Programme & Abstracts

SANUM 2013

The 37th Annual South African Symposium on Numerical and Applied Mathematics

3, 4 and 5 April

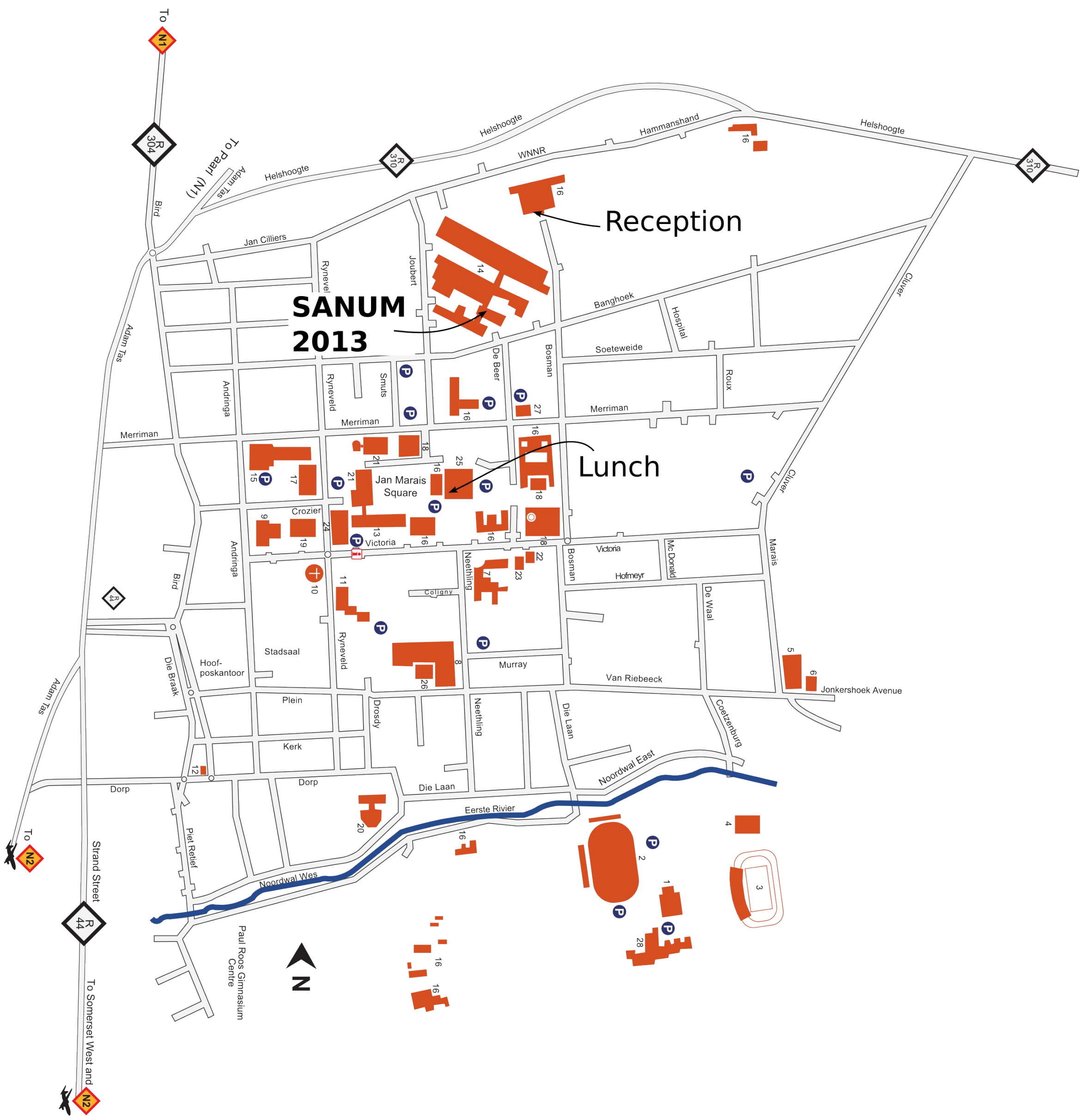
Hosted by the
Department of Mathematical Sciences
Stellenbosch University
South Africa

Welcome to SANUM!

Venue The conference is held on the third floor of Stellenbosch University's Engineering Knowledge Center (see map below). Facing the general engineering building, the entrance is about 50m behind the cafeteria ("Plakkies").

Registration takes place on 3 April from 08:30 to 09:30.

A **special session** will be held on 3 April in celebration of Ben Herbst's 60th birthday.



On the cover: Panoramic view of Stellenbosch from Papegaaiberg. Photo by Francois Malan.

Internet Access, reception, lunches

Wireless internet access is available in the two conference venues:

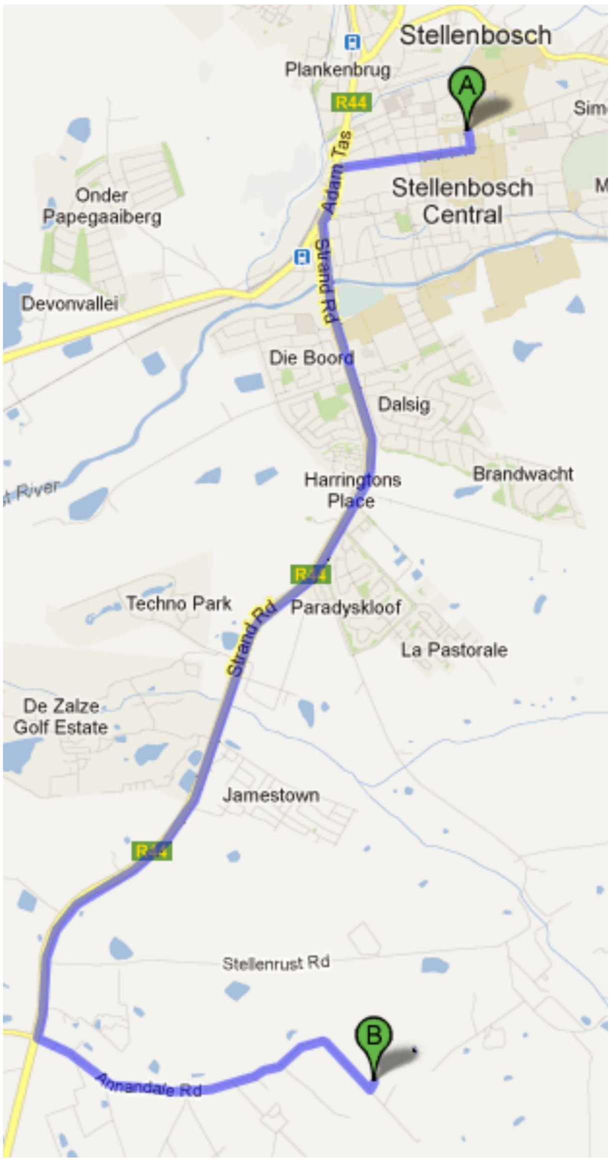
SSID: SANUM2013
Password: s2.71828

As part of the session celebrating Ben Herbst's 60th birthday, please join us for a **reception** on Wednesday afternoon, directly following the talks, on the porch of the Paul Sauer building (see map).

Lunch is served daily at Café Go in the Neelsie student center. Please remember your lunch voucher, allocated during registration.

Conference Dinner

The conference dinner is held on Thursday evening at 19:00 at Long Table restaurant. *If you require transport,* please gather in front of the engineering building at 18:40. Please remember your dinner voucher, allocated during registration.



Directions to GPS coordinates:
S34°0' 0.58" E18°51' 22.12"

- Take the R44 towards Somerset West.
- Turn left onto the Annandale road at the Mooiberge strawberry farm.
- Continue straight, passing Jacana and Bilton Wines.
- The restaurant is at Haskell Vineyards.

Programme for SANUM 2013

	Wednesday 3 April		Thursday 4 April		Friday 5 April	
08:30—09:00	**Registration** (K302/303, General Engineering Building)					
09:00—09:30			*Herbst*	Plenary I (Room K302) ***Jie Shen*** *Some recent advances on phase-field models for multiphase complex fluids*	*Weideman*	Plenary I (Room K302) ***Jan Nordström*** *Initial boundary value problems, summation-by-parts operators and weak boundary conditions with multi-physics applications*
09:30—10:00	***Opening remarks (Room K302)***					
10:00—11:00	*Weideman*	Plenary I ***Mark Ablowitz*** *Nonlinear waves: from beaches to dispersive shocks*		Plenary II ***Susanne Brenner*** *C^0 interior penalty methods*		Plenary II ***Jean-Paul Berrut*** *The linear barycentric rational quadrature method for Volterra integral equations*
11:00—11:30	Tea					
11:30—12:30	*Weideman*	*Plenary II* ***Bengt Fornberg*** *Radial Basis Functions: Freedom from meshes in scientific computations*	*Banda*	*Plenary III* ***Alastair Spence*** *The identification of Hopf bifurcations in problems from computational fluid dynamics*	*Mason*	*Plenary III* ***Siegfried Rump*** *Computer-assisted proofs by verification methods: rigorous results using floating-point arithmetic*
12:30—14:00	Lunch					

	K302	K303	K302	K303	K302	K303
Chair	*Van der Walt*	*Makinde*	*Milewski*	*Abelman*	*Harley*	*Banda*
14:00—14:30	***Ahrens*** *A new angular discretization for the 3D transport equation: initial numerical results*	***Kara*** *Conservation laws of partial difference equations*	***Freitag*** *Fast computation of the stability radius of a matrix*	***Wafy*** *Improvement of visual Cryptography techniques*	***Erickson*** *Stable, high order accurate adaptive schemes for long time highly intermittent geophysics problems*	***Abelman*** *MHD flow of a Sisko fluid past a porous plate*
14:30—15:00	***Maritz*** *Placement of linear accelerometers for angular motion estimation - a systematic approach*	***Moitsheki*** *Nonclassical symmetry analysis of a heat conduction model for heat transfer*	***Ragazzo*** *The turbulent wake of axisymmetric bodies*	***Oliphant*** *Trajectory based method for nonlinear constrained optimization*	***Adegboye*** *Super Runge-Kutta Nyström method for direct Integration of third order ordinary differential equations*	***Mason*** *Hydraulic fractures: conservation laws and exact solutions*
15:00—15:30	***Coetzer*** *Dynamic fusion of human and machine decisions for efficient cost-sensitive biometric authentication*	***Fareo*** *Symmetries and similarity solutions for a pre-existing fracture driven by a power-law fluid in permeable rock*	***Chun*** *How to solve PDEs numerically on Riemannian geometry?*	***Jacobs*** *A novel approach to text binarization via a diffusion-based model*		***Ahmadia*** *PyClaw: An accessible, extensible, and scalable tool for the solution of wave propagation problems*
15:30—16:00	**Tea**					
Chair	*Maritz*	*Kara*	*Moitsheki*	*Laurie*		
16:00—16:30	***Olivier*** *A numerical study of the large-period limit of the Zakharov-Shabat eigenvalue problem*	***Hutchinson*** *Turbulent flow: Lie group analysis*	***Van Rensburg*** *Existence theory for general second order linear hyperbolic equations*	***Swanepoel*** *Writer-specific dissimilarity normalisation for improved off-line signature verification*		
16:30—17:00	***Van der Walt*** *Sparse spherical harmonics kernels for representing diffusion signals on a sphere*	***Molati*** *Group analysis of a generalized KdV equation*	***Basson*** *Convergence of the finite element approximation for the vibration of a beam with a damping tip body*	***Mitchley*** *Orthogonal subspace projection methods for endmember identification in hyperspectral data*		
17:00—17:30	***Weideman*** *A computational exploration of the second Painlevé equation*	***Makinde*** *Computational modelling and optimal control of pest invasion on maize farms*	***Harley*** *A hybrid finite sine transform and the variational iteration method for solving a time-fractional diffusion equation*	***Stoltz*** *Pulse reformation, solving the leakage problem in the Discrete Pulse Transform domain*		
17:30—18:00	***Laurie*** *The interval euclidean algorithm*	***Matebese*** *Improving path quality of RRT-based algorithms*	***Milewski*** *Nonlinear-optics-like behaviour in water waves*	***Sathinarain*** *Numerical investigation of the parabolic mixed derivative diffusion equation via ADI methods*		
	Welcoming Reception at 18:00		Conference Dinner at 19:00			

See http://dip.sun.ac.za/sanum for more information.

Wednesday 3 April

NONLINEAR WAVES: FROM BEACHES TO DISPERSIVE SHOCKS

Mark Ablowitz (University of Colorado at Boulder)

The study of localized waves has a long history dating back to the discoveries in the 1800s describing solitary water waves in shallow water. In the 1960s researchers found that certain equations, including the Korteweg-deVries (KdV) and nonlinear Schrodinger (NLS) equations arise widely. Both equations admit localized solitary wave- or soliton solutions. Via a novel nonlocal formulation of water waves asymptotic reductions can be readily obtained such as the 1d equations mentioned above and the 2d Benney-Luke (BL) and Kadomstsev-Petviashvili (KP) equations. Some solutions will be discussed as well as recent observations. In fluids and optics 'dispersive' shock waves can occur. Applications, properties and interactions of will be discussed.

RADIAL BASIS FUNCTIONS: FREEDOM FROM MESHES IN SCIENTIFIC COMPUTATIONS

Bengt Fornberg (University of Colorado at Boulder)

Most numerical computations in multiple dimensions use either structured grids (e.g. finite differences and spectral methods) or unstructured ones (e.g. finite elements). Radial Basis Functions (RBFs) and, most recently, RBF-generated finite differences (RBF-FD) offer the opportunity of completely meshfree calculations in a large number of application areas, such as geoscience, fluid mechanics, mathematical biology, etc. The approach combines a number of strengths, such as:

- High orders of accuracy,
- High computational efficiency,
- Geometric flexibility (irregular domain shapes, local node refinements, solutions over curved surfaces, etc.),
- Short and simple codes,
- Excellent opportunities for distributed computing.

RBFs were introduced in the 1970s, first as a tool for interpolating scattered 2-D data. Since then, both our knowledge about RBFs and their range of applications have grown tremendously, with the numerical solution of PDE systems arising in the geosciences now becoming a particularly important one.

A NEW ANGULAR DISCRETIZATION FOR THE 3D TRANSPORT EQUATION: INITIAL NUMERICAL RESULTS

Cory Ahrens (Colorado School of Mines)

The discrete ordinate (Sn) equations have been the workhorse of deterministic radiation transport calculations for many years. Here we derive a new angular discretization for the 3D transport equation. The new discretization, derived using Lagrange interpolation on the sphere and collocation, retains the classical Sn structure, with the main difference being how the scattering source is calculated. Because of the formal similarity with the classical Sn equations, it should be possible to modify existing computer codes to take advantage of the new formulation. Moreover, the new Sn-like equations correctly capture delta function scattering, an important property when simulating charged particle transport. Preliminary numerical results will be presented.

PLACEMENT OF LINEAR ACCELEROMETERS FOR ANGULAR MOTION ESTIMATION - A SYSTEMATIC APPROACH

Milton Maritz (Stellenbosch University)

Linear accelerometers are often mounted in satellites (or vehicles) in order to estimate the angular velocity and angular acceleration of the body. How many of these accelerometers must be used, where in the body must one mount them, and in which directions must they point? Is there an optimal placement?
Previous researchers have proposed various placement configurations together with algorithms for extracting the required angular velocity and acceleration.
In this talk we approach the problem in a mathematically systematic way and show the power of this mathematical approach by illustrating that good placement configurations come forth naturally from the analysis. We demonstrate that the previously suggested configurations are special cases of the general 'good' configuration.

DYNAMIC FUSION OF HUMAN AND MACHINE DECISIONS FOR EFFICIENT COST-SENSITIVE BIOMETRIC AUTHENTICATION

Hanno Coetzer (Stellenbosch University)

We present a novel protocol for optimally combining human and machine decisions in a cost-sensitive environment. This should benefit financial institutions where off-line signatures, each associated with a specific transaction value, require authentication. When presented with labelled signatures produced by representative writers, the proficiency of a human workforce and a score-generating machine can be estimated and represented in ROC space. Using different Boolean fusion functions, a specific employee's decision is combined with each threshold-specific machine decision, after which the performance of the candidate ensembles is represented in ROC space. Only the optimal ensembles and associated decision trees are retained. When presented with a questioned signature linked to an arbitrary writer, the system first uses the ROC-based cost gradient associated with the transaction value to select the candidate ensemble that minimises the expected cost, and then uses the corresponding decision tree to make an optimal combined decision for the employee in question. Finally, the optimal combined decisions for every employee in the workforce are fused through majority voting. By considering the average expected cost over all operating conditions, we show that the proposed hybrid classifier consistently outperforms an unaided machine and an unaided human workforce with more than five employees.

A NUMERICAL STUDY OF THE LARGE-PERIOD LIMIT OF THE ZAKHAROV-SHABAT EIGENVALUE PROBLEM WITH PERIODIC POTENTIALS

Carel Olivier (University of Cape Town)

Deconinck and Kutz (2007) developed an efficient algorithm for solving the Zakharov-Shabat eigenvalue problem with periodic potentials numerically. It is natural to use the same algorithm for solving the problem for non-periodic potential (decaying potentials defined over the whole real line) using large periods. In this paper we propose the use of a specific value of the Floquet exponent. Our numerical results indicate that it can produce accurate results long before the period becomes large enough for the analytical convergence results of Gardner (1997) to be valid. We also illustrate the rather complicated path to convergence of some nonlinear Schrödinger potentials.

SPARSE SPHERICAL HARMONICS KERNELS FOR REPRESENTING DIFFUSION SIGNALS ON A SPHERE

Stefan van der Walt (Stellenbosch University)

Magnetic resonance imaging (MRI) allows the observation of protons in water molecules that traverse the human brain. Mapping their motion illuminates white matter fiber pathways, and allows the exploration of the structure of the brain in-vivo ('within the living'). We describe why mathematical modelling of MRI signals is needed, and present a sparse model with desirable properties that fits the data more robustly than equivalent existing methods.

A COMPUTATIONAL EXPLORATION OF THE SECOND PAINLEVÉ EQUATION

JAC Weideman (Stellenbosch University)

The Painlevé equations are nonlinear, second-order differential equations that have become increasingly prominent in applications over the last two or three decades. There are six members in the family, the first two of which are

$$\text{PI: } \frac{d^2u}{dz^2} = 6\,u^2 + z, \quad \text{PII: } \frac{d^2u}{dz^2} = 2\,u^3 + zu + \alpha, \quad z \in \mathbb{C}$$

with α a constant. Solutions cannnot, in general, be expressed in terms of classical special functions and they define new transcendental functions for which numerical methods have to be developed. The solutions are characterized by extended pole fields, sometimes accompanied by pole free sectors in the complex plane (called tronquée solutions). Both these features present their own challenges to numerical computation.

In a talk presented at SANUM2011, an algorithm developed in collaboration with Bengt Fornberg was described. It combines a pole-friendly Padé time stepping algorithm with a novel path selection strategy. In that talk numerical results for PI was presented. Here we describe further results for PII. We shall show computed solutions and pole field patterns, some of which we believe have not been observed before. (Joint work with Bengt Fornberg, University of Colorado at Boulder, USA.)

THE INTERVAL EUCLIDEAN ALGORITHM

Dirk Laurie (Stellenbosch University)

Finding the greatest common denominator of two integers is one of the oldest problems of number theory. Its relevance is undiminininished even in today's world of open-key cryptosystems based on the difficulty of factoring large integers.

When working with very large numbers, say one million digits, the basic idea, first shown by D.H. Lehmer in 1938, is known to numerical analysts as "iterative improvement": approximate answer via low-precision calculation, high-precision evaluation of residual, another low-precision calculation using the residuals as data, high-precision combination of the two results to combine the answers.

This idea still lies at the heart of modern software for the GCD in multiprecision arithmetic. It is common practice to increase the precision exponentially at each step, for example, by doubling the number of digits.
We discuss two variations. The first involves another idea from numerical analysis, namely interval arithmetic. This leads to a simpler and slightly faster algorithm. The second is to increase the precision linearly, but with chunk sizes that are optimized for the desired final number of digits.

CONSERVATION LAWS OF PARTIAL DIFFERENCE EQUATIONS

Abdul Hamid Kara (Wits University)

We present some recently developed notions on conservation laws of Difference Equations and discuss the similarities with Differential Equations (see the works of Hydon and Mansfield & Hydon). The idea of a relationship between symmetries and conservation laws will be pursued.

NONCLASSICAL SYMMETRY ANALYSIS OF A HEAT CONDUCTION MODEL FOR HEAT TRANSFER IN A LONGITUDINAL RECTANGULAR FIN OF A FUNCTIONALLY GRADED MATERIAL

Joel Moitsheki (University of the Witwatersrand)

We consider a heat conduction model arising in transient heat transfer through longitudinal fins of a heterogeneous (functionally graded) material. In this case, the thermal conductivity depends on the spatial variable. The heat transfer coefficient depends on the temperature and is given by the power law function. The resulting nonlinear partial differential equation is analyzed using both classical and nonclassical symmetry techniques. Both the transient state and the steady state result in a number of exotic symmetries being admitted by the governing equation. Furthermore, nonclassical symmetries are also admitted. Both classical and nonclassical symmetry analysis results in some useful reductions and some remarkable exact solutions are constructed.

SYMMETRIES AND SIMILARITY SOLUTIONS FOR A PRE-EXISTING FRACTURE DRIVEN BY A POWER-LAW FLUID IN PERMEABLE ROCK

Adewunmi Gideon Fareo (University of the Witwatersrand)

In this talk we derive group invariant, exact and numerical solution for the evolution of a two-dimensional permeable fracture with non-zero initial length and driven by a non-Newtonian fluid of power-law rheology. The two- dimensional fracture is driven by the injection of an incompressible power-law fluid and the fluid flow in the fracture in considered laminar. With the aid of lubrication theory and the PKN approximation a nonlinear diffusion equation for the fracture half-width is derived. Since the fluid-rock interface is permeable the nonlinear diffusion equation has a leak-off velocity sink term. A condition, in the form of a first order partial differential equation for the leak-off velocity, is obtained for the nonlinear diffusion equation to have Lie point symmetries. The general form of the leak- off velocity is derived. Using the Lie point symmetries the problem is reduced to a boundary value problem for a second order ordinary differential equation. The leak- off velocity is further specified by assuming that it is proportional to the fracture half-width. The effect of power-law rheology on hydraulic fracturing is investigated and the evolution of a two-dimensional fracture with non-zero initial length and driven by a power-law fluid is analysed. In this talk, hydraulic fracturing with shear thinning, Newtonian and shear thickening fluids are compared.

TURBULENT FLOW: LIE GROUP ANALYSIS

Ashleigh Hutchinson (Wits University)

Kolmogorov's theory of turbulence describes turbulent flow as consisting of eddies of varying sizes. Energy is transferred from larger to smaller eddies until sufficiently small scales are reached. The viscosity dominates and energy is effectively dissipated at these small scales. A brief review of Kolmogorov's theory of turbulence is given and it is shown how Lie symmetry methods can be applied. A simple time dependent flow problem is considered and laminar and turbulent solutions are compared.

GROUP ANALYSIS OF A GENERALIZED KDV EQUATION

Motlatsi Molati (National University of Lesotho/North-West University)

In this work the Korteweg-de Vries equation which contains an arbitrary function in the nonlinear term is considered and it is referred to as a generalized KdV. The Lie group analysis approach is employed to obtain the possible forms of the arbitrary parameter.

COMPUTATIONAL MODELLING AND OPTIMAL CONTROL OF PEST INVASION ON MAIZE FARMS

Oluwole Daniel Makinde (Cape Peninsula University of Technology)

In Africa, maize is one of the most commonly cultivated crops in all major agro-ecological zones. It is one of the main food sources for the people in the continent. Maize also finds its way in various forms into animal feed rations and is industrially processed for the production of starches, syrups and alcohol. In spite of its importance, maize yields are generally low due to pest invasion. Among the insect pests associated with maize, Lepidopteran stem borers are the most dangerous threatening for the production. In this paper, a compartmental deterministic model for pest invasion on maize farm is proposed. The nonlinear problem is tackled using stability theory of differential equations and an optimal control strategy is determined computationally. Numerical simulation is performed and the pertinent results are presented graphically and discussed quantitatively.

IMPROVING PATH QUALITY OF RRT-BASED ALGORITHMS

Belinda Matebese (Stellenbosch University)

Sampling-based methods such as Rapidly-exploring Random Tree (RRT) have been successfully used in solving motion planning problems in high-dimensional and complex environments. RRT-based algorithms are the most popular since they have the ability to find a feasible and collision-free path quickly. At the same time, the algorithms are not able to find optimal solutions and moreover produce non-smooth solutions. To improve the quality of these solutions, optimal control based methods are being investigated and will be integrated with the algorithm. This approach will be discussed and preliminary results presented.

Thursday 4 April

SOME RECENT ADVANCES ON PHASE-FIELD MODELS FOR MULTIPHASE COMPLEX FLUIDS

Jie Shen (Purdue University)

I shall present some recent work on phase-field model for multiphase complex fluids. Particular attention will be paid to develop models which are valid for problems with large density ratios and obey an energy dissipation law.
I shall present efficient and accurate numerical schemes for solving the coupled nonlinear system for the multiphase complex fluid, in many case prove that they are energy stable, and show ample numerical results which not only demonstrate the effectiveness of the numerical schemes, but also validate the flexibility and robustness of the phase-field model.

C^0 INTERIOR POINT METHODS

Susanne Brenner (Louisiana State University)

C^0 interior penalty methods are discontinuous Galerkin methods for fourth order problems that use standard finite element spaces for second order problems. These methods have some advantages over classical approaches for fourth order problems. They are much simpler than C^1 finite element methods. The lowest order C^0 interior penalty methods are as simple as classical nonconforming finite element methods. But unlike classical nonconforming finite element methods that only use low order polynomials, C^0 interior penalty methods come in a natural hierarchy and higher order C^0 interior penalty methods can capture smooth solutions efficiently. Compared with mixed finite element methods, the stability of C^0 interior penalty methods can be established in a straightforward manner and the symmetric positive definiteness of the continuous problems is preserved by C^0 interior penalty methods. Moreover, since the underlying finite element spaces are standard spaces for second order problems, multigrid solves for the Laplace operator can be used as natural preconditioners for C^0 interior penalty methods, and problems on smooth domains can be easily handled by isoparametric C^0 interior penalty methods.
In this talk we will discuss the formulation, error analysis, fast solution techniques for these methods and present applications to boundary value problems, obstacle problems and optimal control problems.

THE IDENTIFICATION OF HOPF BIFURCATIONS IN PROBLEMS FROM COMPUTATIONAL FLUID DYNAMICS

Alastair Spence (University of Bath)

The detection of a Hopf bifurcation in a large-scale dynamical system that depends on a physical parameter often consists of computing the right-most eigenvalues of a sequence of large sparse eigenvalue problems. This is not only an expensive operation but many of the common numerical methods for this problem may be unreliable for large sparse matrices. This talk will summarise some of the methods commonly used in applications and discuss their advantages and disadvantages. Next, we shall describe a recent approach that reformulates the problem using Kronecker products of matrices (see Meerbergen & Spence, SIMAX, Vol 31, 2010, and Elman, Meerbergen, Spence & Wu, SISC, Vol 34, 2012). This approach is based on inverse iteration but requires the solution of Lyapunov equations with low-rank right-hand sides at each step of the iteration. Numerical results will be presented for some large- scale problems arising from fluid dynamics and aeroelasticity.

FAST COMPUTATION OF THE STABILITY RADIUS OF A MATRIX

Melina Freitag (University of Bath)

A new fast algorithm for the computation of the distance of a stable matrix to the unstable matrices is provided. The method is based on finding a two-dimensional Jordan block corresponding to a pure imaginary eigenvalue in a certain two-parameter Hamiltonian eigenvalue problem introduced by Byers (SIAM J. Sci. Statist. Comput., 9 (1988), pp. 875-881). The solution is achieved by an extension of the Implicit Determinant Method. Numerical results show the performance of this algorithm for several examples and comparison is made with other methods for the same problem.

THE TURBULENT WAKE OF AXISYMMETRIC BODIES

Clodoaldo Ragazzo (Universidade de São Paulo)

In this talk experimental and theoretical results on axisymmetric selfsimilar turbulent wakes are presented and discussed with emphasis on the wake width growth rate. Some applications of the existing theory to the problem of hydrodynamic interaction between ships is made.

HOW TO SOLVE PDES NUMERICALLY ON RIEMANNIAN GEOMETRY?

Sehun Chun (African Institute for Mathematical Sciences)

Modern scientific era faces unprecedented demands of computational modeling on complex geometry, especially in the area of biological fluid dynamics, image processing, and computational biology. In most numerical schemes, geometry has been regarded as the synonym of 'mesh' and computational techniques have been developed almost independent of the geometrical properties of the domain or its material property. But, these approaches soon revealed its limitations on the order of its convergence on curved surfaces as the simplest Riemannian geometry. In this talk, we briefly review the previous numerical schemes to solve PDEs on curved surfaces and how the incorporation of the concepts of differential geometry can be used to develop or improve the numerical solutions on Riemannian geometry that we face in the actual world. The recently-proposed method of moving frames will be introduced as an example.

EXISTENCE THEORY FOR GENERAL SECOND ORDER LINEAR HYPERBOLIC EQUATIONS

Nic Janse van Rensburg (University of Pretoria)

The existence of solutions for problems involving partial differential equations is not a black or white issue; there exist a number of possibilities. For a start, there is a distinction between so called weak solutions and classical solutions. But there are different "levels of weakness" and definitions also differ. Using the one- dimensional wave equation, it is easy to identify four possibilities. The properties of a solution are not merely of academic interest, since the rate of convergence of numerical approximations depends on the "smoothness" of a solution. Generalized, second order linear hyperbolic partial differential equations and vibration of systems of elastic bodies are of the same type which we refer to as general second order linear hyperbolic equations. In this presentation we consider existence results for this type of problem. Different approaches to the theory are sketched and the usefulness of the results is considered. We argue that there should be at least an awareness of the possibility that a solution may not be smooth enough or not exist at all when computing a numerical approximation. Elementary examples are presented to illustrate some of the possibilities.

CONVERGENCE OF THE FINITE ELEMENT APPROXIMATION FOR THE VIBRATION OF A BEAM WITH A DAMPING TIP BODY

Madelein Basson (University of Pretoria)

A model for the vibration of a beam with a damping tip body that appeared in a recent article is considered. A variational form for the motion of the beam is derived and it is used to prove that the model problem has a unique solution. Using a so called energy method, we prove that the Galerkin approximation converges to the solution and derive error estimates.

A HYBRID FINITE SINE TRANSFORM AND THE VARIATIONAL ITERATION METHOD FOR SOLVING A TIME-FRACTIONAL DIFFUSION EQUATION ON A BOUNDED DOMAIN

Charis Harley (University of the Witwatersrand)

The aim of this research is to introduce a method based on the hybridization of the Finite Sine Transform (FST) and the Variational Iteration Method (VIM). The nature of the boundary conditions informs the choice of transform method: in this case Dirichlet boundary conditions are required by the FST for instance. The VIM considers only the initial condition as an iteration seed. It is this compatibility that suggests this hybridization to be effective. As a source of comparison we implement both the FST-VIM hybrid method and the FST-FFD (Fractional Finite Difference) method and report on the results.

NONLINEAR-OPTICS-LIKE BEHAVIOUR IN WATER WAVES

Paul Milewski (University of Bath)

A sufficiently high intensity beam of light in a medium whose refractive index is intensity dependent (such as air or water) will exhibit self focussing until higher order effects, noise, or plasma generation come into play. The cross-sectional profile of the focussed beam depends on the initial profile. It turns out that a very similar phenomenon occurs in a patch of water waves until nonlinearity arrests the focussing and the patch breaks up into a complex set of localised structures. Whilst water may be too viscous for the phenomena to be observed, computations suggest that the behaviour should occur in mercury.

IMPROVEMENT OF VISUAL CRYPTOGRAPHY TECHNIQUES

Maged Wafy (Aims)

The Visual cryptography scheme (VCS) is a secure method that uses mathematical techniques to encode secret image. The idea is to convert the written material into an image, then breaking this image into N shares. A distinctive property of VCS is that one can visually decode the secret image by superimposing shares without computation.

Recently, research problems in visual cryptography are related to several extension of visual cryptography research that includes VCS for general access structure, contrast optimization and the concept of randomness, Multi secret messages, and meaningful shares. I shall introduce some new ideas to improve contrast, and meaningful shares.

TRAJECTORY BASED METHOD FOR NONLINEAR CONSTRAINED OPTIMIZATION

Terry-Leigh Oliphant (University of the Witwatersrand)

The trajectory-based method for solving nonlinear constrained programming problems is proposed and the convergence properties of this method are investigated. The augmented Lagrangian problem reformulation is used to covert the constrained problem into an equivalent unconstrained problem and a new updating scheme for the penalty parameter is discussed

A NOVEL APPROACH TO TEXT BINARIZATION VIA A DIFFUSION-BASED MODEL

Byron Jacobs (University of the Witwatersrand)

We present here a new approach to document image binarization. The method is based on the dynamic process of diffusion, coupled with a nonlinear Fitzhugh-Nagumo type source term that exhibits binarizing properties. These desirable properties lead to a method that is robust to noise and is able to successfully binarize an input document image. We measure the efficacy of our proposed method against industry standards by two methods, a pixel by pixel comparison with the ground truth image and a standard optical character recognition test. Through these measures we illustrate a progressive method that performs at the highest level in the field.

WRITER-SPECIFIC DISSIMILARITY NORMALISATION FOR IMPROVED WRITER-INDEPENDENT OFF-LINE SIGNATURE VERIFICATION

Jacques Swanepoel (Stellenbosch University)

We present a novel writer-independent off-line signature verification system. This system utilises the discrete Radon transform and a dynamic time warping algorithm for writer-independent signature representation in dissimilarity space. The system also considers writer-specific statistics for dissimilarity normalisation. A discriminant function, either linear or quadratic, is utilised for signature modelling and verification.

We show that the proposed feature extraction and dissimilarity representation framework provides a successful platform for signature modelling and verification. We also show that the inclusion of writer-specific statistics during dissimilarity normalisation improves the proficiency of the proposed writer-independent verification system.

When evaluated on Dolfing's data set, a signature database containing 1530 genuine signatures and 3000 amateur skilled forgeries, we show that the proposed system outperforms existing systems also evaluated on this data set.

ORTHOGONAL SUBSPACE PROJECTION METHODS FOR ENDMEMBER IDENTIFICATION IN HYPERSPECTRAL DATA

Michael Mitchley (University of the Witwatersrand)

Endmembers of a linear mixing model in hyperspectral data span the signal subspace. Using a pseudorank estimation procedure, the sonar- based MUSIC method is used to test endmember candidacy through projection of signal vectors onto the orthogonal noise subspace.

PULSE REFORMATION, SOLVING THE LEAKAGE PROBLEM IN THE DISCRETE PULSE TRANSFORM DOMAIN

Gene Stoltz (TUKS)

The Discrete Pulse Transform is derived from the connected operators, called the LULU operators. Connected operators presents a framework for the extraction of meaningful structures from an image. Two or more of these structures sometimes gets joined to present a nugatory structure; this phenomenon is known as the leakage problem. Pulse Reformation is a proposed solution to undo the union between meaningful structures.

NUMERICAL INVESTIGATION OF THE PARABOLIC MIXED DERIVATIVE DIFFUSION EQUATION VIA ADI METHODS

Melisha Sathinarain (University of the Witwatersrand)

The two-dimensional analog of a two-parameter mixed derivative equation is considered via two numerical methods: the ADI and compact ADI method. These methods are compared and the influence of the kinematic viscosity and viscoelasticity is discussed in relation to the solutions obtained. The two schemes are proven to be conditionally stable and the methods compared via an error analysis.

Friday 5 April

INITIAL BOUNDARY VALUE PROBLEMS, SUMMATION-BY-PARTS OPERATORS AND WEAK BOUNDARY CONDITIONS WITH MULTI-PHYSICS APPLICATIONS

Jan Nordström (Linkoping University)

During the last decade, stable high order finite difference methods as well as finite volume methods applied to initial-boundary-value-problems have been developed. The stability is due to the use of so-called summation-by-parts operators, penalty techniques for implementing boundary and interface conditions, and the energy method for proving stability. In this talk we present the theory by using simple model problems, and exemplify with complex multi-physics applications. By reusing the main ideas behind the development mentioned above, new time-integration procedures and increased accuracy as well as new types of boundary conditions have been derived.

THE LINEAR BARYCENTRIC RATIONAL QUADRATURE METHOD FOR VOLTERRA INTEGRAL EQUATIONS

Jean-Paul Berrut (Fribourg University)

We shall first introduce linear barycentric rational interpolation to the unaware audience: it can be viewed as a small modification of the classical interpolating polynomial. Then we present two direct quadrature methods based on linear rational interpolation for solving general Volterra integral equations of the second kind. The first, deduced by a direct application of linear barycentric rational quadrature given in former work, is shown to converge at the same rate, but is costly on long integration intervals. The second, based on a composite version of the rational quadrature rule, looses one order of convergence, but is much cheaper. Both require only a sample of the involved functions at equispaced nodes and yield a stable, infinitely smooth solution of most classical examples with machine precision.

COMPUTER-ASSISTED PROOFS BY VERIFICATION METHODS: RIGOROUS RESULTS USING FLOATING-POINT ARITHMETIC

Siegfried Rump (Hamburg University of Technology)

The classical mathematical proof is performed by pencil and paper. However, there are many ways in which computers may be used in a mathematical proof, where, however, “proofs by computers” or even the use of computers in the course of a proof are not so readily accepted.
In this talk we discuss how so-called verification methods may assist in achieving a mathematically rigorous result. In particular we emphasize how floating-point arithmetic is used.

STABLE, HIGH ORDER ACCURATE ADAPTIVE SCHEMES FOR LONG TIME HIGHLY INTERMITTENT GEOPHYSICS PROBLEMS

Brittany Erickson (San Diego State University)

We use high-order finite difference methods through summation-by-parts operators and weak enforcement of boundary conditions to derive a provably stable discretization for the elastic wave equation. Time stepping is achieved through the so-called theta-method and the second order BDF which allow us to take large time steps during the intermittent period between earthquakes. Numerical experiments illustrate and corroborate the theoretical results.

SUPER RUNGE-KUTTA NYSTRÖM METHOD FOR DIRECT INTEGRATION OF THIRD ORDER ORDINARY DIFFERENTIAL EQUATIONS

Zamurat Ayobami Adegboye (Kaduna Polytechnic, Kaduna)

In this work, we extend the Runge-Kutta- Nyström (RKN) method which is an extension of the Runge-Kutta (RK) method to Super Runge-Kutta- Nyström (SRKN) method for direct integration of Special and General third order initial valued ODEs via the idea as those invented by Nyström.The theory of Nyström was adopted in the derivation of the method. The proposed method was tested with Numerical experiment to illustrate its efficiency and the method can be extended to solve higher order differential equations. The scheme is simple to implement and converges better with the exact solution.

MHD FLOW OF A SISKO FLUID PAST A POROUS PLATE

Shirley Abelman (University of the Witwatersrand)

We consider magnetohydrodynamic (MHD) flow of an electrically conducting Sisko fluid past a porous plate in the presence of a constant applied magnetic field. The induced magnetic field is not taken into account by considering a small magnetic Reynolds number. The resulting nonlinear problem is solved numerically. Variations of various parameters of interest are examined. A comparative study is made between the results of viscous and Sisko fluids.

HYDRAULIC FRACTURES: CONSERVATION LAWS AND EXACT SOLUTIONS

David Mason (University of the Witwatersrand)

Two conservation laws for the non-linear diffusion equation describing a hydraulic fracture are derived. The Lie point symmetry associated with each conservation law is calculated. The exact analytical solution for the hydraulic fracture generated by the Lie point symmetry is obtained. The analytical solutions are interpreted physically.

PYCLAW: AN ACCESSIBLE, EXTENSIBLE, AND SCALABLE TOOL FOR THE SOLUTION OF WAVE PROPAGATION PROBLEMS

Aron Ahmadia (Continuum)

Development of scientific software involves tradeoffs between ease of use, generality, and performance. We review the design of a general hyperbolic PDE solver using Python that can be operated with the convenience of high-level MATLAB yet achieves efficiency near that of hand-coded Fortran and scales to the largest supercomputers. Our approach uses the petsc4py bindings to PETSc for distributed scalability, while simultaneously allowing for several on-node tuning strategies.

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