We also find that Jacobian regularisation does not guarantee adversarial robustness, but it can lead to larger margin classifiers.

Bipartite Ramsey number pairs involving cycles

Johannes H. Hattingh, *University of North Carolina, USA*
Ernst J. Joubert*, *University of Johannesburg*

SAMS; Subject Classification Number: 11, 21

Let a and b be positive integers with $a \geq b$. For bipartite graphs G_1 and G_2, the bipartite Ramsey number pair (a, b), denoted by $b_p(G_1, G_2) = (a, b)$, is an ordered pair of integers such that for any blue-red coloring of the edges of $K_{r,t}$, with $r \geq t$, either a blue copy of G_1 exists, or a red copy of G_2 exists, if and only if $r \geq a$ and $t \geq b$. In [1], Faudree and Schelp determined bipartite Ramsey number pairs for paths. In this paper we will focus on bipartite Ramsey number pairs that involve cycles. In particular, we will show, for s sufficiently large, that $b_p(C_{2s}, C_{2s}) = (2s, 2s - 1)$.

References

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On the solution to the inverse representation problem for monoids.

Gideo Joubert, *Stellenbosch University*

SAMS; Subject Classification Number: 4

The representation theorem for monoids states that any monoid on a set X is isomorphic to a submonoid of the monoid of endofunctions on X. In the presentation, I will give a solution to the inverse problem. That is, given a submonoid of the monoid of endofunctions of a set X; when is it possible to find a monoid structure $(X, \cdot, a)$, such that the representation monoid of $(X, \cdot, a)$ is the endofunction monoid that was given. As it turns out, a simple diagrammatic argument explains the solution once we view the problem in the language of monoid actions.

This presentation is based on a Honours project under supervision of Prof. Zurab Janelidze.

The number of small weakly connected components in random directed acyclic graphs

Masreshaw Temere Kassaye* and **Dimbinaina Ralaivaosaona**
University of Stellenbosch

SAMS; Subject Classification Number: 6

A directed acyclic graph (DAG) is a digraph with no directed cycles. More formally, a DAG consists of vertices and edges (also called arcs) with each edge directed from one vertex to another in such a way that following those directions will never form a closed loop. These objects are omnipresent in modern science, e.g., they are used as graphical representations of food webs, citation networks, Bayesian networks, etc. We consider a random DAG $\mathbb{D}_{ac}(n,p)$ which is motivated by the well-known binomial model for undirected graphs. The $\mathbb{D}_{ac}(n,p)$ model is obtained from the binomial random digraph model $\mathbb{D}(n,p)$ simply conditioned to be acyclic. We are mainly interested in the sparse regime where p is of the form $\frac{\lambda}{n}$ and λ is a fixed positive constant as it is known that these random structures undergo a phase transition around $p = \frac{1}{n}$.

In the case of general digraphs, it is fundamental to understand the strongly connected components. However, by definition, a strongly connected component of a DAG can only have at most one vertex. Hence, for DAGs it makes sense to consider the weakly connected components instead. An isolated vertex in a digraph can be regarded as a weakly connected component of order one. It was recently proved that the number of isolated vertices in $\mathbb{D}_{ac}(n,\lambda/n)$ is asymptotically normal with mean linear in n as $n \to \infty$. In this talk, we discuss a generalisation of the latter result where we consider the number of weakly components of order bounded by a fixed positive constant. Our approach is analytic which is a combination of symbolic method and complex contour integral techniques.

Numerical investigation of the Rössler attractor by means of the residual function

Kekana MC*, **Shatalov MY, Tong T.O**
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SAMS; Subject Classification Number: 5, 10

In this presentation, A mathematical framework of checking accuracy of numerical methods from mathematical software is been developed and investigated. Rössler is used as case study as its analytical solution is non-existence. The algorithms investigated from Mathcad software includes Adams method, AdamsBDF, Runge-Kutta fixed. The estimation of global and local residuals are shown. The graphical results are plotted on an interval of $0 \leq t \leq 150$.

A study of a generalized Hirota-Satsuma coupled Korteweg-de Vries system

Chaudry Masood Khalique *, *International Institute for Symmetry Analysis and Mathematical Modelling, North-West University*

SAMS; Subject Classification Number: 3

In this talk, a generalized Hirota-Satsuma coupled Korteweg-de Vries system shall be investigated from the group standpoint [1,2]. This system represents an interplay of long waves with distinct dispersion correlations. We present solitary wave solutions for the underlying system.

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Integral conceptions: Transition from single to multivariate integral calculus in engineering

Khemane Thabiso*, **Padayachee Pragashni**, and **Shaw Corrinne**
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SAMS; Subject Classification Number: 16

In their first year at university, engineering students study single variable integration followed by their progression to multivariable integration in second year. This transition from first to second year integral calculus can present challenges that can be attributed to students' conceptions of integrals established in first year. The approach taken to address this transition was to conduct an assessment in the form of a written test administered to second year engineering students at the University of Cape Town, followed by in-depth interviews with six students purposively selected from students who had written the test. Descriptive statistics and qualitative content analysis were used to analyse students' conceptions of integration.

Results show that students had three variations in associating single with double integrals, 1) interpretation of a single integral as an area under a curve associated with a double integral as a volume below a surface, 2) an area under a curve conception of a single integral with the double integral as a summation of vertical area slices, and 3) it was found that students who were able to relate double integrals to the Riemann sum were able to sketch images that represented both single and multivariable integrals.

The results suggest that certain integral conceptions such as an area under the curve may contribute to the difficulties students have in multivariable integrals. These difficulties are likely to affect performance in vector calculus topics like Stokes' theorem. It is important therefore, in the pedagogical approach, to include different integral conceptions that will enable students to understand single and multivariable integrals. Understanding how students' conceptions of single integrals impact their understanding of double integration and other vector calculus topics can enable educators to design instructional instruments that will help students understand double integrals and improve performance in multivariable calculus and in other engineering courses.

Attractors of iterated function system and its applications

Melusi Khumalo* and **Talat Nazir**, *University of South Africa*

SAMS; Subject Classification Number: 11, 21

In this talk, we construct the fractals with the help of a finite family of contractive operators in the setup of distance spaces. Varieties of results for iterated function system are established. Some applications and examples are also obtained from the main results presented therein. Our results unify, generalize and extend various results in the current literature.

Mathematics in the Union of South Africa, 1910-1961

Eder Kikianty*, *University of Pretoria*
Loyiso Nongxa, *University of the Witwatersrand*

SAMS; Subject Classification Number: 16

The first university in Southern Africa, the University of the Cape of Good Hope, was established in the Cape Colony in 1873, which offered only examinations and not tuition. After the formation of the Union of South Africa, two teaching universities were established. Many other universities were later established during this period, and thus marked the beginning of formal higher education in South Africa. We trace the development of mathematics and mathematical activities at the universities in the Union of South Africa.

A new vehicle routing problem for increased driver-route familiarity

Jacobus King*, Jan van Vuuren, and Stephan Nel
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SAMS; Subject Classification: 23

Practical challenges often arise when implementing solutions that stem from solving vehicle routing problem instances. Unplanned external events can result in increased vehicle travel times and subsequent degradations in supply chain operational efficiency. Moreover, drivers tend to get lost and/or often travel on roads that are not suitable for the delivery vehicles utilised when they are unfamiliar with delivery routes, which typically occurs when these routes differ significantly from one day to the next. A possible solution aimed at streamlining the practical implementation of planned delivery schedules is to generate a set of standard delivery routes visiting each customer along different approaches, called *master routes*. These master routes may then be used as blueprints for daily planning purposes when actual delivery routes are computed. Delivery vehicle drivers may thus be afforded the opportunity to become familiar with the master routes, which is anticipated to increase the efficiency with which they perform deliveries if the actual delivery routes do not deviate too much from these master routes. In this presentation, two mathematical models are derived and accompanying approximate solution approaches are proposed for the creation of such master routes, and subsequent actual delivery routes. Furthermore, numerical results obtained during a real case study are discussed.

A framework for intelligent document image enhancement in pursuit of improved OCR performance

Ryno Kleinhans* and Stephan Nel, *University of Stellenbosch*

SAMS; Subject Classification Number: 12, 23, 25

A characteristic trait of the age of digitalisation is the ubiquitous transition from paper-reliant and manual-based business processes to fully digital, computer-assisted and automated versions thereof. Although many industries have already began with this transition away from paper documents, several real-world information chains are still intertwined with downstream paper-based systems. Some of these systems might require several decades to transition into a fully digital version thereof. Consequently, in order to fully automate these processes, the paper-based documents ought to be digitised.

Computerised approaches, *e.g.* optical character recognition engines, have achieved notable success in accurately extracting pixel-based information into machine-encoded information. The performance of these engines are, however, reliant on the quality of the captured document images. Although there are a plethora of image enhancement techniques designed to increase image quality, the implementation of some of these techniques involves a large degree of dependency on human cognition as each document image require a unique set of preprocessing steps. Accordingly, the application of data-driven approaches from the realm of machine learning — more specifically, deep learning — certainly warrants consideration within the presented context.

In this presentation, a high-level overview is provided of a framework that aims to facilitate the text extraction procedure of document images by automating the preprocessing stage through means of intelligently identifying which combination of image enhancement techniques to implement in respect of individual images. Powerful approaches from the domain of computer vision, together with the implementation of transfer learning, are considered.

The mincut graph of a graph

Christo Kriel* and **Eunice Mphako-Banda**, *University of the Witwatersrand*

SAMS; Subject Classification Number: 6

We introduce an intersection graph of a connected graph G, the mincut graph of G, such that every minimum edge-cut in G is a vertex in the mincut graph, $X(G)$, and two vertices in $X(G)$ are adjacent if their corresponding edge-cuts in G share at least one edge. We present the mincut graph as a graph operator. We ask and attempt to answer questions such as 'Which graphs appear as images of graphs?'; 'Which graphs are fixed under the operator?'; 'What happens if the operator is iterated?' Thus, we show that every graph is a mincut graph and furthermore, that non-isomorphic graphs can have isomorphic mincut graphs. We also characterise graphs fixed under the operator and show that no graph diverges under iteration of the operator.

Simulation of earthquake induced oscillations in a vertical structure using the finite element method

Madelein Labuschagne*, **Sonja du Toit**, and **Belinda Stapelberg**
University of Pretoria

SAMS; Subject Classification Number: 23

A new model for earthquake induced oscillations in a vertical structure is presented. The foundation of the structure is considered as a rigid body attached to an adapted Timoshenko beam model and the effect of ground motion is modelled as forces acting on the foundation.

In this presentation the model is investigated numerically using the finite flement method. For the purpose of modal analysis, natural frequencies and modes are computed. The finite flement method is also used for dynamic simulations of the motion and the results are compared.

Conserved vectors and solutions of the two-dimensional potential KP equation

Mduduzi Yolane Thabo Lephoko*, **Chaudry Masood Khalique**,
International Institute for Symmetry Analysis and Mathematical Modelling, North-West University

SAMS; Subject Classification Number: 3

In this talk, we study the potential Kadomtsev-Petviashvili equation [1] which has many applications in fields such as plasma physics, phase imaging and nonlinear mechanics. Using the technique of Lie symmetry analysis [2], we first compute its Lie point symmetries. Thereafter, group-invariant solutions are determined under each symmetry. Finally, conservation laws for this equation are derived using the conservation theorem due to Ibragimov [3].

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Perturbation ideals and Fredholm theory in Banach algebras

Tshikhudo Lukoto*, *University of Limpopo*
Heinrich Raubenheimer, *University of Johannesburg*

SAMS; Subject Classification Number: 11

In 1971, Lebow and Schechter in [1] introduced the notion of perturbation classes where one of the important result discovered is that the perturbation of $\mathcal{A}^{-1}$, the group of invertible elements in a Banach algebra $\mathcal{A}$, is equal to $\mathrm{Rad}(\mathcal{A})$, the radical in $\mathcal{A}$, and we denoted this result as

$$\mathcal{P}(\mathcal{A}^{-1}) = \mathrm{Rad}(\mathcal{A}).$$

We use this notion to characterize perturbation ideals of sets that generate the familiar spectra in Fredholm theory. At first we classify the set in question as either regularity or semiregularity, a concept which is set out in [2].

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The k-Ramsey number for two cycles

Ronald John Maartens, *University of the Witwatersrand*

SAMS; Subject Classification Number: 6

Let F and H be two bipartite graphs with Ramsey number $R(F,H)$. Further, let G be a complete k-partite graph $K(n_1,n_2,\ldots,n_k)$ of order $n=\sum_{i=1}^{k} n_i$ with $n_i \in \{\lceil \frac{n}{k} \rceil, \lfloor \frac{n}{k} \rfloor\}$ for $i=1,\ldots,k$ and $k=2,\ldots,R(F,H)$. The k-Ramsey number $R_k(F,H)$ is then defined as the smallest positive integer n such that for any red-blue coloring of the edges of G there is a subgraph of G isomorphic to F whose edges are all colored red, or a subgraph of G isomorphic to H whose edges are all colored blue. Results on $R_k(F,H)$ is known for two bipartite graphs F and H only. In this talk we investigate the k-Ramsey number of two cycles which are not both bipartite.

A new generalisation of Baer's theorem

Adolfo Ballester-Bolinches, *Universitat de València*
Sesuai Y. Madanha*, *University of Pretoria*
Tendai M. Mudziiri Shumba, *Sobolev Institute of Mathematics*
Maria C. Pedraza-Aguilera, *Universitat Politècnica de València*

Dedicated to the memory of Alexander Grant Robertson Stewart

SAMS; Subject Classification Number: 15

In 1957, Reinhold Baer proved that if G is the product of two normal supersolvable subgroups and G' is nilpotent, then G is supersolvable. There has been several generalisations of this result. In this project the structure of finite groups $G = AB$ which are a weakly mutually sn-permutable product of the subgroups A and B, that is, such that A permutes with every subnormal subgroup of B containing $A \cap B$ and B permutes with every subnormal subgroup of A containing $A \cap B$, is studied. We generalise Baer's theorem.

An elementary proof of semilattice duality

James J. Madden, *Louisiana State University*

SAMS; Subject Classification Number: 4

The category of (discrete) *semilattices* (i.e., idempotent commutative monoids) will be denoted by **SL** and the category of *compact* 0*-dimensional topological semilattices* will be denoted by **ZS**. The Duality Theorem for Semilattices (sometimes called "Pontryagin Duality for Semilattices," see [1] and [2]) states that the hom-functors (suitably enriched)

$$\mathbf{SL}(\_, 2) : \mathbf{SL} \to \mathbf{ZS} \quad \text{and} \quad \mathbf{ZS}(\_, 2) : \mathbf{ZS} \to \mathbf{SL}$$

provide a dual equivalence of categories. A straightforward proof from first principles is difficult to extract from the literature. In this talk, I sketch such a proof.

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Fractional order age structured population dynamics with applications in epidemiology

Madito Gladstone Thabo*, and **Suares Clovis Oukouomi Noutchie**
Pure and Applied Analytics Focus Area, North-West University, South Africa

SAMS; Subject Classification Number: 3, 10

Models describing the dynamics of biological phenomena that evolve in stages have been studied extensively using time-delay mathematical models. Recently, these models have evolved into age structured models with age dependent variables. Various mathematical tools have been employed to study these models and investigate the effects of age structure. This work explores two fractional order age structured models to address the issues of latent infection of cells by HIV and the effects of latent TB infection on the dynamics of HIV. The transmission dynamics of HIV by multiple cell types through two transmission routes within-host is modelled by Caputo fractional order derivatives, whereas, the co-epidemic of HIV and TB is modelled by a system of Caputo-Fabrizio derivatives. The equilibrium points are obtained under certain conditions, and the reproduction numbers of each transmission route and each disease are computed. Furthermore, the local asymptotic stability of the disease-free equilibrium for each model is established.

Solutions of (3+1)-dimensional Gardner-type equation using Lie symmetry analysis.

M. Mafora*, **L.D. Moleleki**, **G. Magalakwe**
North West University

SAMS; Subject Classification Number: 3

In this talk we investigate the solutions of the (3+1)-dimensional Gardner-type equation. This equation and its generalizations are used in many areas of applications, such as hydrodynamics, plasma physics, and quantum field theory. We use the Lie symmetry method to obtain exact solutions of the (3+1)-dimensional Gardner-type equation. Moreover, conservation laws of (3+1)-dimensional Gardner-type equation are derived using Ibragimov's theorem.

On proximity and remoteness in directed graphs

S. Mafunda*, *University of Johannesburg*
J. Ai, *Royal Holloway, University of London*
S. Gerke, *Royal Holloway, University of London*
G. Gutin, *Royal Holloway, University of London*

SAMS; Subject Classification Number: 06

In a strong, finite digraph D of order n, the distance $d_D(u, v)$ from vertex u to vertex v is the length of a shortest $u-v$ path in D. The *average distance* $\bar{\sigma}(x)$ of a vertex x of D is the arithmetic mean of the distances from x to all other vertices of D. The *remoteness* $\rho(D)$ and *proximity* $\pi(G)$ of D are the maximum and the minimum of the average distances of the vertices of D, respectively.

In 2021, Ai et al. [1] showed that for any pair of vertices in D, their average distances can differ by no more than $\frac{1}{2}(n-2)$. This suggests a natural question if there is a simple characterisation of all digraphs where all vertices have the same average distance.

In this talk, we discuss the above characterisation for strong tournaments and we further present an infinite family of non-regular strong digraphs D such that $\rho(D) = \pi(D)$.

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Numerical approximation to a nonlinear equation with global differentiation and intregral operator.

H. Magau*, *Central University of Technology.*
A. Atangana, *University Of the Free State.*

SAMS; Subject Classification Number: 5

The Riemann-Stieltjes integral was established from the works of Riemann and Steiltjes and has been recognized as a more generalized Riemann integral. This integral has opened doors for more theories and applications in the last decades. However, a differential operator associated with this integral has not been defined in the last decades. Very recently, by extending the concept of rate of change, a more general differential operator was suggested. Using the fundamental theorem of calculus, this operator was found to be the derivative associated with the Riemann-Stieltjes integral. In this talk, we present some new theoretical results and applications related to this extension.

Symmetry analysis and exact solutions of extended Kadomtsev-Petviashivili (eKP) equation in fluids

Christina Majola*, **Letlhogonolo Daddy Moleleki**, and **Masood Khalique**
North West University

SAMS; Subject Classification Number: 3

An extended Kadomtsev-Petviashivili equation in fluids [1] is investigated by using Lie symmetry analysis. The similarity reductions and new exact solutions are obtained via the simplest equation method. Exact solutions including solitons are shown.In addition, the conservation laws are derived using multiplier approach.

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Generalized iterated function system for common attractors in partial metric spaces

Vuledzani Makhoshi*, **Talat Nazir**, and **Melusi Khumalo**, *University of South Africa*

SAMS; Subject Classification Number: 11, 21

In this talk, we develop some new common attractors with the assistance of finite families of generalized contractive mappings that belong to the special class of mappings defined on a partial metric space. Consequently, a variety of results for iterated function systems satisfying a different set of generalized contractive conditions are acquired. We present some examples to reinforce the results proved herein. These results generalize, unify and extend a variety of results that exist in current literature.

Multiscale modelling Of norovirus

A.D Maphiri*, **D. Mathebula**, **K. Muzhinji**, **R. Netshikweta**, **W. Garira**
University of Venda

SAMS; Subject Classification Number: 10

Chemical, physical and biological processes interact across multiple scales of organization which are molecular, cellular, tissue, organ systems and whole-body systems. Multiple scales lead to both localised and systematic consequences for physiology, disease progression and medical therapeutics. Disease dynamics emerge from the collective behaviour across multiple scales and hence cannot be understood simply by studying the isolated parts at a single scale. Traditional approaches for modelling infectious diseases have, until recently, been focusing on describing key disease processes (within-host and between-host) separately using single-scale models, hence do not fully explore the linking or coupling of models at multiple scales. These models overlook the fact that transmission is a consequence of process acting on at least two different scales which are within-host and between-host scales. Due to high risk of foodborne diseases and large number of deaths associated with foodborne diseases, it is important to establish a multiscale model that can assist in identifying significant factors that contribute in the spread of foodborne diseases. Therefore, scientific research is required to gather rigorous information on the burden of foodborne diseases which can be vital for supporting policy-makers as well as assisting the process for allocating appropriately resources for food safety control and intervention efforts. This work aims to develop a multiscale model of foodborne diseases caused by norovirus. To achieve the main goal of the study, we develop a within-host sub-model and between-host sub-model of norovirus and link them to form a multiscale model. Sensitivity analysis and numerical simulations show that the influence of within-host dynamics on between-host dynamics has a great effect on disease progression, the results show that targeting within-host parameters will be more helpful in reducing the burden on foodborne disease caused by norovirus.

Congruence lattices of graphs

Klarise Marais* and **Andrew Craig**, *University of Johannesburg*

SAMS; Subject Classification Number: 34

Congruence lattices of algebras are widely used to study their corresponding algebra. In this talk we consider a recent extension of the concept of congruences to graphs, developed by Broere, Heidema and Pretorius [1], and extended to graphs with loops by Broere, Heidema and Veldsman [2]. We study lattice-theoretic properties of these congruence lattices, such as distributivity and modularity. We show that congruence lattices of graphs with three or more vertices are all non-distributive. Further, we investigate conditions for modularity and other connections between graphs and their congruence lattices.

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A numerical solver for some non-linear Poisson equation

G. T. Marewo, *North-West University*

SAMS; Subject Classification Number: 23

The Lane-Emden equation is an example of a non-linear Poisson equation. In this study we consider a boundary value problem for the Lane-Emden equation which describes the behaviour of the density of a gas sphere in hydrostatic equilibrium. The non-linearity of the equation motivates the use of a numerical solver. Our choice of the solver is a Galerkin finite element method coupled with linearisation because finite element methods handle complex geometries well and they have well developed techniques for their mathematical analyses among other desirable properties. The numerical solver is implemented on the computer using `MATLAB`, a computer environment for performing numerical computations and visualisation. Numerical experiments are performed to show that the solver is computationally effective.

References

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Near field structures on a multiplicative group.

Sophie Marques * and **Leandro Boonzaaier**, *Stellenbosch University*

SAMS; Subject Classification Number: 29

In this talk, we will present how exploring the idea of multiple near-field structures on a given multiplicative group leads to interesting concepts. In particular, we will present how they permit to define all the generalized means for any families of complex numbers.

Symmetry analysis of a coupled system of ordinary differential equations

Sivenathi Mbusi*, **Ben Muatjetjeja**, **Abdullahi Adem**
North West University

SAMS; Subject Classification Number: 3

We perform a complete symmetry analysis of a generalized Lane-Emden-Klein-Fock system with central symmetry. Several cases for the non-equivalent forms of the arbitrary elements are obtained. Moreover, a symmetry reduction for some cases is performed.

$(\mathbb{Z}_2)^3$-Graded Contractions of $\mathfrak{g}_2$

Dr Cristina Draper, *Universidad de Malaga*
Thomas Meyer*, *University of Cape Town*
Dr Juana Sanchez-Ortega, *University of Cape Town*

SAMS; Subject Classification Number: 29

We look at the Lie algebras obtained by modifying the structure constants of the complex Lie algebra $\mathfrak{g}_2$ via contractions preserving the fine $(\mathbb{Z}_2)^3$-grading, $\Gamma_{\mathfrak{g}_2}$.

To do so, we consider how we may use a $(\mathbb{Z}_2)^3$-grading of the Octonions to construct the corresponding grading $\Gamma_{\mathfrak{g}_2}$ on the smallest exceptional Lie algebra $\mathfrak{g}_2$, which may be realized as the derivations of the Octonions. We then see how the special properties of this grading allow for the classification of the Lie algebras, obtained from contractions of the grading $\Gamma_{\mathfrak{g}_2}$, to be converted into a combinatorial problem involving the Fano plane, $P(\mathbb{Z}_2^3)$, and collineations of its points. Finally, we investigate some properties (nilpotency, solvability, centre) of the obtained Lie algebras and how these properties relate to the grading.

An optimal portfolio and consumption problem within the framework of viscosity solutions

Farai Julius Mhlanga*, *University of Limpopo*

SAMS; Subject Classification Number: 25

We consider an optimal portfolio and consumption choice problem which incorporates the average past consumption. The investor consumes and allocates her wealth between a risk-free asset and a risky asset. The objective is to find an allocation process and a consumption pattern which optimises the expected utility of the average past consumption. As in [1], [2], we formulate the portfolio optimisation problem as a singular stochastic control problem and is solved using dynamic programming and the theory of viscosity solutions. The value function is characterised as the unique constrained viscosity solution of the corresponding integro-differential variational inequality. For investors having a hyperbolic absolute risk aversion utility functions, we provide explicit consumption and allocation choices.

References

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Highly accurate multi-domain multivariate spectral collocation method for (2+1) dimensional partial differential equations

M. P. Mkhatshwa* and **M. Khumalo**, *University of South Africa*

SAMS; Subject Classification Number: 3,23

The novelty of this work rests upon the use of the domain decomposition technique in time variable when discretizing the domain of solution in spectral collocation algorithm. The single domain multivariate spectral collocation-based methods have been proven to be effective in solving time-dependent partial differential equations (PDEs) defined over small time domains. However, there is a significant loss of accuracy as time computational domain proliferates and also when the number of grid points approaches a definite particular number. Therefore, the establishment of the new innovative multi-domain multivariate spectral quasilinearisation method (MDMV-SQLM) is described for the purpose of solving (2+1) dimensional nonlinear PDEs defined on large time intervals. The main output of this study is confirmation that minimizing the size of time computational domain at each subinterval assures sufficiently accurate results that are attained using minimal number of nodal points and less computational time. To highlight the efficiency and accuracy of the MDMV-SQLM, error estimates, condition numbers and computational time are presented for well known (2+1) dimensional nonlinear Burger's PDEs. The adoption of the domain decomposition technique is effective in suppressing the numerical challenges linked to large matrices and ill-conditioned nature of the resulting coefficient matrix. Also, the obtained results confirm that the numerical scheme is computationally cheap, fast and yield extremely accurate and stable results using fewer number of grid points for large time domains.

Optimal treatment plans for glioblastoma multiforme by PDE constrained optimization

Seyyed Abbas Mohammadi*,
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SAMS; Subject Classification Number: 23, 25

Glioblastoma multiforme is a highly aggressive brain tumor with a life expectancy of 6 to 12 months [1,2]. The recurrence of the tumor, even after radical excision, necessitates the use of postoperative therapeutic strategies such as radiotherapy and chemotherapy to slow tumor progression [2]. To improve conventional treatments and increase patient survival, novel therapies must be developed. Spatial macroscopic models based on PDEs have been widely used to describe the proliferation and invasion of tumor cells in brain tissue; see for example [3–5] and the references therein. In this study, we consider a modification of the reaction-diffusion model proposed in [3] which describes the diffusion and proliferation of tumor cells in addition to the effects of precisely delivered treatment e.g., chemotherapy and radiotherapy. Treatment optimization is still in high demand in order to consider patient-specific biological parameters while avoiding side effects in standard-of-care treatment plans [6–9]. Therefore, a PDE constrained optimization is developed in order to obtain the optimal treatment plan. Using the standard method of the calculus of variations, we shall establish that the optimization problem admits a solution and obtain a necessary condition for the optimizer. The optimizer is explicitly calculated for specific ranges of problem parameters, and its uniqueness is investigated. Then, a gradient-based efficient numerical algorithm is developed to determine the optimizer. For the case of chemotherapy, results suggest a bang-bang chemotherapy strategy in a cycle which starts at the maximum dose and terminates with a rest period. Numerical simulations based upon our algorithm on a real brain image show that this is in line with the maximum tolerated dose (MTD), a standard chemotherapy protocol.

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Designs invariant under simple groups $PSp_4(q)$

Clarence K. Mokalapa, *University of Limpopo*

SAMS; Subject Classification Number: 15

Key and Moori used two methods to construct combinatorial designs and codes from finite simple groups. A transitive permutation group acting on combinatorial designs has important application in coding theory and are useful tools in the decoding procedures. We construct some symmetric designs from the maximal subgroups (namely: stabilizers of points and planes, and parabolic) and conjugacy classes of the Projective Symplectic groups $PSp_4(q)$. Consider the primitive permutation representation of simple groups $PSp_4(q)$ to construct symmetric 1-designs.

Exact solutions and the conservation laws of the (1+1)-dimensional Boussinesq equation

L.D. Moleleki*, **C. Majola**, **C.M. Khalique**, *North West University*

SAMS; Subject Classification Number: 03

In this talk, we study a nonlinear evolution partial differential equation, namely, the (1+1)-dimensional Boussinesq equation. We use Lie symmetry method together with simplest equation method to find the exact solutions of the new (1+1)-dimensional Boussinesq equation. Furthermore, multiplier method will be used to construct the conservation laws of the new (1+1)-dimensional Boussinesq equation.

Students' preparedness for, and response to learning first-year mathematics fully online

Dr Rina Durandt* and **Dr Wilmari Morton**
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SAMS; Subject Classification Number: 16

In this talk we will present empirical results from a study that investigated first-year mathematics students' preparedness for, and response to, online learning. In 2020, due to the COVID-19 pandemic, lecturers and students were faced with a 'sudden' shift from face-to-face to online teaching and learning, and the focus was to save the academic year with no student left behind. Students (from both advantaged and disadvantaged communities) were also expected to adapt to these changes instantaneously. In 2021 the pandemic situation was similar and the first-semester offering of first-year mathematics modules continued fully online at most universities in South Africa. A blended learning environment is not new to first-year mathematics students and has been promoted and encouraged in the years leading up to the pandemic. However, the complete and sudden change in environments, particularly to fully online, was new to most lecturers and students. Several challenges were reported that gave rise to this investigation. We expected students to learn productively when they have (1) regular and reliable access to technological resources, and (2) self-regulated study habits. Quantitative data were collected through a self-designed questionnaire during the first-semester of 2021. The large sample was from a public university in South Africa, where students were registered for one of seven first-year mathematics modules. Main results confirm students are partially prepared for learning first-year mathematics fully online and they have unequal scenarios related to access to technological resources. Further, first-year students' approach to solving mathematics problems while studying online has a significant effect on their performance.

On some Cauchy completions of quasi-pseudo metric spaces and maps that preserves Cauchy sequences

Seithuti Moshokoa, *Tshwane University of Technology*

SAMS; Subject Classification Number: 11

The purpose of the talk is to study further properties concerning complete quasi-pseudo metric spaces, maps that preserves some Cauchy sequences and extensions of such maps to the completion.

Lie group classification of a variable coefficients Gardner equation

Tanki Motsepa*, *School of Computing and Mathematical Sciences, University of Mpumalanga*
Chaudry Masood Khalique, *International Institute for Symmetry Analysis and Mathematical Modelling, North-West University*

SAMS; Subject Classification Number: 3

In this talk, we perform Lie group classification of a variable coefficients Gardner equation [1], which describes various interesting physics phenomena, such as the internal waves in a stratified ocean, the long wave propagation in an inhomogeneous two-layer shallow liquid and ion acoustic waves in plasma with a negative ion. The Lie group classification [2] of the equation provides us with four-dimensional equivalence Lie algebra and has several possible extensions. It is further shown that several cases arise in classifying the arbitrary parameters. Finally, conservation laws are obtained for certain cases using the multiplier approach [3].

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Comparative performance of time spectral methods for solving hyperchaotic finance system

Claude B. Moutsinga*, *Sefako Makgatho Health Sciences University*
Edson Pindza, *University of Pretoria*
Eben Maré, *University of Pretoria*

SAMS; Subject Classification Number: 23

This Talk presents the hyperchaotic finance system (HCFS). The problem is solved by means of spectral methods based on Chebyshev polynomials. One approach uses differentiation matrix and the other considers an integral approach. Numerical simulations are conducted to draw comparison and discussion between the two methods and another numerical method, namely Chebfun.

On closed sets and the k-defect polynomials of certain graphs

Eunice Mphako-Banda, *University of the Witwatersrand*

SAMS; Subject Classification Number: 06

In this talk we begin by giving a brief introduction on closed sets of a graph. We then discuss minors obtained by contracting closed sets of size k and apply this in the computation of the k-defect polynomial. We finally, discuss some values of k for which the k-defect polynomial of a graph is the zero polynomial and pose a few questions which can be explored further.

Note that, analogous to the chromatic polynomial (the 0-defect polynomial), the k-defect polynomials count the number of ways possible to colour a graph with λ colours when allowing k bad edges.

Topology in monoidal categories

Charles Nsukukazifani Msipha, *Tshwane University of Technology*

SAMS; Subject Classification Number: 4

In [2] a subobject of the monoidal unit object in a monoidal category for which the canonical morphism is invertible is called a subunit. Such subobjects parallel the the open subsets of a base topological space in categories like those of sheaves or Hilbert modules. This talk is meant to presents some constructions found in [2] involving subunits. Mentioned constructions endow any monoidal category with some topological intuition. The mentioned constructions are adaptation of ideas whose origins in [1].

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Tertiary mathematics education: Can it be talked about in isolation?

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SAMS; Subject Classification Number: 16

This talk is designed to be an audience engaging discussion, aimed at drawing all our attention to the SAMS Mission Statement especially the following items:

- encourage the maintenance of sound syllabuses, teaching methodologies and assessment standards in mathematical education, and the cooperation in these areas with the organizations and professional societies with similar interests; tertiary mathematics education;
- explore the contributions that Mathematics can make in addressing societal needs and promoting a strong economy that will contribute to an improved quality of life for all South Africans;

The quoted items have a potential to impact positively not only to Tertiary Mathematics Education but across the board. We see the need to significantly contribute to improving the quality of the school system product, delivered to tertiary institutions as one of the societal needs which falls within the ambit of the second quoted item. Every year during the announcements of matric results the Minister of Basic Education and Training does not get tired of reminding us of the serious challenges we have in sciences, in particular in Mathematics. The United Nations having declared this year, 2022 as the International Year for Basic Sciences for Sustainable Development should sensitise us to engage both the problem of our school Mathematics in terms of improving the quality of the product of our schools and the issue of attracting more students to sciences.

This engagement should put us in a position to conceptualise some plan of action firstly to immediate initiate some disruptive interventions and secondly conceptualise relatively long sustained intervention to put a visible dent on the current and ongoing national problem in sciences in general and in particular in Mathematics. Be role players in "*producing our own timber*".

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J-frames

Simo S. Mthethwa, *University of KwaZulu-Natal*

SAMS; Subject Classification Number: 13

In this talk, I shall introduce *J-frames*, which is the pointfree counterpart of the concept of *J-spaces* which was exhibited by E. Michael in 2000. Some characterisations of J-frames via closed and relatively connected sublocales will be furnished. A common property between the remainder of any frame in a perfect compactification and the remainder of a J-frame in any compactification will be discussed and utilised to show that a completely regular frame is a J-frame if and only if its remainder is relatively connected in its Stone-Čech compactification. Among other things, I will show that a non-spatial frame is a J-frame if and only if it is connected.

A generalization of Andrews-Eriksson-Petrov-Romik mapping for MacMahon's theorem

Beaullah Mugwangwavari* and **Darlison Nyirenda**
School of Mathematics, University of the Witwatersrand

SAMS; Subject Classification Number: 6

In the early 20th century, P. A. MacMahon proved that the number of partitions of n wherein no part appears with multiplicity one is equal to the number of partitions of n where parts are even or congruent to 3 modulo 6. The first bijective proof of MacMahon's theorem was provided by G. Andrews, H. Eriksson, F. Petrov and D. Romik. We construct a generalization of their bijection and discuss its consequences.

Average distance, minimum degree, and irregularity index

Simon Mukwembi, *University of the Witwatersrand*

SAMS; Subject Classification Number: 6

We give an upper bound on the average distance of a connected graph of given order and minimum degree where irregularity index is prescribed. Our results are a strengthening of the classical theorems by Dankelmann and Entringer [1], and by Kouider and Winkler [2], on average distance and minimum degree if the number of distinct terms in the degree sequence of the graph is prescribed.**References**

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A parameter-uniform numerical method for singularly perturbed Burgers'equation

Eshetu B. Derzie, *Adama Science and Technology University, Adama, Ethiopia*
Justin B. Munyakazi*, *University of the Western Cape, South Africa*
Tekle Gemechu, *Adama Science and Technology University, Adama, Ethiopia*

SAMS; Subject Classification Number: 23

We propose a parameter-uniformly convergent numerical method for singularly perturbed Burgers' initial-boundary value problem. First, the Burgers' partial differential equation is semi-discretized in time using Crank–Nicolson finite difference method to yield a set of singularly perturbed non-linear ordinary differential equations in space. The resulting two-point boundary value nonlinear singularly perturbed problems are linearized using Newton quasilinearization technique, and then, we apply fitted operator finite difference method to exhibit the layer nature of the solution. It is shown that the method converges uniformly with respect to the perturbation parameter. Numerical experiments are carried out to confirm the parameter-uniform nature of the scheme which is second-order convergent in time and first-order convergent in space.

Iterated function system of generalized cyclic contractive operators

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SAMS; Subject Classification Number: 11

Iterated function systems are based on the mathematical foundations laid by Hutchinson [1]. He showed that Hutchinson operator constructed with the help of a finite system of contraction mappings defined on a Euclidean space $\mathbb{R}^n$ has closed and bounded subset of $\mathbb{R}^n$ as its fixed point, called attractor of iterated function system (see also in [2]). In this context, fixed point theory plays significant and vital role to help in the construction of fractals.

The aim of this talk is to present the sufficient conditions for the existence of attractor of a generalized cyclic iterated function system composed of a complete metric space and a finite family of generalized cyclic contraction mappings. Some examples are presented to support our main results and concepts defined herein. The results proved in the paper extend and generalize various well known results in the existing literature [3,4].

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0-Cauchy completions in strong partial b–metric space

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SAMS; Subject Classification Number: 13

In [2] partial metric spaces, were introduced as generalization of metric spaces. In [1] completions of partial metric spaces was constructed. In this talk we will introduce strong partial b-metric spaces and present 0-Cauchy completions

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Developing higher order unconditionally positive finite difference methods for the advection diffusion reaction equations

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SAMS; Subject Classification Number: 23

Higher order unconditionally positive finite difference (HUPFD) methods are developed to solve linear and non-linear advection-diffusion-reaction (ADR) equations. The stability and consistency of the developed methods are analyzed, which are necessary and sufficient for convergence to the exact solution. The Von Neuman condition is used to analyze the stability since we are dealing with the Cauchy problem. The proposed method's efficiency and effectiveness is investigated by calculating the error, convergence rate, and computational time. A comparison of the solutions obtained by the higher order unconditionally positive finite difference and analytical methods is conducted for validation purposes. The numerical results show that the developed method preserve the solution accuracy. The results also show that increasing the order of the unconditionally positive finite difference leads to the implicit scheme that is unconditionally stable with an increased order of accuracy with respect to time and space.

Spectral domain decomposition

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SAMS; Subject Classification Number: 23

In this project we investigate techniques for solving second-order elliptic ordinary and partial differential equations with variable coefficients on one- and two-dimensional domains. We explore spectral collocation methods and domain decomposition strategies for this task, and derive a Hierarchical Poincaré–Steklov (HPS) based approach, similar to that introduced by Gillman and Martinsson [1]. The HPS method is a recursive domain decomposition. It merges solution and Dirichlet-to-Neumann operators between subdomains, enforcing continuity of the solution and its derivative across domain boundaries. The result is a spectrally accurate discretization with an explicit fast direct solve that can be applied to problems with smooth solutions on any domain which can be decomposed into rectangular subdomains. A major advantage of the HPS strategy is that after the solution and Dirichlet-to-Neumann operators have been constructed, applying the boundary conditions and solving the problem is computationally inexpensive, meaning the scheme is ideal for solving the same problem with different boundary data. An additional benefit of the approach is that it can be parallelised across multiple processors to minimize computation time. We derive and implement the spectral HPS approach, and demonstrate its efficiency on some simple examples.

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New provably energy stable formulations for hyperbolic problems: application to the Euler and shallow water equations

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SAMS; Subject Classification Number: 3, 23

We present the general stability theory for hyperbolic IBVPs developed in [1]. It extends the use of the energy method from linear to nonlinear problems, is easy to understand and leads to L_2 estimates. The only requirements for an energy bound is that a specific skew-symmetric form of the equations exist and that proper boundary conditions are available. We will discuss the key steps to such a formulation and exemplify with the compressible Euler equations.

The new formulation also makes it possible to understand some confusing results obtained from linearisation, where in some case an energy bound exist for the nonlinear problem, but not for the linearised one (or vice versa) [1]. A nonlinear and linear analysis may also lead to different boundary conditions required for a bound [2]. The new formulation shed light on this confusing fact.

The new skew-symmetric formulation was shown to hold for the shallow water equations as well as for the incompressible and compressible Euler equations [1–3]. We will discuss how to determine nonlinear boundary conditions and relate that to a boundary condition analysis for linear problems.

Finally, by discretising using summation-by-parts (SBP) operators [4] which mimic integration-by-parts, we show that nonlinear stability follows automatically if boundary conditions leading to a continuous energy estimate are available.

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Maximal nowhere dense sublocales

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SAMS; Subject Classification Number: 13

Call a closed nowhere dense sublocale of a completely regular locale L maximal nowhere dense if it is not nowhere dense in any closed nowhere dense sublocale of L. This concept is a frame counterpart of maximal nowhere dense subsets which were introduced by Veksler (in [1]) in the category of topological spaces. In this talk, I aim to introduce and study maximal nowhere dense sublocales. I will:

a) show that the concept of maximal nowhere dense subsets is conservative in locales;

b) show that the definition of inacessible points is conservative in locales and define inaccessible sublocales; and

c) present results about maximal nowhere dense sublocales using inaccessible sublocales and remote sublocales.

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How to include policies in disease models

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SAMS; Subject Classification Number: 10

The management of HIV/AIDS has evolved ever since advent of the disease in the past three decades. Many countries have had to revise their policies as new information on the virus, and its transmission dynamics emerged. In this paper, we track the changes in Botswana's HIV/AIDS response and treatment policies using a piece-wise system of differential equations. The policy changes are easily tracked in three epochs. Models for each era are formulated from a **grand model** that can be linked to all the epochs. The grand model's steady states are determined and analysed in terms of the model reproduction number, R_0. The model exhibits a backward bifurcation, where a stable disease-free equilibrium coexists with a stable endemic equilibrium when $R_0 < 1$. The stability of the models for the other epochs can be derived from that of the grand model by setting some parameters to zero. The models are fitted to HIV/AIDS prevalence data from Botswana for the past three decades. The changes in the populations in each compartment are tracked as the response to the disease and treatment policy changed over time. Finally, projections are made to determine the possible trajectory of HIV/AIDS in Botswana. The implications of the policy changes are easily seen, and a discussion on how these changes impacted the epidemic are articulated. The results presented have crucial impact on how policy changes affected and continue to influence the trajectory of the HIV/AIDS epidemic in Botswana.

Error correcting codes from 2-represantation of unitary group U(3,3)

Nyikadzino Tapiwanashe Gift*, *University of Limpopo*

SAMS; Subject Classification Number: 15

In this dissertation, we us modular repesenation theoretic to find error correcting codes admit the unitary group $U(3,3)$ as a primitive permutation group We show that every binary linear code admiting $G = U(3,3)$ as a primitive permutation group is a submodule of the permutation module of the primitive action of G. Our aim is to find the set of all linear codes from the set of 2-representations of a finite simple group. Then explain that, if the Schur multiplier of the group G is trivial and P is a permutation module of degree n, then every binary linear code of length n invariant under G is a submodule of P [1]. We will find the irreducible submodules of the simple group in our case the group will be unitary group $U(3,3)$. We will be using magma [2] to find all those submodules. After finding the submodules we will find the maximal subgroups, each maximal subgroup has a corresponding permutation module we will also find them using magma.

We find submodule of these permutation modules, these submodules are codes from the $U(3,3)$, We Classify some codes and determine properties of some binary codes such as the minimum distance and other properties that we described in the first part such as the weight, minimum weight and the support. We will be able to tell whether the certain code is good error correcting or not based on properties they possess.

We then determine designs from some binary codes using codewords of minimum weight. In order to obtain block designs we must determine the set of points, make a list of all possible base blocks then solve the problem of huge number of constructed designs with the same parameters. We establish the linkages between some linear codes and designs from primitive groups.

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Positive weighted Koopman semigroup on Banach lattice-modules

Tobi David Olabiyi, *Stellenbosch University*

SAMS; Subject Classification Number: 11

In this talk, we will introduce the notion of a positive weighted (Koopman) semigroup representation on a Banach lattice-module over the (Koopman) group representation on a commutative Banach lattice-algebra. For topological dynamics, we obtain the abstract representation of the space of continuous sections vanishing at "infinity" of a *topological* Banach lattice-bundle (over a locally compact space Ω) as an AM m-lattice-module over $C_0(\Omega)$ on which every positive weighted semigroup is isomorphic to the positive weighted Koopman group representation induced by the unique dynamics on the underlying *topological* Banach lattice-bundle.

Do odd discretizations produce higher accuracy for pseudospectral methods?

C. P. Olivier*, *North-West University*
N. V. Alexeeva, *University of Cape Town*

SAMS; Subject Classification Number: 23

The fast Fourier transform (FFT) algorithm was originally developed for discretizations of length $N = 2^j$, for some $j \in \mathbb{N}$ [1]. While this choice ensures optimal performance, the widespread use of the FFT algorithm has led to high efficiency for any sample size N that it can be factorized sufficiently, such as sizes of the form $N = 3^j$. The advantage of using an odd sample size is that all nonzero Fourier coefficient arise in pairs of F_n and F_{-n}. By using an aliasing error analysis, we show that this symmetry reduces high frequency noise associated with the approximation of derivatives. Results are illustrated for the heat equation, the Korteweg-de Vries equation and the Thirring equation. **References**

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On Bourbaki bounded sets on quasi-pseudometric spaces

Olivier Olela Otafudu, *North-West University,*

SAMS; Subject Classification Number: 13

In this talk, we study Bourbaki-boundedness on quasi-pseudometric spaces. It turns out that if a set is Bourbaki-bounded on the symmetrized quasi-pseudometric space, then it is Bourbaki-bounded in the quasi-metric space but the converse need not to be true. We show that an asymmetric normed space is Bourbaki-bounded if and only if it is bounded. Consequently, we prove that every real-valued semi-Lipschitz in the small function on a quasi-metric space is bounded if and only if the quasi-metric is Bourbaki-bounded.

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Binary trees with few ordinary and total dominating sets

Opeyemi Oyewumi*, *Stellenbosch University*
Riana Roux, *Stellenbosch University*
Stephan Wagner, *Uppsala University, Sweden*

SAMS; Subject Classification Number: 6

A subset D of the vertex set $V(G)$ is a *dominating set* of graph G if every vertex of $V(G) \setminus D$ has a neighbor in D. The set D is a *total dominating set* of G if every vertex of G has a neighbor in D. A tree is called a binary tree if all it's internal vertices (i.e., non leaves) are exactly of degree three. Here, we determine the classes of binary trees that has the minimum number of ordinary and total dominating sets. More generally, we extend our results to d-ary trees.

Conservation laws and group-invariant solutions for certain soil water equations

Gontse Simon Pai* and **Chaudry Masood Khalique**,
International Institute for Symmetry Analysis and Mathematical Modelling,
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SAMS; Subject Classification Number: 3

We determine conservation laws for a class of soil water equations that represents a model to simulate soil water infiltration, redistribution, and extraction in a bedded soil profile overlaying a shallow water table and irrigated by a line source drip irrigation system [1–3]. Finally we present some group-invariant solutions of the underlying equations.

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Panel discussion on tertiary mathematics education

Panel Chair: Pragashni Padayachee, *(University of Cape Town)*
Panel Organiser: Rina Durandt, *(University of Johannesburg)*

SAMS; Subject Classification Number: 16

The transition between school and university, especially for disciplines such as mathematics has over several years been a cause for concern. In an everchanging society and in unequal educational environments this discussion seems even more relevant. Furthermore, empirical studies (such as TIMSS) shine a negative light on student performance and retention, particularly for students who plan to enter tertiary mathematics courses. The study of mathematics has an important bearing on the success of students in the STEM field and research studies raise concerns about the transition from high school to university and students' preparedness to study higher education mathematics. Even for students who meet the requirements for STEM courses, mathematics problems still persist, and the lack of mathematical preparedness and mathematical proficiency remain a barrier for student success. So, the question arises: how will possible changes to the Grade 12 curriculum (or trimming of the curriculum) in the South African context influence students' learning of tertiary mathematics? The panel discussion will address this question by incorporating the views and arguments from specialists from different perspectives.

Panellists include: (i) **Mrs Razzia Ebrahim** *(Western Cape Education Department)*, education specialist and curriculum advisor for FET mathematics ; (ii) **Dr Robert Prince** *(University of Cape Town)*, Lead investigator for the DHET funded Diagnostic Mathematics Information for Students Retention and Success Project (DMISRS) ; (iii) **Dr Erna Lampen** *(University of Stellenbosch)*, researcher in mathematics education with experience in curriculum development and the professional development of mathematics teachers; and (iv) **Dr Cerene Rathilal** *(University of Johannesburg)*, researcher in pure mathematics and first-year mathematics lecturer, with experience in community projects for school learners.

The panel discussion will consider views on the content and pedagogy of Grade 12 mathematics on the one hand and tertiary mathematics on the other, suggest ways to address the gap between the transition from school to university, consider how mathematics learning from earlier grades might impact this transition, debate how 'trimming' of the secondary mathematics curriculum might impact learning in the tertiary environment, and make suggestions for possible interventions and changes to the tertiary curriculum and pedagogy needed to address this transition between environments.

Locally connected categories

Samantha Parle, *University of Cape Town*

SAMS; Subject Classification Number: 4

There are various incidences across different areas of mathematics where objects are defined as being connected. In graph theory these are connected graphs, and in topology they are connected spaces. Examining some properties of these connected objects allows us to generalise and come up with a collection of category-theoretic conditions describing a more general form of connectedness. When we apply these conditions to the category of graphs and the category of topological spaces, we get the same objects as when we used the original definitions. We will consider so-called lextensive categories, in which the notion of objects being connected is well-behaved. Such a category is said to be locally connected if every object in it is a coproduct. A locally connected category $\mathbb{C}$ is equivalent to $\mathrm{Fam}(\mathbb{A})$ for some category $\mathbb{A}$, where $\mathrm{Fam}(\mathbb{A})$ denotes the category of families of objects in $\mathbb{A}$. We will consider some properties and examples of such categories.

On asymmetric modular spaces

Tlotlo Phawe* and **Olivier Olela Otafudu**
Department of Mathematics and Applied Mathematics North-West University

SAMS; Subject Classification Number: 13

In this talk, we consider the concept of modular quasi-metric spaces. We look into some fixed-point theorems for nonexpansive mappings on modular quasi-metric spaces.

A comparative study of a multidomain spectral quasilinearization and block hybrid methods for solving evolution parabolic equation

Mapule Pheko*, **Letlhogonolo D. Moleleki**, **Sicelo P. Goqo**,
North-West University

SAMS; Subject Classification Number: 23

In this study, we compare the multidomain spectral quailinearization and spectral block hybrid methods for solving nonlinear evolution parabolic equations. Similar intervals and grid points are used for discretization in both methods. For comparison purpose, we solve the Burgers-Fisher equation and other nonlinear evolution parabolic equations. To verify the approaches' precision, convergence and effectiveness, the results are compared with well-known analytical solutions. The results are discussed in a form of tables and graphs to show the accuracy, convergence and effectiveness of the methods. We found that there is a high degree of accuracy between the two methods.

A new diagonal separation property in the category of locales

Jorge Picado, *CMUC, Department of Mathematics, University of Coimbra, Portugal*

SAMS; Subject Classification Number: 13, 4

Recall that the Hausdorff property of a topological space X is characterized by the closedness of the diagonal in $X \times X$. This is a general phenomenon, the so-called $\mathcal{P}$-separation [2]:

Given a property relevant in the category in question (typically of a topological nature), an object X is $\mathcal{P}$-separated if the diagonal in $X \times X$ has the property $\mathcal{P}$. Besides the Hausdorff property in classical spaces, relevant examples for $\mathcal{P}$ are e.g. the strong Hausdorff axiom or the Boolean property in the category of locales [4,6].

In the context of locales there are the important properties of fittedness and fitness [5]. A sublocale of a locale is *fitted* if it is an intersection of open ones and a locale is *fit* if each of its sublocales is fitted. Since the intersection $S^\circ = \bigcap\{T \mid S \subseteq T,\ T \text{ open}\}$ is an operation of closure type [3], it is natural to ask about fitted diagonals; we will speak of the $\mathcal{F}$-separated locales [1]. This property will be the main topic of this talk.

Taking into account the fact that the subcategory of fit locales is closed under products and subobjects, we have an immediate observation that fitness implies $\mathcal{F}$-separatedness. We will see that $\mathcal{F}$-separatedness is in fact strictly weaker than fitness and we will explore a surprising parallel with the strong Hausdorff axiom, including a Dowker-Strauss type theorem and a characterization in terms of certain relaxed morphisms.

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Solutions and conserved vectors for the Yu-Toda-Sasa-Fukuyama equation of plasma physics

Karabo Plaatjie* and **Chaudry Masood Khalique**, *International Institute for Symmetry Analysis and Mathematical Modelling, North-West University*

SAMS; Subject Classification Number: 3, 22

In this talk, we study the two-dimensional Yu-Toda-Sasa-Fukuyama equation [1] using Lie symmetry analysis [2]. Firstly, Lie point symmetries are obtained and used to perform symmetry reductions. As a result of symmetry reductions, equation is reduced to several nonlinear ordinary differential equations, which we solve with the help of different techniques. Moreover, the derived solutions are illustrated graphically for some parametric values. Furthermore, the conserved vectors are computed using the classical Noether's theorem [3].

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Constructing Fischer-Clifford matrices of a finite extension group from its factor group

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SAMS; Subject Classification Number: 28

Let $\overline{G} = P.G$ be a finite extension where P is a normal p-subgroup of $\overline{G}$. Choose the smallest non-trivial characteristic subgroup K of P such that $\frac{P}{K} = P_1$ is an abelian p-group then it follows that the factor group $\overline{F} = \frac{\overline{G}}{K}$ has structure $P_1.G$ and $\overline{G}$ can be viewed as having the structure $K.\overline{F}$. Since K is normal in $\overline{G}$ the lifts of the ordinary characters $\mathrm{Irr}(\overline{F})$ of $\overline{F}$ are equivalent to ordinary irreducible characters χ_i of $\overline{G}$ such that $K \leq \mathrm{Ker}(\chi_i)$. Using the Fischer-Clifford matrices technique [1] we can obtain the ordinary character table of $\overline{F}$. With the above discussion in mind, the Fischer-Clifford matrices $M(g_i)$ of $\overline{G}$, g_i a class representative of G, can then be constructed from the Fischer-Clifford matrices $\widetilde{M(g_i)}$ of $\overline{F}$ by adding an appropriate number of columns and rows to $\widetilde{M(g_i)}$. In a sense, the matrices $\widetilde{M(g_i)}$ are "lifted" to the matrices $M(g_i)$ in $\overline{G}$. Having obtained the matrices $M(g_i)$ and together with the ordinary or projective characters of the inertia factors H_i of the action of $\overline{G}$ on $\mathrm{Irr}(P)$, the ordinary irreducible character table of $\overline{G}$ is constructed. The approach described above is powerful if P is non-abelian and P_1 elementary abelian and a G-module over the finite field $GF(p)$. In this talk, we will elaborate more about this approach and illustrate it with an appropriate example.

Generalized ergodic domination in ordered banach algebras

A.D. Rabearivony* and **S. Mouton**, *Stellenbosch University*

SAMS; Subject Classification Number: 11

In ordered Banach algebra (OBA) theory, various authors have studied the so-called domination problem (see, e.g., Section 4.2 in [2]): given two elements a and b of an OBA such that $0 \leq a \leq b$, under what hypotheses are properties of b inherited by a? We tackle the corresponding problem where the condition $0 \leq a \leq b$ is replaced by the weaker condition $\pm a \leq b$. (For operators S and T on a Banach lattice E the condition $\pm S \leq T$ means that $|Sx| \leq T|x|$ for all x in E.) We refer to this as the generalized domination problem. Furthermore, it is presented as an open question in [2] whether the (known) ergodic domination theorem (see [1]) can be extended to this setting. We will show that this question has a positive answer, not only for ergodic domination, but also for most of the existing domination results, including those related to Riesz elements, inessential elements and elements of the radical.

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The domination number in Galton-Watson trees

V.R. Misanantenaina, **F. Rakotoniaina**, **D. Ralaivaosaona***
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SAMS; Subject Classification Number: 6

A set $X \subseteq V$ is a dominating set of a graph $G = (V, E)$ if every vertex in $V \setminus X$ is adjacent to a vertex in X. The *domination number* of G is the cardinality of the smallest dominating set of G. We study the distribution of the domination number in a conditioned Galton-Watson model with offspring distribution ξ that satisfies $\mathbb{E}\xi = 1$ and $0 < \mathrm{Var}\xi < \infty$. Given a discrete random variable ξ with support on $\{0, 1, 2, \ldots\}$, the Galton-Watson tree $\mathcal{T}$ with offspring distribution ξ is the random tree generated in the following way: we start with a root and generate a number of children according to ξ. For each of the children of the root, generate a number of children of their own according to ξ independently of the other vertices. Then, we repeat this process until it stops. Now, we consider the model $\mathcal{T}_n$ which is the above random tree conditioned to have exactly n vertices. We show that the domination number in such a random tree is asymptotically normal as the order of the tree tends to infinity. Our method is based on the analysis of the Cockayne-Goodman-Hedetniemi (CGH) algorithm [1,2] and a recent extension of Janson's result on local additive functionals [3]. In fact, the CGH algorithm leads to a finite tree partitioning that allows to construct the additive functional which will be then used to prove our result according to [4], an extension of Janson's result. We also prove that the same result holds for the distribution of the total domination number, i.e., the cardinality of the smallest total dominating set of G. The later is defined similarly as the dominating set by replacing $V \setminus X$ to V. We will discuss these results and their proofs in this talk.

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Spatiotemporal dynamics of mutualist populations in a moving habitat

Andriamihaja Ramanantoanina*, *University of Pretoria*

SAMS; Subject Classification Number: 5

As the global climate changes, different species have limited options for their survival: to move and follow their suitable habitat or to stay and adapt to the changing conditions. The dynamics of populations following a moving habitat have been investigated in the literature using reaction diffusion equations and / or integrodifference equations. Commonly, the focus is on the dynamics of single species and on negative interspecific interactions such as competition. In this talk, we discuss the dynamics of mutualistic populations in a moving habitat using models based on integrodifference equations. We consider the cases in which the habitat quality affects both the growth and dispersal processes. We start with an investigation of the spatiotemporal dynamics the populations when the habitat is moving at a constant velocity, then discuss the frequency of extinction and the time to extinction in a moving habitat with varying velocity.

The nonnegative inverse eigenvalue problem with prescribed zero patterns in low dimensions

André Ran*, *Vrije Universiteit Amsterdam and North-West University*
Emily Teng, *Vrije Universiteit Amsterdam*

SAMS; Subject Classification Number: 11

The nonnegative inverse eigenvalue problem asks for conditions on a set of n complex numbers for it to be the set of eigenvalues of an $n \times n$ entrywise nonnegative matrix. The problem is known to be very difficult in the generality in which it is stated here: solutions exist for $n = 3$ and $n = 4$ only, along with a list of necessary conditions for the higher dimensional cases. In addition, for the case $n = 5$ a complete solution exists under the extra assumption that the trace of the matrix is zero.

Motivated by the latter observation, we considered for $n = 3$ all possible zero patterns in the matrix, and investigated what extra conditions on the set of eigenvalues these give. In addition, some results for $n = 4$ were obtained as well. In the talk we aim to provide a bit of the flavor of the problem and the techniques used for the solution.

Investigation on an integral formula for the quantum 6j symbols

Hosana Ranaivomanana, *Stellenbosch University*

SAMS; Subject Classification Number: 12

I investigate a conjectural integral formula for the quantum 6j symbols suggested by Bruce Bartlett. For that I consider the asymptotics of the integral and compare it with the known formula for the asymptotics of the quantum 6j symbols due to Taylor and Woodward. Taylor and Woodward's formula can be rewritten as a sum of two quantities: *ins* and *bound*. The asymptotics of the integral splits into an interior and boundary contribution. I successfully computed the interior contribution using the stationary phase method. The result is indeed quite similar to although not exactly the same as *ins*. Though I expect the boundary contribution to be similar to *bound*, the computation is left for future work.

Compactifications and separation of the interval topology

Cerene Rathilal*, *University of Johannesburg*
Andrew Craig, *University of Johannesburg*

SAMS; Subject Classification Number: 13, 34.

In 1942, Frink [2] showed that a lattice is complete iff the interval topology of the lattice is compact. We aim to discuss conditions under which the Dedekind–MacNeille completion of a lattice L is a compactification of the interval topology on L. In [1], Ernè established conditions for which the interval topology of L will be Hausdorff and Regular. The concept of finite separabilty plays a central role in understanding the separation axioms of the interval topology of L. As a consequence of investigating compactifications of the interval topology of L, we shall discuss the relationship between separated intervals of a lattice as described in [3], finite separabilty and Ernè's separation conditions from [2].

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A pair of monads in topology

Ando Razafindrakoto, *University of the Western Cape*

SAMS; Subject Classification Number: 4, 13, 34

One of the monads described in [2] by H. Simmons is the prime open filter monad whose algebras were shown to be the class of stably compact spaces together with proper maps. Following Simmons and starting with the fact that the coalgebras of the ideal functor are precisely the stably compact frames and proper frame maps [1], we give an alternative description of the relationship between the prime open filter monad and the ideal functor. We shall give a new proof of the equivalence between stably compact spaces and stably compact frames and how this equivalence reduces to that of compact Hausdorff spaces and compact regular frames.

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Explicit results on the bound of Siegel zeros for quadratic fields

F.B. Razakarinoro, *Stellenbosch University*

SAMS; Subject Classification Number: 22

We consider the Dirichlet L-functions associated with characters defined by the Kronecker's symbol $\chi(n) = \left(\frac{\Delta}{n}\right)$. The study of the zeros of such functions has profound consequences in many results in number theory. The Generalized Riemann Hypothesis states that the zeros of the L-functions with positive real parts lie on the line $\Re(s) = \frac{1}{2}$. However, it is known that there might exist zeros near to 1 in the interval of the form $(1 - C/\log|\Delta|, 1)$ where C is an absolute constant. These hypothetical zeros are called Siegel zeros or Landau-Siegel zeros. In this talk, we first review some elementary definitions and background results, and then give a survey on what we know explicitly on the bounds of Siegel zeros.

A more unified approach to integration

Mangatiana A. Robdera, *Department of Mathematics, University of Botswana*

SAMS; Subject Classification Number: 30

The most important and common features and resemblances in the different approaches to integrals are gathered to obtain a more unified and simpler system of properties suitable for the definition of integration of functions taking values in topological linear spaces with respect to vector-valued integrators. As examples of application, we show that such a novel approach is flexible enough to give a further generalization of the spectral theorem on Hilbert spaces and that such an approach can also be used to push stochastic integration to the more general setting of integrable processes taking values in topological vector spaces.

Analyze and visualize the cathodoluminescence data obtained from images of a photovoltaic cell.

Mienie Roberts*, **Taylor Harvey**, **Aida Torabi**, and **James Sullivan**
Texas A&M University-Central Texas

SAMS; Subject Classification Number: 12

We created a Shiny application using the R software to analyze and visualize the cathodoluminescence data obtained from images of a photovoltaic cell. The Graphical User Interface can import data, create a heatmap to indicate high quality crystal structure, and output several variables of interest. We also developed a spectra data analysis algorithm to functionalize, analyze, and generate statistical measurements of the luminescence data to provide additional insight. This algorithm was coded in the R language program and a set of CdMgSeTe films were studied as an application case study. CL maps were measured for samples with different luminescent responses. A quantitative measure of the heterogeneity of the films was generated by statistical analysis of luminescent intensity and wavelength, spectra type curves, frequency distributions of peak wavelength, and relative intensity maps. The final CL analysis facilitates investigation of the CdMgSeTe films and has potential applications for many semiconductor films.

Representable distributive quasi relation algebras

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SAMS; Subject Classification Number: 34

A relation algebra is an algebraic structure consisting of a non-empty set, the Boolean operations and constants, an associative binary operation that has a unit, and an involutive unary operation, such that these operations and constants satisfy certain equations forming an axiomatization of a calculus of binary relations. The example of a relation algebra that motivated this definition is the algebra of all binary relations on a set, with the associative binary operation interpreted as the usual composition of binary relations, its unit as the identity relation, and the involutive unary operation as the converse relation.

A generalization of the motivating example mentioned above is the power set algebra of an equivalence relation on a set, with the operations defined in the same way. This relation algebra plays an important role in the definition of a representable relation algebra as any relation algebra isomorphic to a subalgebra of the power set algebra of some equivalence relation on some set.

Representable relation algebras have been studied for many decades. In 1950 Lyndon showed that there is a non-representable relation algebra [2]. In 1955 Tarski showed that representable relation algebras is a variety [4]. In 1964, Monk showed that representable relation algbras is not finitely axiomatizable [3], unlike relation algebras, which is finitely axiomatizable by definition.

In the 1940s, Alfred Tarski proved that relation algebras have an undecidable equational theory [5]. To remedy this, Galatos and Jipsen identified a larger variety, called quasi relation algebras, similar to relation algebras that has a decidable equational theory [1].

In this talk we will introduce representable distributive quasi relation algebras. These are defined via a construction involving posets with certain symmetry requirements. This gives rise to many interesting questions about the representability of distributive quasi relation algebras and the properties of representable quasi relation algebras.

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On imprimitive rank 3 permutation groups and binary self-dual codes

Bernardo Rodrigues, *University of Pretoria*

SAMS; Subject Classification Number: 15

One of the questions of current interest in coding theory is the following: given a finite non-solvable permutation group G acting transitively on a set Ω, under what conditions on G are self-dual codes invariant under G existent or nonexistent? In the talk, this problem is investigated under the hypothesis that the group G is an imprimitive rank 3 permutation group.

We will show that if G is a finite imprimitive rank 3 permu- tation group acting transitively on the coordinate positions of a self-dual binary code C then G is one of M_{11} of degree 22; $\mathrm{Aut}(\mathrm{M}_{12})$ of degree 24; PSL(2, q) of degree $2(q+1)$ for $q \equiv 1 \pmod 4$; PSL(m, q) of degree $2 \times \frac{q^m-1}{q-1}$ for $m \geq 3$ odd and q an odd prime; PSL(m, q) of degree $2 \times \frac{q^m-1}{q-1}$ for $m \geq 4$ even and q an odd rime, and PSL(3,2) of degree 14 [1].

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The localization number of outerplanar graphs

Riana Roux, *Stellenbosch University*

SAMS; Subject Classification Number: 6

The localization game is played on a graph by two players: a Cop with a team of k cops, and a Robber. The game is initialised by the Robber choosing a vertex r, unknown to the Cop. Thereafter, the game proceeds turn based. At the start of each turn, the Cop probes k vertices and in return receives a distance vector. If the Cop can determine the exact location of r from the vector, the Robber is located and the Cop wins. Otherwise, the Robber is allowed to either stay at r, or move to a vertex in the neighbourhood of r. Hereafter the Cop again probes k vertices. The game continues in this fashion, where the Cop wins if the Robber can be located in a finite number of turns. The localization number $\zeta(G)$, is defined as the least positive integer k for which the Cop has a winning strategy for the graph G, irrespective of the moves of the Robber.

The localization number of outerplanar graphs have been showed to be either 1 or 2. During this talk we will look at which classes of outerplanar graphs have localization number equal to two.

Effects of rotational modulation on Rayleigh-Bénard convection in ethylene glycol based hybrid nanofluids with internal heating

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SAMS; Subject Classification Number: 23

The effect of rotational modulation on Rayleigh-Bénard convection in ethylene glycol-based hybrid nanofluids with internal heating is studied analytically. The linear and weakly nonlinear stability analysis of the hybrid nanofluid is performed for a single-phase model. The perturbation method is used to obtain the nonautonomous Ginzburg-Landau equation. The Nusselt number expression is obtained using the Ginzburg-Landau solution. The linear stability study indicates that the critical Rayleigh number obtained in the case of hybrid nanofluid is less than the value obtained in the case of mono nanofluid. This accelerates the onset of convection in a hybrid nanofluid. The study further shows that hybridizing the fluid result in an increase of friction coefficient. It is also found that multi nanofluid ethylene glycol-alumina-copper almost doubles the rate of heat transfer as compared to the mono nanofluid in ethylene glycol-alumina, leading to the conclusion that hybrid nanofluids facilitate heat transfer better than the mono nanofluid. Further, the effects of other fluid parameters on heat transfer are presented graphically. The low amplitude of modulation and low modulation frequencies are considered. We find among other results that the amplitude of modulation enhances the heat transfer and the frequency of modulation reduces the rate of heat mass transfer in the hybrid system.

Evaluation of heat irreversiblity in convective flow of reactive third-grade fluid subjected to newtonian cooling

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L Rundora*, *University of Limpopo*
KF Thosago, *University of Limpopo*

SAMS; Subject Classification Number: 20

In this study, the thermal stability and inherent irreversibility analysis for combustible third-grade fluid through a non-Darcian porous medium is examined. The heated fluid flow is assumed to be flowing steadily through parallel Riga plates that are exposed to asymmetrical convective cooling. Neglecting the reactant consumption, the governing equations are formulated and converted to coupled nonlinear Boundary-value problem by dimensional analysis. The dimensionless equations are solved numerically by Shooting-Runge-Kutta method and validated with spectral collocation method. Graphical and tabular illustrations are provided with sufficient explanations for the velocity, temperature, entropy generation and irreversibility ratio profiles.

Some results on reduced designs invariant under finite simple groups

Amin Saeidi, *University of Limpopo*

SAMS; Subject Classification Number: 15

Key-Moori methods 1 and 2 are useful tools to construct 1-designs from finite groups (see [1]). In recent years, many authors have applied these methods to construct 1-designs invariant under finite simple groups. In 2021, Moori in [2] introduced a third method to construct designs from the fixed points of permutation groups. In all these three methods, G is a permutation group acting on the cosets of a given subgroup H. We are mainly interested in the case when G is simple and the action is primitive. Given a simple group G and a maximal subgroup H, we call a design constructed by Method k ($1 \leq k \leq 3$) a $\mathcal{M}_k$- design.

Now let $\mathcal{D}$ be a 1-(v, k, λ) design, and let I_x be the intersection of all blocks of $\mathcal{D}$ containing the point x. In [3], Le and Moori studied the set I_x and defined a 1-$(\frac{v}{|I_x|}, \frac{k}{|I_x|}, \lambda)$ design $\mathcal{D}_I$ called the reduced design of $\mathcal{D}$. The main objective of defining reduced designs is to study the automorphism groups of $\mathcal{M}_2$-designs in general. In this talk, we show that there is a connection between the $\mathcal{M}_3$-designs and the reduced $\mathcal{M}_2$-designs. We proved in [4] that the parameters of $\mathcal{M}_2$-designs can be obtained directly using the parameters of $\mathcal{M}_3$-designs. Here we show that the converse holds for reduced designs. Indeed, we give a explicit formula to find the parameters of the $\mathcal{M}_3$-designs in terms of the parameters of reduced $\mathcal{M}_2$-designs.

As an application, we compute the reduced $\mathcal{M}_2$-designs of two families of finite simple groups, Suzuki groups and PSL$(2, q)$ (q even) and the corresponding $\mathcal{M}_3$-designs. The $\mathcal{M}_2$-designs corresponding to these groups have been computed by the Moori and the speaker in [5,6].

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Probabilistic weak solutions for stochastic Navier-Stokes variational inequalities

Mamadou Sango, *University of Pretoria*

SAMS; Subject Classification Number: 3, 26

In this talk we present an existence result for probabilistic weak (martingale) solutions for stochastic Navier-Stokes variational inequalities involving unilateral boundary conditions and nonlinear forcings driven by Wiener processes. The setting of the problem follows.

Let D be a simply connected domain bounded in $\mathbb{R}^3$ with a sufficiently smooth boundary ∂D (at least C^2). We fix a final time $T > 0$ and denote by Q_T the cylindrical domain $(0,T) \times D$**.** Given a convex, lower semicontinuous function

$$\varphi : \mathbb{R} \times \partial D \to (-\infty, \infty],$$

and letting $u_n = u \cdot n$ be the normal component of u; n being the outward unit normal vector field to ∂D, we are interested in an hydrodynamical problem for the motion of a fluid under random fluctuations governed by the following subdifferential initial boundary value problem for the incompressible stochastic Navier-Stokes equations

$$du + (\nabla \times u \times u - \nu \Delta u + \nabla P)\, dt = f(t,u)\, dt + g(t,u)\, dW \text{ in } Q_T, \tag{6}$$

$$\nabla \cdot u = 0 \text{ in } Q_T, \tag{7}$$

$$u_\tau = 0 \text{ on } (0,T) \times \partial D, \tag{8}$$

$$u(0) = u_0 \text{ in } D, \tag{9}$$

$$P(t,x) \in \partial \varphi (u_n(t,x), x), \text{ for all } (t,x) \in (0,T) \times \partial D, \tag{10}$$

where u is the velocity of the particles of fluid, ∇ denotes the gradient operator in $\mathbb{R}^3$, $u_\tau = u - (u \cdot n)\, n$ is the tangential component of u, $P = p + |u|^2/2$, the total or Bernouli's pressure (p being the usual pressure), W a l-dimensional Wiener process and the right hand-side of (6) represents the force acting on the fluid and consisting of a regular part involving the $3-$vector function f and a chaotic part involving the $3 \times l$ matrix function g and W, the parameter ν is the viscosity of the fluid, $\partial \varphi$ denotes the subdifferential of φ, namely

$$\partial \varphi (v,x) = \{ w \in \mathbb{R} : \varphi(\tilde{v}, x) - \varphi(v,x) \geq w(\tilde{v} - v), \text{ for all } \tilde{v} \in \mathbb{R} \}.$$

The study of stochastic models of fluid dynamics motivated by some aspects of the theory of turbulence in fluids has led to an impressive amount of important works. So far the main focus has been on systems of equations (stochastic Navier-Stokes, magnetohydrodynamic, and their non-newtonian counterparts). Our work undertakes to initiate the study of similar models driven by unilateral boundary conditions which lead to stochastic variational inequalities. The deterministic counterpart has been developed since the 1960's by Lions, Brezis, Stampacchia, Bensoussan and their collaborators. The main result of this research is a stochastic counterpart of the work of Brezis on deterministic Navier-Stokes variational inequalities and generalizes several previous results on Stochastic Navier-Stokes equations to stochastic Navier-Stokes variational inequalities with unilateral boundary conditions.

Lie group analysis of the potential Kortweg-de Vries equation

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SAMS; Subject Classification Number: 3

In this talk, we perform Lie group analysis [1–5] on a third-order non-linear partial differential equation, namely potential Kortweg-de Vries equation [6]. We first compute Lie symmetries and then perform symmetry reductions on it. Thereafter, we use the direct integration method to obtain its travelling wave solutions. Furthermore, conserved vectors for this equation are derived using the multiplier method [7] and the theorem due to Ibragimov [8].

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A generalization of Petersen's matching theorem

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SAMS; Subject Classification Number: 6

One of the earliest results in graph theory is Petersen's matching theorem from 1891 which states that every bridgeless cubic graph contains a perfect matching. Since the vertex-connectivity and edge-connectivity in a connected cubic graph are equal, Petersen's theorem can be stated as follows: If G is a 2-connected 3-regular graph of order n, then $\alpha'(G) = \frac{1}{2}n$, where $\alpha'(G)$ denotes the matching number of G. We generalize Petersen's theorem and prove that for $k \geq 3$ an odd integer, if G is a 2-connected k-regular graph of order n, then $\alpha'(G) \geq \left(\frac{k^2+k+6}{k^2+2k+3}\right) \times \frac{n}{2}$. The case when k is even behaves differently. In this case, for $k \geq 4$ even, if G is a 2-connected k-regular graph of order n, then $\alpha'(G) \geq \left(\frac{k^2+4}{k^2+k+2}\right) \times \frac{n}{2}$. For all $k \geq 3$, if G is a 2-connected non-regular graph of order n and maximum degree k, then we show that $\alpha'(G) \geq \frac{2n}{k+2}$. In all the above bounds, the extremal graphs are characterized.

On members of Lucas sequences which are products of factorials

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F. Luca, *University of the Witwatersrand*

SAMS; Subject Classification Number: 22

We determine upper bounds on n when the nth term of a Lucas sequence is expressible as a product of factorials. As our starting point we use an adaptation of the proof of the Primitive Divisor Theorem. Firstly, a series of lemmas are proven as a means to establishing an upper bound on n. Some of these proofs exploit sieve methods, the properties of cyclotomic polynomials, and results related to linear forms in logarithms, among others. In fact, we show that if $\{U_n\}_{n\geq 0}$ is a Lucas sequence, then the largest n such that $|U_n| = m_1!m_2!\cdots m_k!$ with $1 \leq m_1 \leq m_2 \leq \cdots \leq m_k$, satisfies $n < 62000$. When the roots of the Lucas sequence are real, we have $n \in \{1, 2, 3, 4, 6, 12\}$. As a consequence, we formulate and prove a corollary regarding the X- coordinates of Pell equations which are products of factorials. We show that if $\{X_n\}_{n\geq 1}$ is the sequence of X- coordinates of a Pell equation $X^2 - dY^2 = \pm 1$ with a nonsquare integer $d > 1$, then $X_n = m!$ implies $n = 1$.

Earthquake induced oscillations of high-rise vertical structures

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SAMS; Subject Classification Number: 23

The effect of earthquake induced oscillations on vertical structures such as buildings, masts and chimneys is of considerable interest to structural engineers as well as mathematicians. The relevant structure should be able to withstand these oscillations in order to prevent significant damage to the structure itself, as well as non-structural components.

Reliable mathematical models and data are needed to determine the effect of oscillations on buildings. Beam models, such as the Euler-Bernoulli and Timoshenko beam models, have been extensively used to simulate motion of structures such as high-rise buildings.

A motivation for the use of the Timoshenko model as well as a comparison to other models was done in [2]. In [1], the Timoshenko beam model is used to approximate the dynamic linear elastic behaviour of buildings, but includes so called soil-structure interaction. In this talk the focus will be on high-rise buildings subjected to earthquake induced oscillations, modelled by an adapted Timoshenko beam model, as opposed to the model used in [1]. The adaptation leads to a new and more realistic model for soil-structure interaction, where the foundation of the structure is modelled as a separate entity to the structure. The adapted model will be introduced and modal analysis will be done. Some numerical results will be included.

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Decision support for the integrated vehicle crew rostering problem

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SAMS; Subject Classification: 23

A prominent public transport company which provides a passenger transport service in the form of daily *timetabled bus trips* in and around the city of Cape Town currently performs the assignment of vehicles and crew members to these trips manually. This assignment process is characterised by the solution of three different combinatorial optimisation problems, namely the *vehicle scheduling problem* (VSP), which entails the assignment of vehicles to trips over a scheduling period, the *crew scheduling problem* (CSP), which entails the assignment of crew members to trips over a scheduling period, and the *crew rostering problem* (CRP), which entails the assignment of crew members to trips over a rostering period (whereas a scheduling period typically spans one day, a rostering period typically spans several days). The aforementioned sub-problems are traditionally solved sequentially due to their considerable individual computational complexities. The public transport company in question has, however, launched a research project in which the objective is to develop a method for solving all of these sub-problems simultaneously (known in literature as the *integrated vehicle crew rostering problem*) while attempting to minimise a cost function. Results obtained when adopting the aforementioned solution method are presented and costs are compared with the *status quo* in the context of real timetabled trips data obtained from the transport company.

The Jacobson property in Banach algebras

A Swartz*, *University of Johannesburg*
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SAMS; Subject Classification Number: 11

In this talk A will represent a non commutative and unital Banach algebra. We let $\sigma(a)$ represent the usual spectrum of $a \in A$. It is well known that for $a, b \in A$ we have

$$\lambda \in \sigma(ab) \setminus \{0\} \iff \lambda \in \sigma(ba) \setminus \{0\}.$$

We are interested in subsets of A that have the *Jacobson Property*, i.e. $X \subset A$ such that for $a, b \in A$:

$$1 - ab \in X \iff 1 - ba \in X.$$

Exploring the impact of an undergraduate mathematics intervention at UWC

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University of the Western Cape

SAMS; Subject Classification Number: 16

Prior to 2018, the Natural Sciences Faculty at the University of the Western Cape had experienced an extended time to degree amongst their students doing mainstream mathematics. The cohorts that are enrolled for at least up to second year mathematics (MAT211) include students who are studying towards a BSc degree in either Mathematics and Statistical Science, Physics, Computer Science or Chemistry. In order to assist these students and understand why most students are not graduating in time, the Faculty introduced an intervention directed towards second year mathematics. The reason why this project was directed to second years is that the Faculty identified a problem with the transition from first year mathematics to second year mathematics. The intervention, later named the 'turnaround project', commenced in 2018 with a weeklong bootcamp for MAT211 students at the beginning of the year before the official academic year began. The methods used in the turnaround project to improve students' academic performances included "interactive engagement in lectures, additional seminars, workshops, tutor consultations, and additional training sessions".

In this research the main objective is to determine whether this intervention had an impact on the students' performance and whether it increased the number of in time graduates. Other factors influencing academic performance will be shown. Secondary data extracted from the university system is used to enrich the analysis. Data collected in 2018 is compared to data collected prior to the intervention. Summary statistics and visualisation is used to profile student performance before and after the intervention. Preliminary results indicate that the intervention was successful. Statistical modelling will be used to understand the factors involved in the improved success.

Branching process applied to partial differential equations

Enrique Thomann*, **Radu Dascaliuc**, and **Ed Waymire**, *Oregon State University*
Tuan Pham, *Eastern Oregon University*

SAMS; Subject Classification Number: 3, 26

A survey of recent results obtained in joint work with Radu Dascaliuc, Tuan Pham and Ed Waymire, relating solutions of the Navier Stokes equations of fluid mechanics to a stochastic branching process naturally associated to these equations will be presented. While the equations themselves are deterministic, the scaling properties of the equations suggest a branching structure of the Fourier transform of the equations. First noted by LeJan and Sznitman in 1997, the branching structure provides a representation of the solution provided this branching process is non explosive. Open problems remain, including the Clay foundation Millennium challenge, but simpler questions related to the structure of the branching process will be highlighted.

COVID-19 disease and its economic impact – A mathematical model

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SAMS; Subject Classification Number: 10, 23, 25

The rapid global spread of coronavirus disease in 2019 (COVID-19) which is a severe acute respiratory infection, has become a major public health crisis and a calamity to global economy. In this paper, a compartmental deterministic model of COVID-19 disease transmission dynamics, its economic impact and appropriate control strategies are proposed and analysed both qualitatively (using the stability theory of differential equations, Pontraygin's maximum principle and cost effectiveness analysis) and numerically via variation iteration method (VIM) and 4th order Runge-Kutta integration scheme. Obtained pertinent results are displayed graphically and discussed quantitatively.

On ideals of rings of continuous integer-valued functions on a frame

Batsile Tlharesakgosi, *University of South Africa*

SAMS; Subject Classification Number: 2

Let L be a zero-dimensional frame and $\mathfrak{Z}L$ be the ring of integer-valued continuous functions on L. We associate with each sublocale of ζL, the Banaschewski compactification of L, an ideal of $\mathfrak{Z}L$, and show the behaviour of these types of ideals. The socle of $\mathfrak{Z}L$ is shown to be always the zero ideal, in contrast with the fact that the socle of the ring $\mathcal{R}L$ of continuous real-valued functions on L is not necessarily the zero ideal. The ring $\mathfrak{Z}L$ has been shown by B. Banaschewski to be (isomorphic to) a subring of $\mathcal{R}L$, so that the ideals of the larger ring can be contracted to the smaller one. We show that the contraction of the socle of $\mathcal{R}L$ to $\mathfrak{Z}L$ is the ideal of $\mathfrak{Z}L$ associated with the join (in the coframe of sublocales of ζL) of all nowhere dense sublocales of ζL. It also appears in other guises.

Dual solutions and temporal stability analysis for nanofluid flow past a heated shrinking surface

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SAMS; Subject Classification Number: 10, 20, 23

The advent of nanotechnology together with one of its offspring known as nanofluid has revolutionized the thermal management, efficiency, reliability, and performance of various engineering and industrial systems. In this paper, the effects of magnetic field, surface slipperiness, thermophoresis, and Brownian motion on stagnation point flow of an incompressible nanofluid towards a convectively heated shrinking surface is theoretically examined. Under the flow assumptions and similarity transformation, the balanced governing equations of continuity, momentum, energy, and nanoparticles concentration transport are obtained. Shooting numerical technique coupled with the Runge-Kutta-Fehlberg integration scheme are employed to tackle the model problem. The existence of dual solutions in the specific range of shrinking surface parameter is found. Temporal stability analysis is performed, and upper solution branch is found to be a stable and physically realistic solution to the problem. Pertinent results depicting the effects of various emerging thermophysical parameters on nanofluid velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number are obtained graphically and discussed.

On the construction of noetherian forms for algebraic structures

Francois van Niekerk, *Stellenbosch University*

SAMS; Subject Classification Number: 4

A noetherian form is a self-dual axiomatic context in which the Noether isomorphism theorems can be established. A noetherian form consists of objects, morphisms between objects and subobject lattices for each object. By design, any group-like variety together with its homomorphisms and subalgebra lattices is a noetherian form. In this talk we will show that even though a non-group-like variety together with its homomorphisms and subalgebra lattices is not a noetherian form, the subalgebra lattices can be suitably extended such that we have a noetherian form. The construction of these noetherian forms over a variety can be described categorically. The main result in this talk, is a characterization of when this more general categorical construction results noetherian form.

A model framework for optimising multi-period cross-functional team selection in terms of anticipated performance

Van Zyl Venter* **and Jan van Vuuren**
Department of Industrial Engineering, Stellenbosch University

SAMS; Subject Classification: 23

Team formation has become an integral part of modern-day organisations and the procedure for selecting appropriate candidates to fulfil specific roles in a team can become complex. This process of *cross-functional team selection* (CFTS), where members from different departments in an organisation are to be selected to form a team whose members are required to work together to complete a project, is further complicated if team compositional decisions have to be made at multiple points in time over the lifetime of a project. Predicting the timing of performance peaks anticipated for available team candidates over the entire lifetime of the project may therefore prove valuable when composing the initial project team.

A model framework is proposed in this presentation in support of effective CFTS over multiple time periods within the lifetime of the project. The efficacy of the model framework is demonstrated in a case study involving team selection for the popular *Fantasy Premier League* (FPL) 2020/2021 soccer season, which is based on the well-known *English Premier League* (EPL). The case study takes the form of a comparison of the weekly cross-functional FPL teams recommended by the framework with the teams actually selected by top-ranked players participating in the 2020/2021 FPL season.

The energy and number of indepdependent subsets under degree restriction

Mr Sinoxolo Xhanti* and **Dr Eric O.D. Andriantiana**, *Rhodes University*

SAMS; Subject Classification Number: 6

Let G be a graph with n vertices. The energy $E_n(G)$ of G is defined as the sum of the absolute values of its eigenvalues. The Hosoya index $Z(G)$ of G is the number of independent edge subsets of G. And the Merrifield-Simmons index $\sigma(G)$ is the number of independent vertex subsets of G, including the empty set. The three above-mentioned closely related graph invariants are studied in mathematical chemistry.

The talk will be discussing extremal caterpillars, relative to the Energy, Hosoya index and Merrifield-Simmons index. Since the Energy and Hosoya index can be redefined in terms of an auxiliary invariant, $M(G,x) = \sum_{k\geq 0} m(G,k)x^k$, where $m(G,k)$ is the number of independent edge subsets of order k in G, we focus on finding extremal caterpillars, relative to $M(.,x)$. We use similar methods as with $M(.,x)$ to conclude on extremal caterpillars, relative to the Merrifield-Simmons index.

Optimisation for decision support in retail inventory management

Jurie Zietsman* and **Jan van Vuuren**,
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SAMS; Subject Classification: 25, 26

Globalisation and the growth of e-commerce have led to retail companies having to manage and control larger and larger numbers of *stock keeping units* (SKUs). The success of any retail company depends on how well it can satisfy demand while remaining financially viable. Inventory management systems are typically aimed at balancing the conflicting objectives of achieving good customer service levels and minimising inventory and operating costs. As the number of SKUs a company holds increases, however, so too does the complexity of this balancing problem. The main decisions in respect of SKU inventory management in a retail warehouse, which affect this balance, are related to (1) which SKUs need to be replenished, (2) when to place replenishment orders for these SKUs, and (3) the appropriate volumes of SKUs to include in these orders.

In this presentation, a generic framework is proposed for the development of decision support systems in the context of SKU inventory replenishment in a retail warehouse. This includes a time series forecasting approach which utilises supplementary data on customer demand and generates forecasts of demand distributions. It also includes an inventory replenishment model according to which orders for SKUs may be placed at discrete, equi-temporal points in time. The objective is to batch SKU replenishment orders together, while accounting for lead times, minimum order quantities and backlogged orders. Model performance is evaluated on the key performance indicators of customer service level and cost, which form the typical trade-off in inventory management. The model is solved exactly. The functionality of an instantiation of this framework is presented in the context of a case study involving real demand data in the South Africa retail sector.

Asymptotics of the eigenvalues of fourth order boundary value problems

Bertin Zinsou, *School of Mathematics, University of the Witwatersrand*

SAMS; Subject Classification Number: 30

Fourth order problems with the differential equation $y^{(4)} - (gy')' = \lambda^2 y$, where $g \in C^1[0, a]$ and $a > 0$, occur in engineering on stability of elastic rods. They occur as well in aeronautics to describe the stability of a flexible missile. Fourth order Birkhoff regular problems with the differential equation $y^{(4)} - (gy')' = \lambda^2 y$ and eigenvalue dependent boundary conditions are considered. These problems have quadratic operator representations with non self-adjoint operators. The first four terms of the asymptotics of the eigenvalues of the problems are evaluated explicitly.

On de Finetti's theorems

Paola Zurlo*, *Università degli studi di Bari "Aldo Moro", Italy*

SAMS; Subject Classification Number: 24

The investigation of distributional symmetries was initiated by de Finetti's celebrated theorem, which shows that any finite joint distribution of sequences of two-point valued exchangeable random variables is obtained by randomization of the binomial distribution. This result has since found several generalizations both in classical and noncommutative settings. Also motivated by the key role played in physics by the CAR algebra, we carry out a careful study of the (minimal) infinite graded tensor product of a given C^*-algebra with itself, which is acted upon in a natural way by the group of finite permutations. Invariant states for this action turn out to be automatically even and extreme invariant states are characterized as infinite products of a single even state on the C^*-algebra. As a consequence, the extreme symmetric states of the (minimal) graded tensor product are sufficiently many to separate its points, allowing us to prove weak ergodicity of the permutation action. Finally, a version of de Finetti's theorem for graded processes is established, for in this case invariant states correspond to exchangeable quantum stochastic processes.

The talk is based on joint work with V. Crismale and S. Rossi.

On convex structures in quasi metric spaces

Mcedisi Sphiwe Zweni* and **Olela Olivier Otafudu**
North-West University

SAMS; Subject Classification Number: 33

Künzi and Yilzid introduced the concept of convexity structures in the sense of Takahashi in quasi-pseudometric spaces. In this talk, we continue the study of this theory, introducing the concept of W-convexity for real-valued pair of functions defined on an asymmetrically normed real vector space. Moreover, we show that all minimal pairs of functions defined on an asymmetrically normed real vector space equipped with a convex structure which is W-convex is translation-invariant.

Extreme point methods in the study of isometries on certain noncommutative spaces

Pierre de Jager*, *University of South Africa*
Jurie Conradie, *University of Cape Town*

SAMS; Subject Classification Number: 11

Characterizations of the extreme points (of the unit balls) of various Banach function spaces have often proved useful in obtaining structural descriptions of isometries on those spaces (see for example, [1] and [2]). In this talk we look at how this technique can be employed in the setting of the noncommutative space $L^1 + L^\infty(\mathcal{A})$, where $\mathcal{A}$ is a semi-finite von Neumann algebra.

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Configurational peridynamics

Andie de Villiers*, *Stellenbosch University*,
Ali Javili, *Bilkent University*,
Paul Steinmann, *University of Erlangen-Nuremberg*,
Andrew McBride, *University of Glasgow*

SAMS; Subject Classification Number: 23

Peridynamics (PD) is a non-local continuum mechanics theory established by Stewart Silling [1]. PD was originally developed to overcome challenges encountered in the classical continuum mechanics formulation when discontinuities such as cracks are present. PD is a continuum formulation characterised by material points associated with a finite volume. The material points that lay within a certain cut-off radius influence each other. The resulting governing equation of PD is an integro-differential equation, thereby side-stepping issues with evaluating a spatial derivative in the presence of a discontinuity. PD have evolved over the years and there are different approaches to overcome some of the initial shortcomings of the theory. Continuum-kinematics inspired peridynamics (CPD) uses basic kinematic entities from CCM by including deformation measures for line, area and volume of neighbouring points in the determination of material behaviour [2]. CPD is variationally consistent and the internal energy of the system is decomposed into parts due to one-neighbour, two-neighbour, and three-neighbour interactions.

Configurational mechanics is concerned with energetic changes that accompanies a variation of the material configuration. In contrast to deformational mechanics where the stationary point of a spatial variation of the total potential energy yields equations for equilibrium, the material variation of the total potential energy employed in configurational mechanics does not in general renders a stationary point but rather defines a change of potential energy. If this release of energy is sufficient, it can be dissipated in the creation of new crack surfaces [3]. Configurational PD has the potential to offer a novel and energy driven model for damage and fracture that is non-local and fully non-linear. This contribution will present the theoretical framework for configurational PD and configurational CPD along with numerical examples that illustrate what configurational forces can tell us.

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Large displacements and rotations for a local linear elastic rod

S du Toit*, *University of Pretoria*
M Labuschagne, *University of Pretoria*
A J van der Merwe, *Auckland University of Technology*

SAMS; Subject Classification Number: 20

The Local Linear Timoshenko (LLT) model for the planar motion of a rod that undergoes flexure, shear and extension, was recently derived in a paper by Van Rensburg et al. In this presentation an algorithm developed for this model is considered. The algorithm is based on the mixed finite element method, and projections into finite dimensional subspaces are used for dealing with nonlinear forces and moments. The algorithm is used for an investigation into elastic waves propagated in the LLT rod. Interesting properties of the LLT rod include the increased propagation speed of elastic waves when compared to the linear Timoshenko beam, and the appearance of buckled states or equilibrium solutions for compressed LLT beams. It is also shown that the LLT rod is applicable to a wide range of slender elastic objects; from beams to highly slender flexible rods.

Equivalence after extension and Schur coupling for Fredholm operators

Sanne ter Horst, *School of Mathematical and Statistical Sciences, North-West University*

SAMS; Subject Classification Number: 11

The relations between two bounded linear Banach space operators known as Equivalence After Extension (EAE) and Schur Coupling (SC) originated from the study of holomorphic operator functions and integral operators and provide information about the relative spectral properties of the operators near the origin, e.g., Fredholm properties, level of compactness, etc. In the application of these relations it is essential that they coincide, i.e., the operators in question are EAE if and only if they are SC. It was observed in the 1990s that at the level of bounded linear Banach space operators SC implies EAE, while the converse implication was posed as an open question that was only resolved in the negative in 2019 by considering Fredholm operators on Banach spaces that are essentially incomparable. In this talk we investigate the question whether EAE and SC coincide for Fredholm operators without the essential incomparability assumption. Surprisingly, some of the observations from the case of essentially incomparable Banach spaces remain in the general case, while, on the other hand, we show that the results for essentially incomparable Banach spaces do not extend in general when weakening the assumption to projective incomparability.

The talk is based on joint work with N.J. Laustsen.

Local linear Timoshenko rod

NFJ van Rensburg*, *University of Pretoria*
S du Toit, *University of Pretoria*
M Labuschagne, *University of Pretoria*

SAMS; Subject Classification Number: 20

In this presentation a mathematical model for large planar motion of elastic rods which undergo flexure, shear and extension but not torsion is derived. (We use the term rod for one-dimensional solids, i.e. beams, cables, ropes, hoses. etc.) Since the Timoshenko theory provides an excellent approximation for three-dimensional elastic behaviour with plane stress, we adapted the constitutive equations for application to large rotations to complete the model, which we call the the Local Linear Timoshenko rod (LLT) model. We demonstrated that this model serves as a framework for a class of simpler mathematical models for slender solids in various applications with the advantage that the more general model can be used to evaluate and compare the simpler models.

Each mathematical model in this presentation consists of a system of nonlinear partial differential equations.

Square roots of H-nonnegative matrices

M van Straaten*, **DB Janse van Rensburg**, and **F Theron**, *North-West University*
C Trunk, *Technische Universität Ilmenau, Germany*

SAMS; Subject Classification Number: 11

Consider an $n \times n$ matrix B in the indefinite inner product space generated by an invertible Hermitian matrix H. The matrix B is called H-nonnegative if the indefinite inner product $[Bx, x]$ is nonnegative for all $x \in \mathbf{C}^n$. Then B has a simple structure as B is also H-selfadjoint, B has only real eigenvalues and the Jordan normal form of B is almost diagonal since all of the Jordan blocks have size 1 except for the Jordan blocks corresponding to the zero eigenvalue which have size at most 2.

We will take a look at conditions for the existence of square roots (with and without structure) of H-nonnegative matrices as well as the description of the roots in the nilpotent case.

A generic framework for aspect-based sentiment analysis

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SAMS; Subject Classification: 17, 26

With the increasing volume and complexity of user-generated content shared *via* the Internet, the need has arisen for automated methods capable of extracting meaningful insights from unstructured text data. Sentiment analysis is a form of text analysis involving the process of computationally identifying the polarity of an opinion expressed by an author of a given piece of text. While much of the existing work in this field focusses on document-level or sentence-level analysis, in which an entire document or sentence, respectively, is viewed as a single information unit and is assumed to contain a maximum of one expression of sentiment, *aspect-based sentiment analysis* involves a more fine-grained approach, facilitating the discovery of any number of topics, and the sentiment polarities towards these topics, present in a document of text data [1]. The most promising approaches to aspect-based sentiment analysis to date are those based on supervised machine learning. Many of the methodologies in the literature are, however, focussed on the application of specific machine learning models, on only specific sub-tasks of the problem, or on a specific domain of application.

In this presentation, a generic framework for aspect-based sentiment analysis is proposed, the aim of which is to guide a user through the process of gaining insights from an unstructured text data set from any domain, and by utilising any appropriate machine learning models. The generality of the framework is demonstrated through its application to benchmark data sets from the literature, as well as on a case study involving a South African retail bank.

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Convex invertible cones and Nevanlinna-Pick interpolation: The suboptimal case

Alma van der Merwe*, *University of the Witwatersrand*
Sanne ter Horst, *North-West University*

SAMS; Subject Classification Number: 11

Nevanlinna-Pick interpolation developed from a topic in classical complex analysis to a useful tool for solving various problems in control theory and electrical engineering. Over the years many extensions of the original problem were considered, including extensions to different function spaces, nonstationary problems, several variable settings and interpolation with matrix and operator points. We discuss a variation on Nevanlinna-Pick interpolation for positive real odd functions evaluated in real matrix points. This problem was studied by Cohen and Lewkowicz using convex invertible cones and the Lyapunov order, but was never fully resolved. We present a solution to this problem in a special case that we refer to as 'suboptimal' based on connections with the classical case. The solution requires a representation of linear matrix maps going back to R.D. Hill and an analysis of when positive linear matrix maps are completely positive, on which we reported in earlier work.

CONTRIBUTED POSTERS

Investigating the colouring space of a graph

Louise Beyers, *Stellenbosch University*

SAMS; Subject Classification Number: 6

How many non-equivalent k-colourings of a graph are there? A framework is developed which describes the solution space of this question for three different solution representations: k-colourings, k-partitions and partition classes of a graph. The framework exploits relationships between these three solution representations so that information obtained about one representation may be used to obtain information about another. Partition classes are defined and integrated into the existing theory of the framework. An algorithm is developed and implemented to find partitions of odd cycles and another algorithm is developed and implemented to obtain partition classes from those partitions. The partition classes found are used to formulate conjectures which apply to general odd cycles.

A bio-economic framework for modelling the impact of pest outbreaks on agricultural commodities

Fuzail Dawood* and **Jan van Vuuren**
Department of Industrial Engineering, Stellenbosch University

SAMS; Subject Classification: 23Number

As the world's population continues to soar, there is an urgent need to increase the capacity of agricultural production, as well as to decrease food losses and waste globally. To this end, the second *Sustainable Development Goal* (SDG) identified by the *United Nations* (UN) seeks to "End hunger, achieve food security and improved nutrition, and promote sustainable agriculture." A fundamental requirement to achieving this goal is enriching farmer knowledge with respect to the latest scientific findings related to sustainable land, water, and pest management with a view to ensure resilient food producing systems. As of 2019, the *Food and Agriculture Organisation* (FAO) of the UN estimates that as much as 40% of the world's agricultural crop production is lost annually to pest invasions, costing the global economy approximately $70 billion per annum, and contributing to the estimated 2.37 billion people suffering from food insecurity.

Several studies have been dedicated to model the spatio-temporal spread of pests, or to economic impact assessments at field or market level aimed at estimating the losses incurred due to pest outbreaks. Few quantitative studies, however, have integrated biological spread models with economic impact assessments. For this reason, quantifying the economic impact of pest outbreaks on crops represents one of the most important research questions for agricultural simulation modelling.

The primary aim in this poster is to describe the design and to demonstrate the practical workability of a generic bio-economic framework capable of modelling the spatio-temporal spread of pest species outbreaks on a spatial extent primarily utilised for cultivating agricultural commodities. The framework explores fundamental scientific and mathematical approaches within the domains of GIS spatial analysis, machine learning, simulation, and economic modelling in order to quantify the impact of pest outbreaks as accurately as possible.

A framework for foreign exchange algorithmic trading

Spencer de Wit* and **Jan van Vuuren**
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SAMS; Subject Classification: 10, 26

The term *financial market* refers broadly to any marketplace in which the trading of securities occurs. These markets provide an opportunity for realising profitable returns, although, it is well known that humans are unable to trade efficiently in these markets due to inherent limitations of the human brain. This problem has led to the rise of algorithmic trading and the implementation of machine learning in pursuit of more consistent returns. *Reinforcement learning* (RL) is a class of machine learning algorithms that has recently achieved notable results when implemented in a trading environment. A framework for evaluating RL algorithmic trading strategies, tailored to the Forex market is presented in this poster. The framework is partitioned into different stages, most notably, a pre-processing phase of market data, an implementation phase of trading strategies in an RL context, and back-testing and optimisation phases of applying the trained RL models. The proposed framework is then utilised to establish an optimised algorithmic trading model which forms the basis of a flexible *decision support system* (DSS) capable of identifying a suitable algorithmic trading strategy for each of a specified set of market conditions.

An interactive MATLAB® live script textbook for pre-graduate engineering students at Universities of Technology

M Ellis Erasmus, *Central University of Technology, Bloemfontein, Free State*

SAMS; Subject Classification Number: 16

The Central University of Technology (CUT) is in possession of a MATLAB® total academic headcount (TAC) license, enabling students to download the MATLAB® and Simulink software. This presents a teaching intervention not usually associated with engineering mathematics at universities of technology. Students can interact with MATLAB® software on three platforms, the Command window (plain coding), Live script editor and Simulink of which the last two were found to be best suited to the blending of coding and traditional engineering mathematics especially when face-to-face lectures are compromised. MATLAB® live script allows hands on interactive coding together with traditional subject content with instant visualization of solutions. Although Simulink, a graphic programming environment which uses a set of blocks to simulate dynamical systems, is not available on Live script, its uncomplicated and pleasing presentation makes it a student favourite. Simulink is useful when reverse engineering system outputs back to the equations they originated from, an aspect usually not addressed in classic mathematics teaching. Live script manuals for all pre graduate engineering mathematics courses were compiled between 2020 and 2022 with additions made as student interaction with code, theory and Simulink necessitated. All traditional mathematics topics such as vector theory, wave theory, linear algebra, complex numbers, differential calculus, integration, basic statistics, Fourier series, Laplace transform, differential equations and numerical methods are covered in the live script manuals. There are a number of plain code text books demonstrating MATLAB® coding with engineering mathematics but they do not allow interaction and they are not pitched at pre graduate engineering students at universities of technology. The Live script manuals provide a pleasant non-intrusive way to not only make mathematics useful, but they also serve as an introduction to plain coding for when this is required at post graduate level in their respective engineering disciplines.

Modelling blood flow during syringing

Jaimé Goedhals*, **Dr A. de Villiers**, and **Prof. G.J.F. Smit**
Stellenbosch University

SAMS; Subject Classification Number: 23

In low-resource settings, such as in rural hospitals, emergency rapid blood transfusions are often administered via syringing. However, this has been shown to cause significant hemolysis as compared to the use of a pressure bag, which can have negative effects on the patient receiving the transfusion. It is therefore the aim of this project to develop guidelines for performing rapid blood transfusions via syringing to make this option safer and more viable. This will be achieved by using numerical simulations together with various hemolysis models to determine under what flow conditions hemolysis is most likely to occur.

The blood flow is initially modelled as a Newtonian fluid flowing through a sudden contraction, which represents the syringe. The problem is then extended to non-Newtonian flow. Various non-Newtonian models are explored for this application. The problem is solved using the finite element software deal.II.

The matrix taxonomy of finitely complete categories

Michael Hoefnagel, *Stellenbosch University*

SAMS; Subject Classification Number: 4

This poster details some recent work on what together with my collaborators we have termed "Matrix taxonomy", a system (based on matrices of positive integers) for classifying finitely complete categories according to their so-called matrix properties.

A decision support framework for the selection of delivery modes during the last-mile of a retail supply chain

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Christa Searle,
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SAMS; Subject Classification: 25, 26

The last-mile of a retail supply chain in which customers can place online orders for commodities entails delivering parcels from a brick-and-mortar store to end-user customers. This portion of the supply chain poses challenging logistical problems related to the pursuit of acceptable trade-offs between operational cost minimisation (as a result of eliminating last-mile delivery inefficiencies) and minimising the environmental impact of traditional commodity delivery modes (as a result of CO2 emissions and the use of fossil fuels).

The aim in this poster is to showcase the design of a decision support framework for managing the multi-modal last-mile delivery logistics of a retailer. The working of the framework is based on an agent-based simulation-optimisation model which takes as input

- time-stamped commodity demand volumes for a number of customers pertaining to a single retail store,
- time windows during which the aforementioned demand has to be satisfied at customers,
- details about a set of delivery modes available to service customer demand,
- data describing the fleets of available delivery vehicles for each of the aforementioned modes (including fixed and variable costs, rates of CO_2 emission, vehicle speeds, and vehicle capacities), and
- information related to the delivery transport infrastructure and expected travel times within this infrastructure for each of the aforementioned delivery modes.

The delivery modes include trucks, cars, motorcycles, bicycles, and walkers. The model produces as output an assignment of vehicles of the appropriate transportation modes to service the set of customers in pursuit of the objectives mentioned above. The model also suggests together delivery routes and schedules to be followed by each of the delivery vehicles. The poster includes a demonstration of the working of a computerised instantiation of the framework which is capable of analysing the solutions to the agent-based model formulated over a rolling planning horizon (discretised into multiple planning periods) during which both customers and delivery vehicles are modelled as stochastic agents exhibiting autonomous behaviour (in terms of subjective preferences and the perceived value of time). The instantiation includes a virtual, real-time bidding platform for registering and evaluating delivery alternatives and corresponding occasional driver incentives offered.

Jacobian norm regularisation and conditioning in neural ODEs

Shane Josias*, *Applied Mathematics & School for Data Science and Computational Thinking, Stellenbosch University*
Willie Brink, *Applied Mathematics, Stellenbosch University*

A recent line of work regularises the dynamics of neural ordinary differential equations (neural ODEs), in order to reduce the number of function evaluations needed by a numerical ODE solver during training. For instance, in the context of continuous normalising flows, the Frobenius norm of Jacobian matrices are regularised under the hypothesis that complex dynamics relate to an ill-conditioned ODE and require more function evaluations from the solver. Regularising the Jacobian norm also relates to sensitivity analysis in the broader neural network literature, where it is believed that regularised models should be more robust to Gaussian and adversarial perturbations in their input. We investigate the conditioning of neural ODEs under different Jacobian regularisation strategies, in a binary classification setting. Regularising the Jacobian norm indeed reduces the number of function evaluations required, but at a cost to generalisation. Moreover, naively regularising the Jacobian norm can make the ODE system more ill-conditioned, contrary to what is believed in the literature. As an alternative, we regularise the condition number of the Jacobian and observe a lower number of function evaluations without a significant decrease in generalisation performance. We also find that Jacobian regularisation does not guarantee adversarial robustness, but it can lead to larger margin classifiers.

An intraday ensemble trading framework for the Forex market

Coetzee Koegelenberg* and **Jan van Vuuren**
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SAMS; Subject Classification: 10, 26

Price movements in financial markets have been compared to random walks which exhibit chaotic and unpredictable behaviour [2]. Traders continually attempt to overcome this market randomness phenomenon, often unsuccessfully. Reasons for this lack of success include an inability to model market randomness, prevailing market information inefficiencies, the presence of trader cognitive bias, and extreme market events [1, 3].

In this poster, decision support is provided to traders in the form of a holistic *intraday ensemble trading framework* (IETF) focused on the development of a Forex market trading algorithm which reduces human error. The framework is partitioned into three sub-components. The first component is responsible for all data preparation and processing. This framework component is designed to create a logical pipeline of reformatted data of an acceptable structure. The second component of the IETF is geared towards forecasting future market movements in an attempt to identify possible trading windows. The inclusion of multiple heterogeneous forecasting methods (statistical methods or machine learning methods) is emphasised in an attempt to incorporate generalisation capabilities and improve market forecasting capabilities. Lastly, the third component is aimed at providing the user with an intelligible road map for designing and executing a trading strategy based on information gathered in the first two components in an attempt to maximise profitable returns while minimising risk. Component modules responsible for entry strategies, exit strategies, and risk management are illustrated in this poster.

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A note on the representation of BSDE-based dynamic risk measures

L. Mabitsela*, **R Kufakunesu**, **C Guambe**
University of Pretoria

SAMS; Subject Classification Number: 12

We derive a representation for dynamic capital allocation when the underlying asset price process includes extreme random price movements. Moreover, we consider the representation of dynamic risk measures defined under Backward Stochastic Differential Equations (BSDE) with generators that grow quadratic-exponentially in the control variables. Dynamic capital allocation is derived from the differentiability of BSDEs with jumps. The results are illustrated by deriving a capital allocation representation for a dynamic entropic risk measure and a static coherent risk measure.

A model framework for the design and operational management of a wind/solar photovoltaic hybrid power generation system

Désirée Taliane Mpassi Mahinga* and **Jan van Vuuren**
Department of Industrial Engineering, Stellenbosch University

SAMS; Subject Classification: 23

The rapid depletion of non-renewable fuel sources has sparked interest in the development of renewable energy technologies. This interest has led to an increase in the use of *Hybrid Renewable Energy Systems* (HRESs) for power generation purposes. In this poster, a model framework for generating robust quantitative decision support with respect to both the strategic design and operational management of HRESs is proposed. The design aspect of the model is aimed at recommending the number of power generating components included in the hybrid system. The operational management aspect, on the other hand, is concerned with the maintenance and availability of each component of the system during the operational phase, while meeting the load demand for a customer base. The hybrid system is composed of solar photovoltaic panels, wind turbines and a battery bank as an energy storage system. The objective is to minimise the total system cost over a period of twenty years.

Analytical determination of the permeability and specific surface area of open-celled foams with nodal lumps

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SAMS; Subject Classification Number: 12

Numerous studies in the literature have been devoted to the prediction of geometric and transport properties of open-celled foams, such as metal and ceramic foams, including the foam Representative Unit Cell (RUC) model developed at Stellenbosch University. The RUC modelling approach is an attractive modelling method due to the simple rectangular geometry, as well as its satisfying performance in comparison to other models and experimental data as proven elsewhere in the literature for porous media. Foams often contain solid lumps at the intersection of struts, known as nodes. Several authors from the literature have taken these lumps into consideration in their geometric models. In this study the foam RUC model is adjusted in a similar manner. Predictive equations are provided for the permeability and specific surface area prediction for high porosity foams based on the node adjusted RUC model, i.e. the adjusted RUC model where the lumps are incorporated at the nodes of the foam. Both the Darcy and Forchheimer permeability coefficients are presented and the permeability and specific surface area expressions are provided in terms of the porosity and pore diameter. A comparative analysis is furthermore included between the modified foam models found in the literature and the MRUC model. The models are evaluated through comparison with available experimental data in the literature.

Enstrophy in Burgers' Equation

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SAMS; Subject Classification Number: 23

A nonlinear partial differential equation (PDE) called the Burgers equation

$$u_t + uu_x = vu_{xx} \tag{11}$$

can be found in areas of applied mathematics like fluid mechanics or traffic flow. A simplified version of the Navier Stokes equation is thought to be represented by this equation [2]. It preserves the three important components of the Navier-Stokes equation, i.e, the evolution in time, the nonlinearity and parabolic nature, and the presence of the viscosity term. For these reasons, it has drawn considerable interest from the scientific community as it is used as a "toy model" to learn more about the theory of fluid motion through numerical methods [4]. The ν in equation (1) is called the viscosity coefficient, and when $\nu = 0$ then equation (1) is called the inviscid Burgers equation. The solution of the equation forms a discontinuous shock wave or a singularity. When $\nu > 0$, equation (1) is the general form of the viscous Burgers equation and the diffusion term u_{xx} prevents the shock from happening [2].

The enstrophy is defined as

$$E(t) = \int_a^b (u_x(x,t))^2 \, dx. \tag{12}$$

It serves as a condition for determining the uniqueness and regularity of solutions of the Navier-Stokes equation in global time [3]. The best estimates for its growth to date do not rule out the possibility of enstrophy becoming unbounded in finite time, indicating a loss of regularity of solutions [1]. Investigating the sharpness of such bounds, in the context of the Burgers equation, for enstrophy growth is what is of interest. To achieve this, we will numerically maximize the quantity E(t) given a periodic given a periodic initial condition $u(x, 0) = \sin(x)$, and investigate the boundedness of the quantity. It is guaranteed that the solution will be regular in global time as long as this quantity is finite.

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Stability analysis of a virulent code in a network of computers

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SAMS; Subject Classification Number: 23

Network of computers are vulnerable to attack by virulent codes which can halt an organization's activities and in the process resulting in loss of revenue. The need to understand their dynamics is utmost importance and hence there is a need to develop mathematical models. These models will help us understand the impact these virulent codes have on the network of computers. In this article, we develop and solve numerically a mathematical model which can be used to understand the dynamics of a virulent code in a network of computers. This model is called the Immune, susceptible, exposed, infectious, quarantine, and recovered (MSEIQR).The model is solved using a very robust spectral method called the piecewise pseudospectral relaxation method (PPRM). PPRM accuracy is validated by comparing the results with the standard Runge-Kutta method. Stability analysis is also performed on the modified MSEIQR model for malicious code. Results generated are in agreement with the stability analysis performed.

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Developing higher order unconditionally positive finite difference methods for the advection diffusion reaction equations

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SAMS; Subject Classification Number: 23

Higher order unconditionally positive finite difference (HUPFD) methods are developed to solve linear and non-linear advection-diffusion-reaction (ADR) equations. The stability and consistency of the developed methods are analyzed, which are necessary and sufficient for convergence to the exact solution. The Von Neuman condition is used to analyze the stability since we are dealing with the Cauchy problem. The proposed method's efficiency and effectiveness is investigated by calculating the error, convergence rate, and computational time. A comparison of the solutions obtained by the higher order unconditionally positive finite difference and analytical methods is conducted for validation purposes. The numerical results show that the developed method preserve the solution accuracy. The results also show that increasing the order of the unconditionally positive finite difference leads to the implicit scheme that is unconditionally stable with an increased order of accuracy with respect to time and space.

Towards improved healthcare using knowledge graphs

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SAMS; Subject Classification Number: 12, 23, 25

Consultations between doctors and patients are described as "the central act of medicine" which "deserves to be understood." It forms the basis of the relationship that develops between both parties and lays the groundwork upon which appropriate treatment can be administered. Advancements in artificial intelligence, information, and communication technologies have enabled healthcare practitioners to revaluate the manner according to which both rudimentary medical procedures, such as doctor-patient consultations, as well as advanced medical interactions are performed.

The true potential of clinical data can only be realised if clinical data sources are integrated in an appropriate data representation and modelling approach.. One such model is a so-called *knowledge graph* (KG). A KG data model is a graph database in which information may be visualised as a graph structure, where a graph is defined as a representation of a network comprising a collection of nodes, representing real-world entities, that are inter-connected by means of edges, representing real-world relationships. KGs and machine learning techniques can equip healthcare practitioners with the ability to leverage health-related data in a manner that enhances the decision making process, ultimately improving the general standard of healthcare.

The aim in this poster is to describe an appropriate method for constructing a KG pertaining to data obtained from real-world medical conversations and electronic health records. Furthermore, appropriate graph machine learning approaches, including both shallow and deep learning methods, are to be described in the context of clinical KGs. The combination of these two tasks aims to serve as the foundation for the development of a framework for constructing KGs from doctor-patient consultation data that may in turn be used to provide decision support for healthcare practitioners.

A decision support system for carbon- and cost-effective transportation

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SAMS; Subject Classification: 10, 23, 25

Climate change has become one of the, if not the most, pre-eminent environmental issues threatening the human population and natural ecosystems. The levels of greenhouse gases emitted, mainly due to burning fossil fuels, have drastically increased due to the successive industrial revolutions, increasing the likelihood of irreversible consequences. The greenhouse gas emissions of logistics processes and supply chain activities account for a majority of companies' overall greenhouse gas emissions and carbon footprint. Most of these emissions are engendered by logistics activities related to transportation.

The carbon emissions from transportation logistics vary among supply chains and are a function of the fuel consumption of the vehicles used for transportation and distribution. Influential factors and trends contributing to fuel consumption — *e.g.* transport mode, vehicle attributes, driver behaviour, environmental factors, and operations — must be taken into account when attempting to minimise the environmental impact of transportation activities. There is, however, a trade-off in respect of minimising cost.

In this thesis, the aim is to investigate the utility of applying agent-based modelling and mathematical-based programming to model real-world transportation conditions and decisions related to the South African fruit export supply chain. An agent-based modelling approach is necessary so as to model the complexities associated with the various vehicle characteristics, environmental factors, traffic conditions, driver behaviour, and operational decisions. Different bi-objective mixed integer programming models are to be formulated so as to improve various aspects pertaining to the aforementioned, *e.g.* facility location and fleet scheduling. A *decision support system* (DSS) is to be designed, developed, and implemented to facilitate the decision-making process in order to aid stakeholders towards minimising both cost and carbon emissions.

Achieving mathematical maturity through mathematical identity

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SAMS; Subject Classification Number: 16

In the last few years, South African universities have been experiencing a significant drop in the mathematical skills of incoming students. The decrease in the number of students writing mathematics in Grade 12 and the decline in performance in the mathematics results have also been echoed by the Department of Basic Education for several years. This causes incoming students to experience problems when transitioning from school to university mathematics, hence a drop in the university throughput in science and mathematics-related fields. The issue of mathematics transition is global, as many countries internationally experience similar challenges and obstacles in the transition from school to university mathematics. The increasingly weaker mathematics background of university entrants and its consequences have been reported worldwide. Due to this reason, universities nationally and internationally introduced diagnostic tests as an initial assessment used to assess the basic mathematical knowledge and skills of incoming students.

There is no question that there is a distinctive breach between secondary school and university mathematics. One of the major difficulties experienced by universities is improving the mathematics skill base of the underprepared students entering tertiary education. However, universities cannot exonerate themselves from this crisis. This research aims to improve struggling students' performances in mathematics at the first-year level by implementing strategies to bridge the gap between school and university mathematics. The goal will be achieved by developing a pragmatic intervention programme which will focus on components that are believed to develop students' mathematical proficiency.

Finite element analysis of the effects of the viscoelastic properties of the Achilles tendon on stress and strain

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The human Achilles tendon is prone to injuries, especially amongst athletes. This tendon consists of fibres and has an intricate hierarchical structure Approximately 70% of its composition is made up of water, with the majority of its dry weight consisting of Type I Collagen.

Collagenous tissues, like tendons and ligaments, display time dependent mechanical behaviours This means that they do not have constant stress-strain relationships, but that these relationships are dependent on displacement time or on applied load. This property is called viscoelasticity.

The eventual objective of this study is to obtain a validated finite element model of the Achilles tendon, which incorporates the effects of its highly hydrated nature, as well as its viscoelastic properties.

The surface geometry of the model is generated from point cloud data, obtained from three-dimensional freehand ultrasound imaging, provided by Griffith University, Australia. Some material parameters are assigned by optimising parameters to tensile experimental data, whilst others are obtained from the literature.

A fast and simple method for solving quasimonotone variational inequalities

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SAMS; Subject Classification Number: 11

In this work we seek to establish a simple iterative method for solving quasimonotone variational inequalities, that are more general than the classical monotone version. While the few existing results are only weak convergence methods under quite restrictive conditions, we present a new Mann-type inertial projection and contraction method for solving quasimonotone variational inequalities as well as variational inequalities without monotonicity. Under standard assumptions, we prove a strong convergence result for our method. We also present several numerical examples to illustrate the simplicity and computational advantages of our proposed scheme over existing methods in literature.

A decision support framework for algorithmic functionality selection in the optimisation of plant breeding problems

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SAMS; Subject Classification: 25, 26

The demand for agricultural development to build stable and sustainable food systems has resulted in the need for plant breeding and genetic optimisation. The significant crop improvements required for adapting to the growing global population demands development of innovative optimisation methods within plant breeding programs. In practice, however, as crop populations (*cultivars*) increase in size, problem complexity increases exponentially. To approach the optimisation of these problems with significant complexity, the implementation of metaheuristics is required to yield high-quality solutions with high computational efficacy. The diversity of plant breeding programs and crop scenarios calls for investigation into developing a *decision support framework* for selecting appropriate algorithmic designs with suggested functionality to improve optimisation performance. The framework is aimed at providing breeders with improved performance over generic algorithm designs available in the literature and typically implemented in practice.

A generic framework for algorithm design selection in the context above is proposed in this poster, and further follow-up work anticipated is described; including a computerised instantiation of the framework in the form of a practical decision support system.

Algorithmic trading of securities – A machine learning approach

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SAMS; Subject Classification Number: 10, 25, 26

The incorporation of quantitative strategies into classical investment management has resulted in the ability to not only access data at unprecedented speeds, but also analyse vast amounts of data at significant scale and varying levels of abstraction. Aside from the unrivalled speed advantage, quantitative investing— especially those based on data-driven, machine learning approaches— holds the potential to recognise patterns in security pricing data automatically, without the pitfalls typically associated with humanistic approaches. Apophenia is described as the human tendency to infer patterns from (mostly random) data — akin to, for example, humans observing elephant-shaped clouds, traders are also susceptible to recognising supposed patterns within trading data.

Whilst the vast majority of quantitative research involves supervised machine learning approaches, this study, on the other hand, focuses on the application of unsupervised- and self-supervised learning methods in the context of algorithmic trading. The pursuit of identifying non-trivial hidden patterns within the data to better predict market movement is further convoluted by the vast amount of technical indicators available in the domain of trading. To aid in the selection of technical indicators as input features, auto-encoders (a subset of unsupervised machine learning) are investigated towards identifying a reduced set of features, representative of the security price being predicted.

Due to the rapid and significant advances in the field of computer vision, more specifically *convolutional neural networks* (CNNs), a focus is placed on the encoding of time-series data as images by means of *Gramian Angular Fields* (GAFs). The GAF images, *i.e.* arrays, are automatically labelled according to the price movement that follows the time-span over which the images were encoded, and is then utilised for predicting price movement.

The main research objectives pursued are: (1) To investigate the application of unsupervised, and self-supervised learning methods in the context of security markets, and (2) to develop an optimisation-based trading strategy that seeks to maximise profit whilst effectively managing portfolio risk.