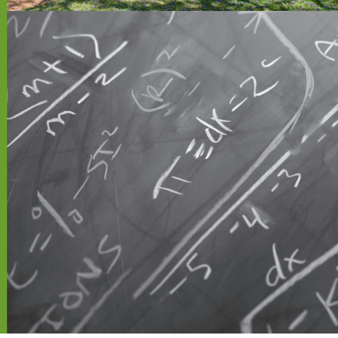
SANUM

2015

South African Symposium on Numerical and Applied Mathematics

Programme & Abstracts

The 39th South African Symposium on Numerical and Applied Mathematics (SANUM)



Hosted by the Department of Mathematics and Applied Mathematics, University of Pretoria

30 March to 1 April 2015

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Natural and Agricultural Sciences

Fakulteit Natuur- en Landbouwetenskappe
Lefapha la Disaense tša Tlhago le Temo

www.sanum.co.za
www.up.ac.za/maths/sanum2015

NRF National Research Foundation

MaSS DST-NRF CENTRE OF EXCELLENCE IN MATHEMATICAL & STATISTICAL SCIENCES

UP Natural Hazard Centre

DST/NRF SARChI M³B²

CASIO

ABSA Chair in Actuarial Science

Organising Committee

Prof Roumen Anguelov (Chair), roumen.anguelov@up.ac.za

Prof Nic van Rensburg (Chair), nic.vanrensburg@up.ac.za

Dr Inger Fabris-Rotelli (Administrator), inger.fabris-rotelli@up.ac.za

Prof Mapundi Banda

Dr Madelein Basson

Dr Michael Chapwanya

Dr Alta Jooste

Prof Jean Lubuma

Welcome to SANUM 2015

Table of Contents

Welcome message from the organisers

The SANUM symposium has a long standing tradition as an event in the scientific life in South Africa. Following in this tradition, the scope of SANUM 2015 includes the topics of ordinary and partial differential equations, mathematical modelling, numerical analysis, biomathematics, image analysis, optimization and approximation theory. The conference also features 2 special sessions: Risk Analysis and Quantitative Finance, and Approximation Theory and Applications.

The conference attracted 74 participants from all corners of South Africa, with 46 presentations and 7 posters included in the scientific programme. We thank all participants for their contribution to the success of this conference.

We warmly welcome all participants in SANUM 2015 who come to Pretoria from other cities, provinces and countries. We are particularly grateful to the plenary speakers who have accepted the responsibility of delivering keynote talks, jointly covering a wide spectrum of the scope of the conference.

Growing a new generation of researchers is important for any area of science. A special event in the programme of this SANUM conference is the Young Mathematicians Function to take place on 31 March from 18:00. Thank you to Prof Ingrid Rewitzky for accepting to be the guest speaker at the function.

The financial support of

- National Research Foundation
- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS)
- UP Natural Hazard Centre
- DST/NRF SARChI Chair in Mathematical Models and Methods in Bioengineering and Biosciences
- ABSA Chair in Acturarial Science
- University of Pretoria
- CASIO

is acknowledged with gratitude.

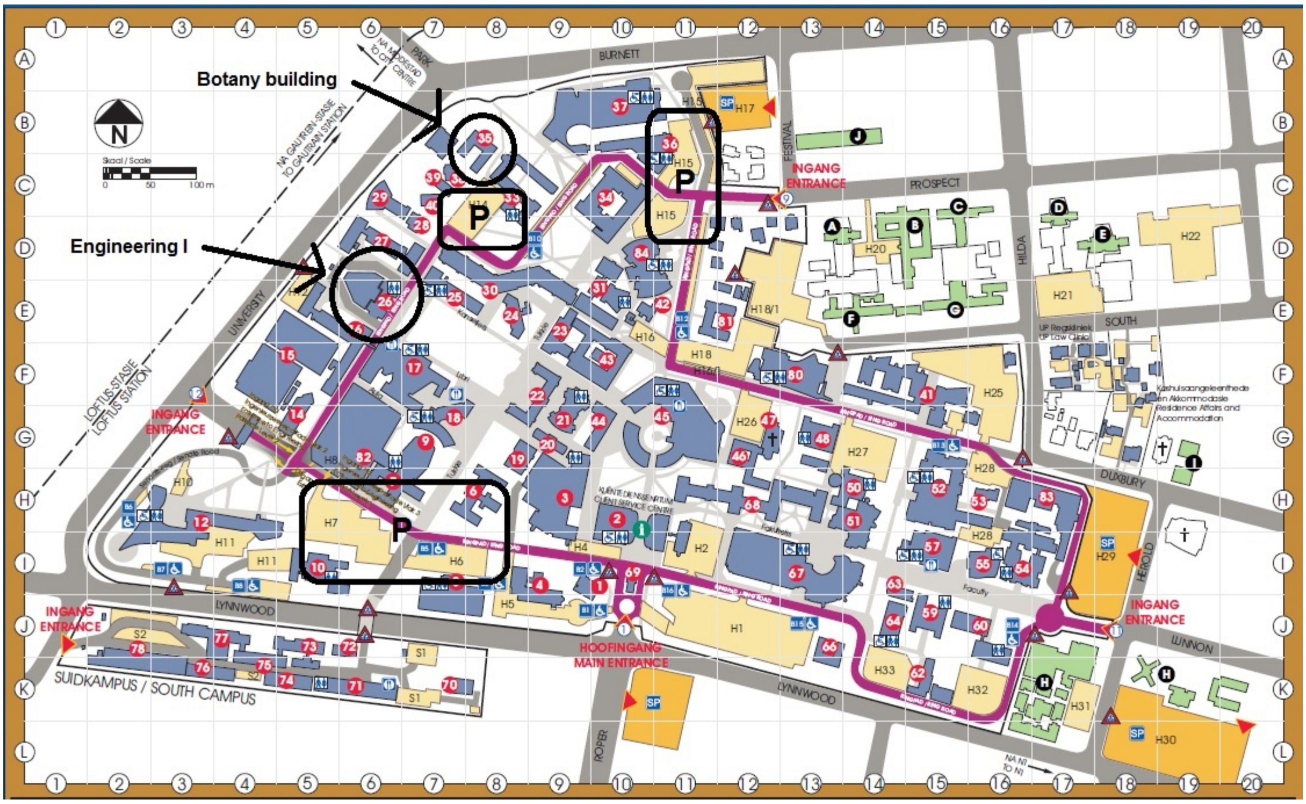
We wish all participants productive time at the conference. We hope that SANUM 2015 will have a stimulating effect on your research and future career.

Roumen Anguelov and Nic van Rensburg
For the Organising Committee

General information

Venues

- **Talks:** All talks will be in Engineering I, room 1-1 or room 1-4.
- **Tea and Lunch:** In the Exhibition Hall at the entrance of Engineering I.
- **Functions:** In the Botany building (more information under "Social events").



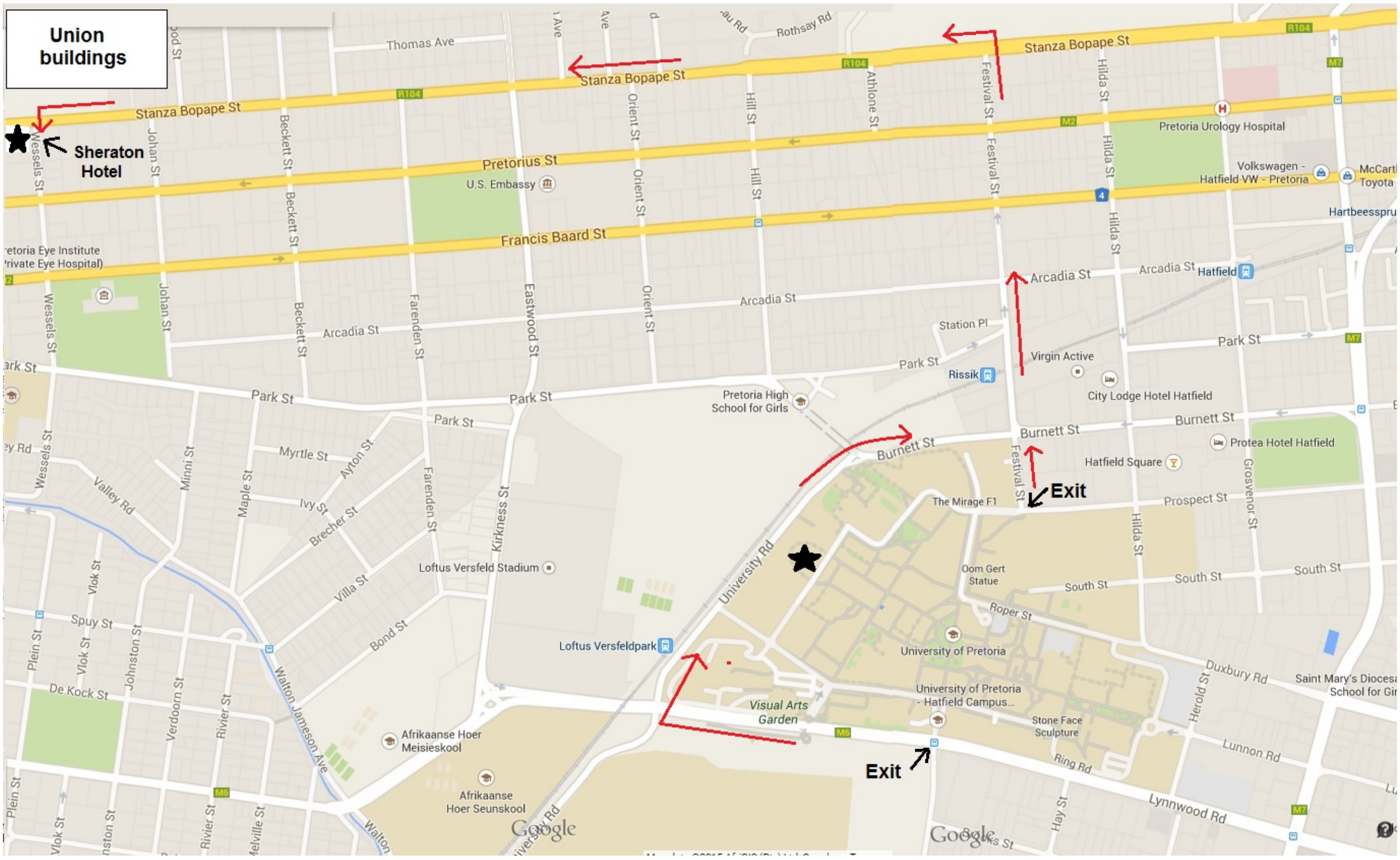
The **parking** lots are indicated on the map. Please note that the undercover parking is reserved.

Social events

Welcome evening: Monday at 18:00 in the Botanical Gardens.
The Botanical Gardens are next to the Botany Building on campus.

Young mathematicians function: Tuesday at 18:00 in the Botany building.
A special function for all the students and delegates who recently (in the last 5 years) obtained their PhD's.
Prof Ingrid Rewitzky, Professor of Mathematics at the University of Stellenbosch will give a talk titled "*Advice to a Young Mathematician*".

Conference lunch: Wednesday at 12:30 at the Sheraton Hotel.
The Sheraton Pretoria Hotel is located at 643 corner of Stanza Bopape (Church) & Wessels Streets in Arcadia. Opposite the Union Buildings. See map below.



Other

- The **registration desk** will be active in the Exhibition Hall the whole day Monday. Any additional information needed can be found there, or ask any of the LOC members.
- There is **Wi-fi** available on campus. Log in to Tuks Guest - delegates will have to register each day and set up their own username and password every day. Details on how to connect to the Guest network can be found at http://www.up.ac.za/up-wireless-network.

Programme: Sessions and Plenary speakers

Monday 30 March 2015	
8:00-8:45	*Registration, Tea and Coffee (Eng I Exhibition Hall)*
8:45-9:30	**Opening (ENG I 1-1):** Prof A Ströh (Vice-Principal: Institutional Planning) Prof JM-S Lubuma (Dean: Faculty of Natural and Agricultural Sciences) Prof R Anguelov (HOD: Mathematics and Applied Mathematics)
9:30-10:30	**Plenary Talk: Prof Andre Weideman (ENG I 1-1)** Contour Integral Methods for PDE's
10:30-11:00	*Tea Break (Eng I Exhibition Hall)*
11:00-12:00	**Plenary Talk: Prof Abba Gumel (ENG I 1-1)** Mathematical Assessment of the Role of Climate Change on Vector-borne Diseases
12:00-12:30	Parallel Sessions Special session: Risk analysis and Quantitative Finance
12:30-13:30	*Lunch (Eng I Exhibition Hall)*
13:30-14:30	**Plenary Talk: Prof Andrzej Kijko (ENG I 1-1)** "Black Swan" events -beyond the realm of expectations
14:30-15:30	Parallel Sessions Special session: Risk analysis and Quantitative Finance
15:30-16:00	*Tea Break (Eng I Exhibition Hall)*
16:00-18:00	Parallel Sessions Special session: Risk analysis and Quantitative Finance
18:00 –	Welcome Function (Botanical Gardens Behind Botany Building)

Tuesday 31 March 2015	
7:30 -	*Tea and Coffee (Eng I Exhibition Hall)*
8:00-9:30	Parallel Sessions
9:30-10:30	**Plenary Talk: Prof Montaz Ali (ENG I 1-1)** Transformation-based and differential Equation-based Approaches for Mixed Integer Nonlinear Programming Problems
10:30-11:00	*Tea Break (Eng I Exhibition Hall)*
11:00-12:00	**Plenary Talk: Prof Alfred Stein (ENG I 1-1)** Advanced remote sensing image analysis with super resolution mapping
12:00-12:30	Parallel Sessions Special session: Approximation Theory and Applications
12:30-13:30	*Lunch (Eng I Exhibition Hall)*
13:30-14:30	**Plenary Talk: Prof Claude Brezinski (ENG I 1-1)** Survey of extrapolation methods for linear algebra problems
14:30-15:30	Parallel Sessions Special session: Approximation Theory and Applications
15:30-16:10	Poster Session *and Tea Break (Eng I Exhibition Hall)*
16:10-17:40	Parallel Sessions Special session: Approximation Theory and Applications
18:00 –	Young Mathematician's Function (Botany 2-26 and 2-27) **Guest speaker: Prof Ingrid Rewitzky** Advice to a Young Mathematician

Wednesday 1 April 2015	
7:30	*Tea and Coffee (Eng I Foyer outside ENG I 1-1)*
8:00-9:30	Parallel Sessions
9:30-10:30	**Plenary Talk: Prof Jacek Banasiak (ENG I 1-1)** Singularly perturbed systems with non-isolated limit manifolds and applications in mathematical biology
10:30-11:00	*Tea Break (Eng I Foyer outside ENG I 1-1)*
11:00-11:30	Parallel Sessions
12:30-15:30	*Conference Lunch, Prizes, Closing* *(Sheraton Hotel)*

Abstracts - Plenary speakers

Transformation-based and differential Equation-based Approaches for Mixed Integer Nonlinear Programming Problems

M. M. Ali
School of Computational and Applied Mathematics,
University of the Witwatersrand
montaz.ali@wits.ac.za

Topic: Mixed Integer Nonlinear Programming Problems

This talk begins with the new definition of a local minimizer of MINLPs (mixed integer nonlinear programming problems). Two solution techniques for MINLPs are then presented. The first technique is based on transforming the MINLPs into an equivalent potential energy of the motion of a moving particle in an n-dimensional conservative force field. The corresponding relaxed differential equation is then solved, producing a lower bound of the original MINLP. The new definition of the local minimizer is then used to identify potential initial conditions using which a number of differential equations, with integer variable being fixed, are solved.

The second technique is based on some transformations for solving linearly constrained mixed integer quadratic programs (MIQPs). The technique can be applied to problems with various types of Hessian of the continuous part, and is based on a carefully chosen linear transformation which is used to reduce the number of bi-linear terms in the objective function. Results are presented for both techniques.

In the last part of the talk, an outer approximation algorithm for a class of mixed-integer nonlinear programming problems (MINLPs) with partial differentiability is considered. Theoretical results are then further extended to design an outer approximation algorithm with non-differentiable data. By dropping the differentiability assumption, we substitute gradients with subgradients obtained from KKT conditions, and use the outer approximation method to reformulate convex MINLP as one equivalent MILP master program. The convergence of this algorithm is also presented.

Singularly perturbed systems with non-isolated limit manifolds and applications in mathematical biology

J. Banasiak
School of Mathematics, Statistics and Computer Science, UKZN
banasiak@ukzn.ac.za

Topic: Ordinary differential equations, modelling, associated numerical analysis

Increasing complexity of models that describe many natural and technological processes, which often stretch across multiple time and size scales, often prevents their robust analysis. Thus, there is a need for developing methods allowing for simplification of such models preserving, however, salient features of their dynamics, which are relevant at the chosen, say, time scale. In models described by differential equations, different time scales often are represented by parameters that express the ratios between typical duration times of processes in the system under consideration. If such a parameter is close to zero, then the processes occurring at one of the time scales are dominant. Then a natural question is whether we can focus on processes at this time scale, discarding, or somehow aggregating, the other ones – this would correspond to letting the small parameter equal to zero. It turns out that such a simplification not always is possible. Moreover, even if it is, it often cannot be achieved by simply setting the small parameter to be zero, but requires an involved limit process with a nontrivial theory to show it availability. In the presented lecture we shall discuss one such theory, based on the Tikhonov theorem. In particular, we shall show how the standard Tikhonov theorem can be applied to derive a model with the Allee effect from simpler population models with interactions based on the law of mass action. The main part of the lecture is devoted to the cases in which not all assumptions of the Tikhonov theorem are satisfied. Often the limit manifolds, obtained by passing with the small parameter to zero, are not uniquely determined and we observe unexpected behaviour of solutions passing close to intersection points of the limit manifolds. Though, save for one-dimension, there is no exhaustive theory of such phenomena, we present a theory which can be applied to monotone systems and we shall illustrate it on epidemiological and ecological models.

Survey of extrapolation methods for linear algebra problems

Claude Brezinski
Laboratoire Paul Painlevé, UMR CNRS 8524
UFR de Mathématiques Pures et Appliquées
Université des Sciences et Technologies de Lille
59655–Villeneuve d'Ascq cedex, France.
Claude.Brezinski@univ-lille1.fr

Joint work with M. Redivo Zaglia and others

In the first part of this talk, we give a general idea about what sequence transformations are, in particular those for vector sequences. We explain why these sequence transformations are extrapolation methods and why some of them are also projection methods.

In the second part of the talk, we show how these procedures can be use for solving a system of linear equations and for computing the eigenvalues of a matrix. We focus, in particular, on the ε–algorithms.

The last part of the talk is devoted to applications of extrapolations methods in the context of some linear algebra problems. In particular, we show that they can be used for estimating the norm of the error for the solution of systems of linear equations, and we apply these estimates to the choice of the best parameter in Tikhonov regularization method. Another application is the computation of the trace of powers of matrices and another one concerns web search and the PageRank algorithm. Finally we show how to accelerate the convergence of Kaczmarz method for linear systems. Numerical examples illustrate these applications.

Mathematical Assessment of the Role of Climate Change on Vector-borne Diseases

A Gumel
Arizona State University, USA
agumel@asu.edu

Climate drivers, such as temperature, rainfall, vapor pressure, humidity etc., are known to influence the incidence of vector-borne diseases (VBDs). This talk introduces the main concepts and challenges associated with the mathematical modeling of the role of climate change on the transmission dynamics of VBDs, with emphasis on malaria. Some new results will be discussed.

“Black Swan” events -beyond the realm of expectations

A Kijko
University of Pretoria Natural Hazard Centre, Africa
andrzej.kijko@up.ac.za

Humans constantly face catastrophes on different scales in time and space. Extremes in nature, geological, hydrometeorological, geophysical and financial hazards adversely affect human life and health as well as having a dramatic impact on sustainable development of society. There exists an inherent threat to vulnerable lifelines and infrastructure such as reservoirs, pipelines and power plants. Dramatic changes in overall socio-economic states and financial crashes also belong to this category. The financial repercussions of catastrophic events can disrupt entire markets and regional or even global economies. Within minutes these extreme events can have severe repercussions on the life and economic development of affected nations which can set them back for decades.

The location, size and time of occurrence of the next disaster is highly unpredictable. Some events are even unthinkable. What would happen if, for some reason, New York City was flooded by a monster tsunami, or San Francisco was razed to the ground by a mega-earthquake? Although events like these are highly improbable, they may not be totally impossible and are therefore known as “Black Swans”. The concept behind “Black Swans” are not new but was recently given new life by the author Nassim Taleb in his book of the same name. Taleb defines the Black Swan as “a high impact, hard-to-predict, extreme and rare event beyond the realm of normal expectations.” These events are so rare that they are almost beyond imagination and often leave legions of “experts” to explain why they were unable to predict it based on existing data.

Recently “Black Swan” events appear to occur more frequently. Most recent events include the 9/11 terrorist attack in New York City, Hurricane Katrina in the south-eastern part of the USA and the Tohoku-Oki mega-earthquake and tsunami in Japan which caused the disastrous explosion of the Sendai Nuclear Power Plant. Recent financial extreme events include the chain of extreme losses triggered by the collapse of Lehman Brothers in the USA, the crisis at banking giant Socit Gnrale in France due to a single rogue trader’s actions and the ongoing financial crisis in Greece. Each of these events resulted in adverse impacts on human life and health as well as having a dramatic impact on the sustainable development of impacted society.

From an analytical point of view, most extreme events can be seen as statistical random processes with the time of event occurrence and its magnitude (size) as random variables.

It is commonly believed that the first step in the mitigation of these mega-disasters is the careful statistical analysis of these extreme phenomenon. The purpose of this lecture is to highlight and discuss frequent obstacles which one can face during such studies. The potential difficulties include:

i How to take into account the incompleteness of extreme event records. More specifically, how to combine mega historic events with recent, more complete sets of observations, into the hazard and risk assessment when each catalogue exhibits separate time-dependent levels of completeness. For financial risks such as Operational Risk losses, incompleteness can arise due to fraudulent activities not being detected or not being measured at all (due to predetermined minimum thresholds).

ii How to incorporate the uncertainties associated with time and size of mega-events. This includes uncertainties surrounding the dependence between events within a risk category or even between different risk categories.

iii How to incorporate the inherent discrepancy inherent between the selected mathematical model and reality.

Thus this lecture shall attempt to address the above problems, to provide tools for their solution and to present a coherent methodology for the efficient assessment of hazard and risk required for the successful mitigation of natural catastrophes.

Advanced remote sensing image analysis with super resolution mapping

Alfred Stein, Valentyn Tolpekin, Benson Kipkemboi Kenduiywo and Milad Mahour
University of Twente
a.stein@utwente.nl

Topic: Image analysis

Recently, progress has been made on remote sensing image analysis with super resolution mapping (SRM). This presentation gives an overview of some essential steps. Class separability is accounted for by means of controlling the balance tuned by a smoothness parameter λ between the prior and the likelihood terms in the posterior energy function. A generally applicable procedure has been developed to estimate λ, based on local energy balance analysis. The study shows how the optimal value of the smoothness parameter depends quantitatively and monotonically upon the class separability and the scale factor. Effects are studied on an image synthesized from an agricultural scene with field boundary subpixels. The accuracy of the resulting land-cover-map image is assessed by means of the κ statistic at the fine-resolution scale and the class area proportion at the coarse-resolution scale. The optimal value of λ exists for each combination of scale factor and class separability. This allows us to reach a high classification accuracy ($\kappa = 0.85$) even for poorly separable classes, i.e., with a transformed divergence equal to 0.5 and a scale factor equal to 10. The developed procedure agrees with the empirical data for the optimal smoothness parameter. The study shows that SRM is now applicable to a larger set of images with class separability ranging from poor to excellent. The paper further includes two application.

- The methodology is applied to precision agriculture in Iran, where grape trees were identified within a vineyard. SRM was applied to a high resolution GeoEye image of a vineyard in Iran with the aim to determine the Actual EvapoTranspiration (AET) and Potential EvapoTranspiration (PET). The Surface Energy Balance System (SEBS) applied for that purpose requires the use of a thermal band, provided by a Landsat TM image of a 30 m resolution. Grape trees in the vineyard were planted in rows and three levels were distinguished: the field, rows and individual trees. The study showed that AET values were obtained close to the value derived by standard calculations at the field

scale. The study concluded that modern satellite derived information in combination with recently developed image analysis methods is able to provide reliable AET values at the row level, but not yet for every individual grape tree.

- The second application considers conditional random fields (CRFs) to classify built-up area around the city of Nairobi, Kenya. A feature selection approach including a novel data dependent term was designed to discriminate classes. Mean, standard deviation and the variogram slope at small distances characterized training areas, taking spatial class dependencies into account. Accuracy was stable for a varying smoothness parameter while preserving class boundaries and aggregating similar labels. The delineated built-up area increased by 98.9 ha, whereas 26.7 ha was changed from built-up to non-built-up areas. The study concluded that the new procedure can be used to detect and monitor the extent of built-up area. In this way it provides timely spatial information to urban planners and other professionals.

References

Tolpekin, V., and Stein, A. 2009. Scale effects and parameter optimization for super resolution mapping from satellite images with Markov Random Fields. IEEE Transactions in Geoscience and Remote Sensing 47 (9), 3283 3297. Kenduiywo, B. Tolpekin, V. and Stein, A. 2014 Detection of built-up area in ASTER and SAR images using Conditional Random Fields. Journal of Applied Remote Sensing 8(1), 083672 (Feb 27, 2014). doi:10.1117/1.JRS.8.083672 Mahour, M., Stein, A., Sharifi, A., Tolpekin, V. Subm. Integrating Super Resolution Mapping and SEBS modeling for evapotranspiration mapping at the field scale. Precision Agriculture.

Contour Integral Methods for PDEs

J.A.C. Weideman
Applied Mathematics, Stellenbosch University, South Africa
weideman@sun.ac.za

Topic: Numerical Analysis

Contour integral methods for the time integration of parabolic PDEs have been tried and tested by many authors. This includes the speaker, who gave talks on this topic at earlier SANUM conferences. The new topic discussed here is the extension of these ideas to elliptic PDEs. The literature for this case is much more sparse, and almost exclusively limited to the pen of I.P. Gavrilyuk and co-workers. The papers from this group provide powerful convergence proofs in a general setting but do not emphasize practical numerical considerations. We focus on the latter aspect here, and discuss implementation issues such as the choice of conformal mapping that defines the contour, the optimal step size of the trapezoidal rule that is ultimately applied on the contour, and the numerical linear algebra involved in solving the resolvent systems at each quadrature node. One of our test problems is the "particle-in-a-box" problem, the 3D version of the last of the famous Ten Digit Problem Set proposed by L.N. Trefethen in 2002. (Joint work with Nick Hale, Stellenbosch University.)

Abstracts - Contributed talks & Posters

Special sessions

Approximation Theory and Applications

Risk analysis and Quantitative Finance

Other topics

Biomathematics

Image analysis

Partial differential equations, modelling, associated numerical analysis

Other

Posters

Nonstandard finite difference scheme for a model of nerve conduction

AA Aderogba
Dept. of Maths. & Appl. Maths., Univ. of Pretoria
aderogba.adebayo@up.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

Most physical models give rise to nonlinear partial differential equations and most of these equations are not amenable to closed form solutions. Numerical approximation of these equations therefore, becomes important. The generalized Nagumo reaction diffusion equation models excitation and propagation of nerve axon and therefore requires reliable numerical approximations. In this work, we design explicit schemes that preserve the qualitative properties of this differential model. We obtain conditions under which the schemes are elementary stable and we present some numerical results.

Laplace transform-homotopy perturbation method for pricing under affine pure diffusion models

CR Bambe Moutsinga
Tshwane University of Technology
moutsingacb@tut.ac.za

Topic: Risk Analysis and Quantitative Finance

Most existing multivariate models in finance are based on affine diffusion models. These typically lead to the need to solve a system of Riccati differential equations. In this paper, we introduce an efficient method for solving system Riccati differential equations. In this technique, combination of Laplace transform and new homotopy perturbation methods (LTHPM) are considered as an algorithm to the exact solution of the nonlinear Riccati equations. tThe new technique is applied to solving stiff affine model problems that include interest rates models, two and three factor stochastic volatility models. We show that the present approach is relatively easy, efficient and highly accurate.

Implementation of the standard Finite Element and Mixed Finite Element methods to the vibration of a Timoshenko beam

M Basson
University of Pretoria
madelein.basson@up.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

We compare the Finite Element and Mixed Finite Element method for implementation to the vibration in a Timoshenko beam. We consider the following three types of implementation: eigenvalue problem, equilibrium problem and dynamic problem. The theory is not new, but does not explain in practical terms why the Mixed Finite Element Method produces more accurate results with few elements. In this talk we attempt to explain this and show some impressive comparisons.

Analysis of power-law Stokes equations: A penalty finite element approach

J. Djoko Kamdem
University of pretoria
jules.djokokamdem@up.ac.za

Topic: Numerical Analysis, Partial differential equations.

In this joint work with M. Mbehou, we study the penalty finite element approximation of the stationary power-law Stokes equations. We established uniform convergence of the finite element solution with respect to the penalized parameter under classical assumptions on the weak solution. We also discuss the implementation of the nonlinear saddle point problem formulated by adopting a particular algorithm based on vanishing viscosity approach and long time behavior of an initial value problem. Finally, the predictions observed theoretically are validated by numerical experiments.

Turing instability in SI model due to infective avoidance

R. Anguelov, H.M. Tenkam
Department of Mathematics and Applied Mathematics
University of Pretoria
roumen.anguelov@up.ac.za, michel.djouosseu@up.ac.za

Topic: Biomathematics

We study the effect of infective avoidance on the spacial distribution of susceptible and infective on a SI model. The avoidance of infective by susceptible and the avoidance of overcrowding are both represented via advection terms so that a conservation law holds on any open subset of the domain. More precisely the model is a boundary value problem on a bounded domain $\Omega \subset \mathbb{R}^2$ with a piece-wise smooth boundary as follows

$$\begin{cases} \dfrac{\partial S}{\partial t} = f(S,I) + \gamma \Delta S + \sigma \nabla (S \nabla N) + D \nabla (S \nabla I) \quad \texttt{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial I}{\partial t} = g(S,I) + \gamma \Delta I + \sigma \nabla (I \nabla N) \quad \texttt{in} \quad \Omega \times (0,\infty), \\ \left.\dfrac{\partial S}{\partial \eta}\right|_{\partial\Omega} = \left.\dfrac{\partial I}{\partial \eta}\right|_{\partial\Omega} = 0 \quad \texttt{on} \quad \partial\Omega, \\ S(x,0), I(x,0) \geq 0 \quad \texttt{in} \quad \Omega, \end{cases}$$

where $N = S + I$. We show that for sufficiently large D the spatially homogeneous steady state is unstable. Thus, spatial patterns are formed via the Turing mechanism.

Beam models for vertical high-rise structures

S Du Toit
University of Pretoria
sonja.dutoit@up.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

In high-rise structures or buildings it is necessary to reduce earthquake- or wind-induced vibrations in order to increase the level of structural safety and occupant comfort. To study these structures, it was found that researchers simplify models drastically. In fact, most authors use the simplistic Euler-Bernoulli theory to model the vibration of high-rise structures. We investigate whether this simplification is justified.

The focus of this presentation will be model derivation. We consider the conservation laws (conservation of momentum and angular momentum) to derive general equations of motion. We derive a linear approximation with careful consideration of the necessary assumptions. Finally, the models are adapted to obtain the Timoshenko and Euler-Bernoulli models for a vertical structure incorporating the effect of gravity.

Finite Element Approach to Trap-Insect Model

CC Dufourd
Department of Mathematics and Applied Mathematics,
University of Pretoria
claire.dufourd@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

The trap-insect model considered in this presentation comprises a system of two advection-diffusion-reaction equations. We develop finite element approximation of the solution of the model in order to produce accurate numerical simulations using a non-uniform triangulation. The algorithm is used for computing estimates of the parameters of the insect population. Particular attention is paid to estimating the insect population, including the case of heterogeneous distributions. Using traps is the common practice to gain knowledge on the presence of a particular insect population and its density. This work aims to contribute to optimizing field protocols for accurate parameters estimation.

Motion Deblurring of Images Using Tomographic Reconstruction

J Engelbrecht and MF Maritz
Stellenbosch University
16265262@sun.ac.za, mfmaritz@sun.ac.za

Topic: Image analysis

Motion blur in images can be corrected by deconvolution methods if the blur kernel is known. Often, only the blurred image is available and the kernel must be estimated from the blurred image itself, called 'blind' deconvolution. Previous work on blind deconvolution makes use of optimization techniques to estimate the blur kernel.

In this presentation we present a method based on tomographic reconstruction of the kernel. Edges in the blurred image are found with standard edge detection methods and a sinogram of the blur kernel is obtained by differentiating across edges in the image for a sufficient number of edge orientations. Application of the inverse radon transform yields the kernel.

Some options for improving the proposed tomographic method will be discussed, among others, sinogram interpolation. Typical results will be shown.

Spatial randomness of Discrete Pulse Transform constructs

I Fabris-Rotelli
Department of Statistics, University of Pretoria
inger.fabris-rotelli@up.ac.za

Topic: Image analysis

The *LULU* operators have been developed in one dimension and more recently in higher dimensions, such as images, and act as smoothers for discrete data. In addition their recursive implementation results in the so-called Discrete Pulse Transform (DPT), and provides a consistent decomposition of the data resulting in additional insight into the structure of the data. The constructs present in the different scale layers of the DPT can be modelled. We investigate the nature of their spatial positioning for various image types.

Spurious resonance in semidiscrete methods for the Korteweg–de Vries equation

M Fasondini
University of the Free State
fasondinim@ufs.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

A multiple scales analysis of semidiscrete methods for the Korteweg–de Vries equation is conducted. Methods that approximate the spatial derivatives by finite differences with arbitrary order accuracy and the limiting method, the Fourier pseudospectral method, are considered. The analysis reveals that a resonance effect can occur in the semidiscrete solution but not in the solution of the continuous equation. It is shown for the Fourier pseudospectral discretization that resonance can only be caused by aliased modes. The spurious semidiscrete solutions are investigated in numerical experiments and we suggest methods for avoiding spurious resonance.

Symmetries and Conservation Laws of Difference Equations

MK Folly-Gbetoula
University of the Witwatersrand, Johannesburg
Mensah.Folly-Gbetoula@wits.ac.za

Topic: Other

A number of nontrivial conservation laws of some difference equations, viz., the discrete Liouville equation and the discrete Sine-Gordon equation, are constructed using first principles. Symmetries and the more recent ideas and notions of characteristics (multipliers) for difference equations are also discussed.

Optimal C^3 Interpolatory Subdivision Schemes

Mpfareleni Rejoyce Gavhi & Bin Han
African Institute for Mathematical Sciences, University of Alberta
rejoyce@aims.ac.za,

Topic: Approximation theory

Subdivision; a branch of Computational Mathematics is a method for generating smooth curves and surfaces. It has, over the last two decades, developed into a powerful tool in several computer-aided design (CAD) application areas, for example, aircraft design and animation movie production. Comparing subdivision with other possible modeling approaches for smooth curves and surfaces, subdivision is easy to implement and is computationally efficient. It is also a fundamental mathematical tool in wavelet analysis which has many proven successful applications such as image/video processing and computer vision. The underlying mathematical analysis of subdivision and wavelets is strongly related via the concept of refinable functions. In this talk, I will introduce subdivision for curves, and provide conditions on which subdivision convergence and regularity is guaranteed. Graphical illustrations will be provided.

Multilevel Finite Volume Method for the Convective Cahn-Hilliard Equation

HH GIDEY
University of Pretoria
hagoshigd@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

In this work, we consider the convective Cahn-Hilliard Equation (CCHE) and implement an implicit multilevel finite-volume discretization. The numerical stability of this method is proved. The stability analysis shows that the traditional one-level method is more restricted than the multilevel method; i.e., the multilevel method can allow larger time steps than the traditional method. Some numerical simulations of the convective Cahn-Hilliard Equation are presented using the multilevel and one-level methods to elucidate the methods.

Computing with Legendre polynomials

N. Hale
University of Stellenbosch
nickhale@sun.ac.za

Topic: Numerical Analysis

Polynomial expansions and polynomial interpolation are ubiquitous in algorithms of numerical analysis and scientific computing. Whilst monomial series and/or equally-spaced interpolation points are conceptually the most straightforward, these usually lead to ill-conditioning and unstable algorithms. Instead, one typically uses Chebyshev polynomials and Chebyshev points, in no small part because of their close connection with the discrete Cosine transform (DCT) and hence the fast Fourier transform (FFT).

But are Chebyshev expansions always the best choice? For one thing, they are associated with the weighted inner-product space,

$$\langle f, g \rangle = \int_{-1}^{1} \frac{f(x)g(x)}{\sqrt{1-x^2}} dx,$$

whereas the standard L_2 inner-product might be more natural in some situations. Conversely, the Legendre polynomials *are* orthogonal in L_2, but lack the connection to the FFT and associated fast algorithms.

At last year's SANUM meeting I described a fast transform between Chebyshev and Legendre series, making efficient computation with the latter feasible. In this talk we shall look at some recent algorithms made possible by the ready computation of Legendre series, including: fast L_2 projection; convolution of polynomial series; computing fractional derivatives; and the discrete Legendre transform (DLT).

Dynamical analysis and consistent numerics for a delay model of viral infection in phytoplankton population

A. S. Hassan
University of Pretoria
Adamu.Hassan@tuks.co.za

Topic: Biomathematics

In this article, we consider the effects of delay (on stability and viral infectivity) in the dynamics between susceptible and viral phytoplankton by transforming the system of three ordinary differential equations in Dhar and Sharma [2010], into a system of two delay differential equations, by incorporating delay as an incubation period of the viral phytoplankton. The equilibria are rigorously analyzed and the phenomenon of stability switches are shown. The standard numerical schemes and methods sometimes fails to preserves the basic qualitative properties of the continuous models they represents. To buttress this claim, we present and compare two numerical schemes for the continuous model. The ecological implications and interpretations are also provided.

Numerical simulations of Isothermal Collapse

R. S. Herbst
University of the Witwatersrand
sheldon.herbst@wits.ac.za

Topic: Other

Numerical simulations of gravitationally collapsing isothermal clouds is considered in one and two dimension. The isothermal clouds are assumed to be spherically (for 1D) and axially (for 2D) symmetric. Results of the simulations in relation to star formation are important due to the structures formed during the collapse process. Comparisons to analytical solutions for the late stages of collapse are also investigated.

Frequency Analysis of wind- and earthquake induced oscillations in vertical structures

A. Hoogkamer
University of Pretoria
anyahkr@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

Consider a simplified model for the effect of an earthquake on a vertical structure (e.g. building). The solution of the model problem can be obtained by the superposition of a particular solution of the non-homogeneous problem and a complementary function. Of particular interest is to analyse the particular solution to investigate the possibility of resonance. For this analysis we use an eigenfunction series solution. In applications it is necessary to calculate approximate eigensolutions and in this presentation finite element approximations are discussed.

Inverse Heat Conduciton Problem in the Mixed Derivative Heat Equation

B. A. Jacobs
University of the Witwatersrand
byron.jacobs@wits.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

This research presents the approximate solution to the ill-posed inverse heat conduction problem using the mixed derivative heat equation to describe the phenomenological transfer of heat in a one and two dimensional bounded domain. Inverse problems are inherently ill-posed and can be formulated in many ways, we consider here the transient model of hear conduction based on the mixed derivative heat equation with an unknown initial heat distribution. The fuzzy inference system is then used to solve the inverse problem through successive updating of an initial guess based on the fuzzy categorisation of the error in a heat measurement at some time $T \neq 0$.

Big data: a compressed sensing approach

C Janse van Rensburg, I Fabris-Rotelli
Department of Statistics, University of Pretoria
churlybear@gmail.com

Topic: Image analysis

Compressed sensing(CS) is an application of statistics and mathematics in the field of signal processing. More specifically, we will look at an application in image processing and big data. Today's presentation serves as an introduction of the heart of CS and why it is an exciting research field. The problem that CS addresses can be phrased as following: is it possible to reconstruct a signal f exactly, having no a priori information regarding the signal besides that it is sparse, and only having a sample of limited number of linear measurements of f in some domain? CS is in some sense the opposite of data compression and mines potential that goes beyond the limits of the famous Nyquist/Shannon sampling theorem.

Simulations of wind and earthquake induced oscillations in vertical structures.

L. Janse van Rensburg
University of Pretoria
liesljvrensburg.ljvr@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

In this presentation we focus on the application of the finite element method to vertical structures. We consider wind and earthquake induced oscillations using the Timoshenko model for a cantilever beam. Since we consider a vertical beam, it is important that the weight of the beam should be taken into account. We also need to define our "load" in terms of the force generated by the wind and or the earthquake.

We derive the variational form and approximate the solution using the Galerkin approximation with piecewise linear basis functions. The problem is then rewritten as a system of ordinary differential equations, which can be discretized using many different finite difference schemes such as Central differences or Central difference average acceleration finite element method.

Lastly we discuss some numerical results.

Conservative finite difference schemes for the KdV equation

O. A. Jejeniwa
University of Pretoria, Pretoria, South Africa.
u13419545@tuks.co.za

Topic: Numerical Analysis

The KdV equation, $u_t + u_x + uu_x + u_{xxx} = 0$, describes the elongation of water waves. It is known that the integrability of this equation guaranties an infinite invariants. In this work, we propose two conservative numerical finite difference schemes for this equation: one which conserves both the mass and momentum and the other which conserves mass and energy. A numerical comparison of these schemes is provided.

New bounds for the extreme zeros of Meixner polynomials

AS Jooste
University of Pretoria
alta.jooste@up.ac.za

Topic: Approximation theory

Consider the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$. If the polynomials p_n and p_{n-m}, $m = 2, 3, \ldots, n-1$, have no common zeros, there exists a real polynomial of degree $m-1$, completely determined by the coefficients in the three term recurrence relation satisfied by the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$, whose real simple zeros provide a set of points that, together with the zeros of p_{n-m}, completely interlace with the zeros of p_n, a property we refer to as completed Stieltjes interlacing. The conditions under which completed Stieltjes interlacing holds between the zeros of polynomials from different orthogonal sequences are studied and this leads to a set of points that can be applied as bounds for the extreme zeros of the polynomials. We apply our results to the Meixner and Krawtchouk polynomials, that are orthogonal with respect to a discrete measure, and we identify new bounds for the extreme zeros of these polynomials.

Polynomials orthogonal with respect to symmetric moment functionals

AS Kelil
University of Pretoria
abey.kelil@tuks.co.za

Topic: Approximation theory

Polynomials orthogonal with respect to a symmetric moment functional can be generated via quadratic transformation from the classical orthogonal polynomials as shown by Chihara in [1]. Laguerre polynomials generate a class of generalised Hermite polynomials while Jacobi polynomials give rise to a class of generalised Ultraspherical polynomials. In this presentation, we discuss various properties of these and other polynomials orthogonal with respect to symmetric moment functionals which arise by quadratic transformation of orthogonal polynomials.

References

[1] T.S. Chihara, *An introduction to orthogonal polynomials*, Gordon and Breach Science Publishers, New York, 1978.

Distributions of the *LULU* operators in images

Kwok Ho Lau, I Fabris-Rotelli, A Bekker
Department of Statistics, University of Pretoria
alexlau1712@gmail.com

Topic: Image analysis

The *LULU* operators have been developed in one dimension and more recently in higher dimensions, such as images, and act as smoothers for discrete data. In addition their recursive implementation provides a consistent decomposition of the data providing additional insight into the structure of the data. The distributions for these smoothers were developed by Conradie et. al. [2] and output distributions investigated by Butler [1]. The distributions for higher dimensions have not been investigated. We illustrate some ideas towards this.

References

[1] Pieter-Willem Butler. *The transfer of distributions by LULU smoothers.* PhD thesis, Stellenbosch: Stellenbosch University, 2008.

[2] WJ Conradie, T De Wet, and M Jankowitz. Exact and asymptotic distributions of lulu smoothers. *Journal of Computational and Applied Mathematics*, 186(1):253–267, 2006.

Quantitative guidelines for portfolio safety during retirement

E. Maré
Department of Mathematics and Applied Mathematics
University of Pretoria
eben.mare@up.ac.za

Topic: Risk Analysis and Quantitative Finance

In this contribution we analyse drawdown (i.e., spending rates) of retirees in a South African context. We demonstrate that the current practice of drawing more than 6% annually from a living annuity is unsustainable. We also investigate the success rate of various asset allocation profiles and show the use of a risk management framework for longer-term investments.

Adaptive Function Approximation using a Wavelet Basis

M R Mitchley
University of the Witwatersrand, Johannesburg
michael.mitchley@wits.ac.za

Topic: Approximation theory

Within reinforcement learning, it is advantageous to define a *value* for each state, which must be approximated in Markov decision processes (MDP) with continuous state variables. Approximation via fixed bases has the disadvantage of using a set level of detail which must be chosen *ad hoc* in the absence of information about the MDP, and such fixed bases scale exponentially with dimension. We discuss a novel wavelet-based adaptive function approximation procedure which solves these two problems.

On a multi-domain bivariate Lagrange polynomial based spectral collocation method for solving non-linear evolution partial differential equations

S.S. Motsa
University of KwaZulu-Natal
sandilemotsa@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

In this work we present a multi-domain approach for the solution of non-linear evolution partial differential equations (PDEs). The solution method is based on Chebyshev spectral collocation that employs bivariate Lagrange interpolation polynomials as basis functions. The time domain of the PDE is decomposed into many non-overlapping sub-intervals that are connected by patching based on the assumption that solutions in the different intervals are can be matched using the continuity condition. The solutions of Fisher equation and Burger-Fisher equations are presented to illustrate the effectiveness of the method. The proposed method is validated, in respect of accuracy, against known exact solutions. The evaluation of the advantages of the proposed multi-domain method is conducted by comparison against the performance of the equivalent method in single domains.

Mechanical basis of seashell morphology

D.E. Moulton
University of Oxford
moulton@maths.ox.ac.uk

Topic: Biomathematics

Seashells have intrigued scientists and mathematicians alike for centuries. Over the past several hundred years paleontologists have amassed a huge body of observations on shell form and its evolutionary variation and diversification. However, though the patterns in shells have been well documented, the formation of those patterns has only been explained in functional terms, while the developmental mechanisms underlying shell morphogenesis have remained largely elusive. I will present work on a mechanical basis for shell morphogenesis, using models based on the fundamental principles of the growth process.

We first present a mathematical framework to describe the accretionary growth of shells, based on local shell geometry and utilising differential geometry of curves[1]. We then turn to the nearly ubiquitous presence of shell ornamentation such as spines and ribs. Working within the framework of elasticity, we demonstrate how the mechanical interactions of the accreting tissue of the mollusc and the rigid shell to which it adheres provides a mechanism for the formation of shell ornamentation, and indeed seems to account naturally for the great diversity of ornamentation patterns[2, 3].

References

[1] Moulton, D. E., and Goriely, A., 2012. Surface growth kinematics via local curve evolution. *J Math Biology*, **68**(1-2), 81-108.

[2] Moulton, D. E., Goriely, A., and Chirat, R. (2015). The morpho-mechanical basis of ammonite form. *J Theor Biology*, **364**(C), 220?230.

[3] Chirat, R., Moulton, D. E., and Goriely, A. 2013. Mechanical basis of morphogenesis and convergent evolution of spiny seashells. *PNAS*, **110**(15), 6015–6020.

Towards a single curve to rule them all

N. Nokwara, M. M. Harvey, D. L. Hendricks, D. L. Wilcox, T. J. Gebbie
University of the Witwatersrand
Nkululeko.Nokwara@wits.ac.za

Topic: Risk Analysis and Quantitative Finance

The relationship between price changes associated with the volumes of executed orders is an important property of financial markets. By analysing the transaction data from the Thomson-Reuters Tick History (TRTH) database for the 40 largest stocks on the JSE over a period of 2 years we seek to better understand the behaviour of the response of stock prices to volume across different types of stocks. In particular, we investigate the ability to collapse the different price impact curves into a single impact curve indicative of the broad features of the JSE.

Lie symmetry treatment for pricing options with transactions costs under the fractional Brownian model

B F Nteumagne
University of Pretoria
feuganteu@gmail.com

Topic: Risk Analysis and Quantitative Finance

In the past four years, Option pricing under fractional Brownian motion (fBm) has received a good deal of research. At first, the discussion around the fBm was rather technical and seemingly oblivious. Fractional Brownian motion provides some interesting dynamics and the decision whether this is an applicable model in finance is rather overdue. The interesting question is that of finding what is the suitability of the fBm in finance. i.e., which economic conditions suit the fBm. While finding a good market condition to apply the model is an issue, the solution to the model equation is greater issue. Numerical methods have so far managed to provide a viable approximate to the solution in one dimension. However there remains a relentless quest for a method that can circumvent the curse of dimensionality. Lie theory of group classification provides a systematic transformation of a fractional Black-Scholes partial differential equation (PDE). In this paper, we show how one can apply Lie symmetries to solve the model analytically.

Discrete singular convolution for the generalized variable-coefficient Korteweg-de Vries equation

E Pindza
University of Pretoria
pindzaedson@yaho0.fr

Topic: Partial differential equations, modelling, associated numerical analysis

Numerical solutions of the generalised variable-coefficient Korteweg-de Vries equation are obtained using a discrete singular convolution and a fourth order singly diagonally implicit Runge-Kutta method for space and time discretisation, respectively. The theoretical convergence of the proposed method is rigourously investigated. Test problems including propagation of single solitons and interaction of solitary waves are performed to verify the efficiency and accuracy of the method. The numerical results are checked against available analytical solutions. We find that our approach is a very accurate, efficient and reliable method for solving nonlinear partial differential equations.

Variations around Padé approximation

Michela Redivo-Zaglia
Dept of Mathematics, University of Padua, Italy
E-mail: Michela.RedivoZaglia@unipd.it

Topic: Numerical Analysis, Approximation theory

We consider a particular case of the general rational Hermite interpolation problem where the value of a function f is interpolated at some points, and where the formal series expansions at the origin of the rational interpolant and of f agree as far as possible.

There are two different ways for constructing such a rational interpolant.

Since the denominator of a Padé–type approximant can be freely chosen, we will choose it in order that it also interpolates the function at some points. We thus obtain a Padé–type rational interpolant (PTRI).

Rational interpolation can be achieved by a barycentric formula where the weights can be freely chosen. We will choose them in order that this rational function matches the series f as far as possible. Thus, the Padé–type barycentric interpolant (PTBI) constructed in this way possesses a Padé–type property at the origin. A partial knowledge on the poles and/or the zeros of f can also be took into account.

We can also consider the well known Padé approximants usually written under the form of a rational fraction or as the convergent of a certain continued fraction. But we will show that a Padé approximant can also be written under (at least) two different barycentric rational forms which depend on arbitrary parameters. Such an approximant will be called a *barycentric Padé approximant* (BPA). Then, we will write a Padé approximant under a partial fraction form, called a *partial fraction Padé approximant* (PFPA), with the possibility to impose some poles and/or zeros.

Numerical examples show the interest of these new representations, all based in the computation of their coefficients by solving a linear system.

This is a joint work with Claude Brezinski (Univ. of Lille, France).

Eigenvalues and eigenfunctions for symmetric bilinear forms on a Hilbert space

T Schmid
University of Pretoria
TanjaS@Nedbank.co.za

Topic: Partial differential equations, modelling, associated numerical analysis

Second order hyperbolic partial differential equations are used to model vibration as well as wave motion. In this talk our concern is the usefulness of modal analysis (often referred to as spectral analyis). The idea is that a solution of a model problem can be written as a series using eigenfunctions. Without damping, the associated eigenvalue problem is symmetric (or selfadjoint). For these cases modal analysis proved to be extremely useful and the mathematical theory is well established. In this talk we consider a general symmetric eigenvalue problem in variational form and prove the properties of eigenvalues and eigenfunctions using the Hilbert-Schmidt theory.

Finite time extinction models in population biology and epidemiology

P Sivakumaran
University of Pretoria
preshanthisivakumaran@gmail.com

Topic: Biomathematics

It is common in differential equation models that solutions approach a stable equilibrium without actually reaching it. In many situations this is not a problem since the solutions are treated as approximations. However, in instances when the equilibrium is zero and is related to extinction (of the population) one may obtain qualitatively unrealistic behaviour e.g. a population which is practically extinct but can grow. Here we consider ODE and PDE models with the property that solutions approaching zero at least exponentially are non-zero only on a finite time interval. These include basic population models, space and age structured epidemiological models and models of vector borne diseases.

Comparing the 2π periodic solutions of the nonlinear oscillators obtained by Poincare-Linsdtedts method and second order Van der Pol plane method

MP Skhosana
Vaal University of Technology
spskhosi@yahoo.com / marias@vut.ac.za

Topic: Ordinary differential equations, modelling, associated numerical analysis

The presentation will illustrate one of the perturbation method. The emphasis will be on the direct expansion method as the source of irregularities of the expansion and how to counter act that effect regularization of the expansion. The method chosen to remove any irregularities called secular terms caused by direct expansion in the system will be Poincare-Lindstedts method usually refered to as method of small parameters. Outlining the techniques of obtaining the $2\pi-$periodic time solution of a nearly second order differential equation subjected to harmonic forcing term. The Duffing equation will be used to illustrate the methods of small parameter.

How students can be more efficient in learning Newton-Kantorovich methods

Margaret Spicer
PhD student in Applied Mathematics at UCT Research
Supervisor Dr. Nora Alexeeva
E-mail Margaret.Spicer789@gmail.com

Topic: Numerical Analysis

As a newbie to Matlab, after learning the basics I went on to coding Newton-Kantorovich method and using a finite difference schemes for discretization. As working examples I solve four nonlinear ordinary differentiation equations. This poster deals with ways ways students can learn Newton-Kantorovich methods more effectively and analyses the necessary skills.

The discontinuous Galerkin finite element approximation for the multi-dimensional wave equation

B Stapelberg
University of Pretoria
belinda.stapelberg@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

In this presentation error estimates for the symmetric interior penalty discontinuous Galerkin approximation of the second order multi-dimensional wave equation (Grote et al. 2006) are derived. First a semi-discrete error estimate in the $\mathcal{L}^2$-norm is derived and then a fully discrete estimate is derived by applying the Newmark method to the semi-discrete Galerkin approximation. The results are then combined with the triangle inequality to obtain a final error estimate. The advantages of this approach are that proofs are simplified and less regularity is assumed of the exact solution.

Pattern formation in a coupled reaction-diffusion system

SM Stoltz
Univesity of Pretoria
stoltzstep@gmail.com

Topic: Partial differential equations, modelling, associated numerical analysis

We present a numerical investigation into the pattern formation mechanism in the Brusselator model focusing on the interplay between the Hopf and Turing bifurcations. The dynamics of a Coupled Brusselator model is studied in terms of wavelength and diffusion and provide insight into generation of stationary and oscillatory patterns. The expected asymptotic behaviour is confirmed by numerical simulations. The observed patterns include inverse labyrinth oscillations, inverse hexagonal oscillations, dot hexagons and parallel lines.

Efficient Mathematical methods for Subsurface Energy Recovery

A. Tambue
AIMS South Africa and CERECAM (UCT)
antonio@aims.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

The required exploration and production of hydrocarbons incorporate great technological challenges for the oil and gas industry. Indeed, about 70 hydrocarbon fields that are more than 30 years old. But several of these fields are exhibiting a significant production decline. In order to meet the worlds future demand for oil and gas, further technological advances are essentially needed, where new developments should aim at efficiency and accuracy in subsurface mapping, monitoring of reservoir depletion, and numerical simulation of production scenarios. This requires research across multiple disciplines, including mathematics, geophysics, geology, petroleum engineering, signal processing, and computer science. In this talk, mathematical models for subsurface energy recovery (oil,gas or geothermal energy) will be provided, along with some efficient numerical methods for their approximations. We will end by presenting some numerical simulations of production scenarios.

Approximating Nehari interpolants: The rational matrix function case

S. ter Horst
North West University
sanne.terhorst@nwu.ac.za

Topic: Approximation theory

The matrix-valued (optimal) Nehari problem asks for an (best) approximation of a matrix-valued L^∞ function by a matrix-valued H^∞ function, with respect to the supremum norm. Various methods to obtain such solutions exist, yet, if the initial L^∞ function is irrational or rational but of a large McMillan degree, one needs additional methods to make computation efficient, possibly relaxing the interpolation or the norm condition. Various model reduction methods exist, where typically the interpolation condition is relaxed. In this talk we consider an interpolatory method, where the optimal solution of the Nehari problem in question is approximated by an exact optimal interpolant with respect to an alternative norm.

Solid Rocket Motor Thrust Modelling Using Level Set Methods

MH Tshokotsha and MF Maritz
Stellenbosch University
17772842@sun.ac.za, mfmaritz@sun.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

Solid Rocket Motors (SRM's) are propulsive systems providing thrust from gas exiting through a nozzle, obtained by burning a solid fuel (the grain) inside a chamber. Various thrust profiles can be obtained by different initial grain geometries. The purpose of numerical modeling, is to predict the thrust curve, given an initial geometry.

In this presentation we discuss a simplified model for SRM behaviour. The grain geometry is represented by a signed distance function, and the burn-back is calculated using a numerical solution of the burning process based on level-set methods. The gas flux is modelled using a 1-D approximation of the gas dynamics inside the chamber, following a method proposed by Lamberty.

Examples of typical thrust curves that were modelled will be shown.

One-dimensional models for the motion of an elastic solid

NFJ van Rensburg
University of Pretoria
nic.vanrensburg@up.ac.za

Topic: Partial differential equations, modelling, associated numerical analysis

Although objects are three-dimensional, one-dimensional models are often used. Ideally the model should be as simple as possible but at the same time sufficiently realistic. A significant number of different models are possible varying from a complex model involving a system of nonlinear partial differential equations to the well known linear wave equation. In this presentation we discuss the mathematics involved in the modeling process.

The usefullness of a particular model is determined by the properties of solutions. Therefore the modeling process is not complete without further analysis or numerical experiments. In this presentation we consider comparison of mathematical models.

On some developments with regards to the Finite Element Method in Option Pricing

A.J. van Zyl
University of Pretoria
gusti.vanzyl@up.ac.za

Topic: Risk Analysis and Quantitative Finance

The partial differential equations (PDEs) arising in the pricing of financial options pose problems that are often not within the assumptions made in general expositions of the theory of parabolic PDEs. For instance, the spatial domain is unbounded, and the initial condition is typically not square integrable.

For this and other reasons, the literature that is potentially relevant to the use of the Finite Element Method in the pricing of options is quite extensive. In this talk we wish to draw attention to some theoretical developments that either came after, or are outside of the scope, of the main current survey papers on this topic. For example, the question of what errors arise when one truncates the domain to construct the grid, now has some good answers, at least in the Geometric Brownian Motion world.

List of Participants

Surname and Initials	Affiliation	Email
A Aderogba	University of Pretoria	Aderogba.Aderogba@up.ac.za
MM Ali	WITS	montaz.ali@wits.ac.za
R Anguelov	University of Pretoria	Roumen.anguelov@up.ac.za
CR Bambe Moutsinga	Tshwane Univ of Technology	MoutsingaCB@tut.ac.za
J Banasiak	University of KwaZulu-Natal	banasiak@ukzn.ac.za
M Banda	University of Pretoria	Mapundi.banda@up.ac.za
JJH Bashingwa	WITS	jeanjuste@aims.ac.za
M Basson	University of Pretoria	madelein.basson@up.ac.za
A Bogaers	CSIR	ABogaers@csir.co.za
N Botha	CSIR	Nbotha@csir.co.za
C Brezinski	Université des Sciences et Technologies de Lille	claude.brezinski@univ-lille1.fr
M Chapwanya	University of Pretoria	Michael.chapwanya@up.ac.za
S Das	CSIR	SDas@csir.co.za
J Djoko Kamdem	University of Pretoria	jules.DjokoKamdem@up.ac.za
HM Djouosseu-Tenkam	University of Pretoria	michel.djouosseu@up.ac.za
S Du Toit	University of Pretoria	sonja.DuToit@up.ac.za
CC Dufourd	University of Pretoria	Claire.Dufourd@up.ac.za
NKK Dukuza	University of Pretoria	Kenneth.Dukuza@up.ac.za
J Engelbrecht	Stellenbosch University	16265262@sun.ac.za
I Fabris-Rotelli	University of Pretoria	inger.fabris-rotelli@up.ac.za
M Fasondini	University of the Free State	fasondinim@ufs.ac.za
MK Folly-Gbetoula	WITS	Mensah.Folly-Gbetoula@wits.ac.za
S Garba	University of Pretoria	salisu.garba@up.ac.za
MR Gavhi	AIMS	joygavhi@gmail.com
JD Gertenbach	North-West University	Jan.Gertenbach@nwu.ac.za
HH Gidey	University of Pretoria	hagoshigd@gmail.com
A Gumel	Arizona State University	agumel@asu.edu
N Hale	Stellenbosch University	nick.p.hale@gmail.com
AS Hassan	University of Pretoria	Adamu.Hassan@tuks.co.za
DL Hendriks	WITS	737654@students.wits.ac.za
RS Herbst	WITS	Sheldon.Herbst@wits.ac.za
A Hoogkamer	University of Pretoria	anyahkr@gmail.com
N Hussaini	University of Pretoria	nafiu.hussaini@up.ac.za
BA Jacobs	WITS	byron.jacobs@wits.ac.za
L Janse van Rensburg	University of Pretoria	liesljvrensburg.ljvr@gmail.com
C Janse van Rensburg	University of Pretoria	churlybear@gmail.com
OA Jejeniwa	University of Pretoria	jejeniwaayo@yahoo.com
AS Jooste	University of Pretoria	alta.jooste@up.ac.za
KH Jordaan	University of Pretoria	Kerstin.jordaan@up.ac.za
M Kekana	CSIR	Mkekana@csir.co.za
A Kelil	University of Pretoria	u14073073@tuks.co.za
A Kijko	University of Pretoria	andrzej.kijko@up.ac.za
KH Lau	University of Pretoria	alexlau1712@gmail.com
JM-S Lubuma	University of Pretoria	jean.lubuma@up.ac.za

Surname and Initials	Affiliation	Email
E Mare	University of Pretoria	Eben.Mare@stanlib.com
MF Maritz	Stellenbosch University	mfmaritz@sun.ac.za
MR Mitchley	WITS	Michael.Mitchley@wits.ac.za
N Mofoka	CSIR	Nmofoka@csir.co.za
S Motsa	University of KwaZulu-Natal	sandilemotsa@gmail.com
DE Moulton	Unversity of Oxford	moulton@maths.ox.ac.uk
N Nokwara	WITS	Nkululeko.Nokwara@wits.ac.za
BF Nteumagne	University of Pretoria	feuganteu@gmail.com
V Pavlenko	University of Pretoria	pavlenko.vasily@gmail.com
E Pindza	University of Pretoria	pindzaedson@yahoo.fr
M Redivo-Zaglia	University of Padova	michela@math.unipd.it
I Rewitzky	Stellenbosch University	rewitzky@sun.ac.za
T Schmid	University of Pretoria	tschmid92@gmail.com
P Sivakumaran	University of Pretoria	preshanthisivakumaran@gmail.com
N Siyakatshana	CSIR	NSiyakatshana@csir.co.za
MP Skhosana	Vaal University of Technology	marias@vut.ac.za
A Smit	University of Pretoria	ansie.smit@up.ac.za
M Spicer	University of Cape Town	margaret.spicer789@gmail.com
B Stapelberg	University of Pretoria	belinnie.stap@gmail.com
A Stein	University of Twente	a.stein@utwente.nl
SM Stoltz	University of Pretoria	stoltzstep@gmail.com
A Tambue	AIMS and University of Cape Town	antonio@aims.ac.za
S Ter Horst	North-West University	Sanne.TerHorst@nwu.ac.za
Y Terefe	University of Pretoria	yibeltal.terefe@up.ac.za
B Tsanou	University of Pretoria	Berge.Tsanou@up.ac.za
MH Tshokotsha	Stellenbosch University	17772842@sun.ac.za
NFJ Van Rensburg	University of Pretoria	nic.vanrensburg@up.ac.za
AJ Van Zyl	University of Pretoria	gusti.vanzyl@up.ac.za
JAC Weideman	Stellenbosch University	weideman@sun.ac.za
GY Weldegiyorgis	University of Pretoria	WeldegiyorgisGY@tuks.co.za