

41st South African Symposium of Numerical and
Applied Mathematics (SANUM) 2017
March 28 - March 30 2017

University of the Witwatersrand



WITS
UNIVERSITY



Welcome Address

The School of Computer Science and Applied Mathematics and Faculty of Science at the University of the Witwatersrand is proud to welcome our plenary speakers and conference delegates to the 41st annual South African Symposium of Numerical and Applied Mathematics (SANUM).

Mathematical sciences play an important role in the development of South Africa through the training of the next generation of mathematicians and applied mathematicians. SANUM has been established as an important component of this development, through engagement of scientists from across the country, continent and the world.

Year after year the SANUM community grows and the range of topics grows with it. This range includes, ordinary and partial differential equations, numerical methods and analysis, image processing, industrial problems, population dynamics, biomedical applications, astrophysics and more.

We hope that your engagement at SANUM will be productive and stimulating.

Byron Jacobs
on behalf of the SANUM local organising committee.

Logistics

Wifi

Wifi is available at the venues, with the credentials below.

SSID: Wits-Conference

Password: P@\$\$word_123

Session Chairs

We ask that the second speaker in a session chair for the first speaker and the first speaker chair for the remainder of each session.

Conference Organising Committee

If you have any questions, comments or concerns please feel free to contact any of the conference organising committee members:

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		Tuesday		28 March		Wednesday		29 March		Thursday		30 March	
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8:00	8:30	Tea, coffee, and registration				Tea and coffee: 8:30 - 9:00				Tea and coffee: 8:30 - 9:00			
8:30	8:45	Welcome:											
9:00	9:30	Plenary: Robert Weiss				Plenary: Luca Gerardo-Giorda				Fabris-Rotelli			
9:30	10:00									Banda			
10:00	10:30	Plenary: Nigel Bishop				Plenary: Ebrahim Momoniat				Netshivha			
10:30	11:00									Zongo			
11:00	11:30	Tea and coffee				Tea and coffee				Tea and coffee			
		Special Session		Special Session		Special Session		Special Session		Special Session			
11:30	12:00	Weideman	Kara			Chipoyera	Harley			Oke			
12:00	12:30	Hale	Mason			Platt	Stapelberg			Egbelowo			
12:30	13:00	Anguelov	Carrim			Olupitan	Tladi			Mohammed			
13:00	13:30		Nchabeleng				Magan			Herbst			
13:30	14:00	Lunch				Lunch				Lunch			
14:00	14:30												
14:30	15:00												
15:00	15:30	Ojo				Van Rensburg				Key	Mins		
15:30	16:00	Ojewola				Labuschagne				Plenary talk		60	
16:00	16:30	Nuugulu				du Toit				Room 1		30	
16:30	17:00	Amima				Ngnotchouye				Room 2		30	
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Plenary Abstracts

The Numerical Calculation of Gravitational Waves

Nigel Bishop
Rhodes University

Einstein's equations form a nonlinear system of hyperbolic partial differential equations, and can be solved exactly only in cases of high symmetry. As a first step towards obtaining numerical solutions, the equations need to be recast from 4-dimensional form into one where the evolution is explicit. There are various ways of doing this, developed in the 1960s and subsequently. The numerical method applied to these equations has normally been finite differencing, but recently spectral methods have been widely used for certain types of problem. The numerical solution of Einstein's equations requires substantial computer resources, and so in practice such methods are used only outside the weak-field limit when there is no alternative. This includes the most powerful burst sources of gravitational waves, such as supernova explosions and compact object coalescence, where compact object can be either a black hole or a neutron star.

Some Mathematical and Numerical Aspects of Brain Activity Modeling

Luca Gerardo-Giorda
Basque Center for Applied Mathematics

Among the most complex problems in Biomedical science, understanding the functioning of the human brain is still one of the biggest challenges of science. Several dynamics coexist in a working brain, each more or less understood in its own, from the electrophysiological activity of the neurons, to the metabolic cycle of the neuron-astrocyte complex and the blood flow in the circulatory system of the Circle of Willis. In order to draw a more comprehensive and coherent picture of the working human brain, computational models are today more relevant than ever. In this talk I will discuss the modeling (and associated challenges) of the local electro-metabolic dynamics in the neuron-astrocyte complex, and the macroscopic propagation of the Cortical Spreading Depression (CSD). The former is a lumped model, coupled through a feedback process of energy supply/demand, that is inherently multiscale due to the different temporal scales of the electrophysiological and metabolic dynamics. The second is a distributed model, describing the progression,

from the visual cortex to the peripheral areas, of a depolarisation wave that has been deemed a correlate of visual aura, a neurological phenomenon preceding migraine. As of today, little is known about the mechanisms that can trigger or stop CSD, but the complex and highly individual characteristics of the brain cortex suggest that the geometry might have a significant impact in its propagation. I will introduce patient-specific computational models to cope with variability and anisotropy of cortical geometries, and discuss our simulation results highlighting the importance of model personalization.

Sliding contacts for thin films on a cylindrical coordinate system

Ebrahim Momoniat

University of the Witwatersrand

In this talk I derive a Reynolds equations for chemical mechanical polishing (CMP) and pin-on-disk testing from the Navier-Stokes equations. Coriolis force effects are included in the derivation of the model equations. Half-Sommerfeld boundary conditions are imposed on the numerical solutions of the model equation. The results will be discussed and their implications to the industrial processes of CMP and pin-on-disk testing discussed.

Tsunamis from Asteroid Impacts and Submarine Landslide: Numerical Simulations of the Loudest and Quietest Tsunami Hazards

Robert Weiss

Virginia Polytechnic Institute and State University

Tsunami waves generated during asteroid impacts into oceans and submarine landslides create waves whose characteristics (wave period and amplitude) is significantly different from those generated by more common tsunami sources, such as earthquakes. Earthquake-generated tsunami waves at least are 20 times longer than the ocean depth. Tsunami waves generated by asteroid impacts or submarine landslides are not; they are shorter, but often exhibit much larger amplitudes. Due to the difference in wave period and amplitude, tsunami waves generated by asteroid impacts and submarine landslide experience physical processes, such as dispersion, that require higher-order derivatives in the governing equations to simulate the respective physical processes. Obviously robust and stable numerical solutions of governing equations with higher-order derivatives require smarter numerical methods and stronger computational resources. While I am going to touch upon these issues in my talk, I will focus on the generation mechanisms of tsunami waves by asteroid impacts and submarine landslides. I will introduce the physical processes that govern the characteristics of both generation mechanisms, present examples where they unleashed their destructive power, and introduce a numerical tool to simulate both processes for tsunami simulations to present some quantitative analysis.

Conference Abstracts

Topological dynamic consistency of numerical schemes

Roumen Anguelov
University of Pretoria

There has been a significant research interest in characterizing numerical methods with respect to how well they replicate qualitatively the properties of the model approximate. The interest is from different areas and aspects of numerical analysis. Particularly, there has been such interest from researchers in the Nonstandard Finite Difference Method, since replicating essential properties of the original model is a primary goal of these methods. In the specific setting of dynamical systems, the concept of dynamic consistency is widely used, often with different meaning, to depict the qualitative links between the dynamical system and its numerical method. In this talk we introduce the concept of topological dynamic consistency as a topological conjugacy between the flow of a dynamical system and its numerical discretization. Considering that the properties of interest in a dynamical system are in essence topological properties, this concept indeed captures well the expectations from a qualitative point of view that a numerical method should satisfy. We discuss appropriate tools and techniques for practical application of this definition to characterize existing numerical methods and to constructing numerical schemes which are topologically dynamically consistent.

On optimization in gas networks using the space-mapping approach

Mapundi Kondwani Banda

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In a gas network, different domains of the network can be modelled by different models depending on the requisite qualitative detail required: a full Euler system of equations; an isothermal Euler system of equations; a linearized model derived from the isothermal Euler system or a steady state model of gas flow also referred to as an algebraic model. A proposal was made to improve the efficiency of optimisation in gas networks based on a space-mapping approach [1]. Therein a model hierarchy was exploited to define an appropriate multi-fidelity approach. In this talk the mathematical formulation and algorithmic aspects of a space-mapping technique for an

optimization problem, with evolutionary partial differential equations as constraints, are developed. Some issues that may pose as challenges as well as opportunities will be highlighted.

- 1 M. K. Banda, M. Herty, "Towards a Space Mapping Approach to Dynamic Compressor Optimization of Gas Networks", *ptim. Control Appl. Meth.* (2011), **32**(3), pp. 253 - 269, DOI: 10.1002/oca.929

Unsteady magnetohydrodynamic flow of a fourth grade fluid caused by an impulsively moving plate in a Darcy porous medium: A group-theoretical analysis

Abdul Hamid Carrim¹, Taha Aziz¹, Fazal Mahomed¹ and Chaudry Masood²

¹University of the Witwatersrand,²North-West University

The effects of non-Newtonian fluids are investigated by means of an appropriate model studying the flow of a fourth grade fluid. The geometry of this model is described by the unsteady unidirectional flow of an incompressible fluid over an infinite flat plate within a porous medium. The fluid is electrically conducting in the presence of a uniform applied magnetic field. The classical Lie symmetry approach is utilized in order to construct group invariant solutions to the governing higher-order nonlinear partial differential equation (PDE). The conditional symmetry approach has also been utilized to solve the governing model. Some new classes of conditional symmetry solutions have been obtained for the model equation in the form of closed-form exponential functions. The invariant solution corresponding to the nontraveling wave type is considered to be the most significant solution for the fluid flow model under investigation since it directly incorporates the physical behavior of the flow model.

Comparison of various linear and nonlinear beam models

Sonja du Toit

University of Pretoria

Beam models are frequently used in engineering applications, but research on theoretical issues and numerical methods is still an ongoing and active field.

We consider a nonlinear model for possibly large motion of a linearly elastic beam. The theory is incomplete at this stage. However, a numerical method based on finite elements has been developed and the results are more than satisfactory.

In this talk numerical results are used to illustrate convergence for the nonlinear beam model. We also compare approximations obtained for the nonlinear model to the exact solution of the linear model. Lastly, simulations are used to determine whether properties of the beam models are acceptable.

Nonstandard finite difference method of solving a nonlinear one-compartment pharmacokinetic models

Oluwaseun F. Egbelowo, Charis Harley and Byron A. Jacobs
University of the Witwatersrand

We provide nonstandard finite difference (NSFD) schemes to one-compartment nonlinear pharmacokinetics models. NSFD rules based on Mickens' idea was use to transfer the nonlinear models into discrete schemes. The method was compared with established methods to verify the efficiency and the accuracy of the method. Simulation of the one-compartment pharmacokinetic model with different routes of administration was made.

The LULU-based connected median filter to remove noise from images

Inger Fabris-Rotelli¹, Alfred Stein^{1,2} and Magnus van Niekerk¹
¹University of Pretoria, ²University of Twente, Netherlands

A new median smoothing filter for images is proposed. This median filter combines the idea of level sets/connected components in the LULU filters in images with an algorithm similar to that of the adaptive median filter. An algorithm is given and we compare the new filter to the adaptive median for images.

A fast and well-conditioned spectral method for Fredholm and Volterra integro-differential equations with convolution kernels

Nick Hale
Stellenbosch University

We introduce a Legendre–Petrov–Galerkin spectral method for the solution of integro-differential equations with convolution kernels, for example

$$y'(t) + a(t)y = f(t) - \int_0^t K(t-s)y(s) ds, \quad y(0) = y_0,$$

although the approach extends naturally to higher-order derivatives.

If the kernel $K(t)$ and the ODE coefficients $a(t)$ are sufficiently smooth then the computed solution converges geometrically fast in the degrees of freedom (i.e., the polynomial degree of the approximation) to the true solution. Furthermore, the resulting almost-banded matrix systems can be solved in linear time.

The algorithm combines the (a) Legendre-Ultraspherical spectral method [1,2] with (b) a convenient formula for the convolution of two Legendre series [3]. In this talk we briefly reintroduce (a) and (b), show how they may be combined to solve integro-differential equations, and provide examples to demonstrate the claims of accuracy and efficiency above.

- 1 S. Olver and A. Townsend, “A fast and well-conditioned spectral method”, SIAM Review, 2013.
 - 2 N Hale and S. Olver, “A fast and spectrally convergent algorithm for fractional integral and differential equations with half-integer order terms”, Submitted to SISC, Nov 2016.
 - 3 N Hale and A. Townsend, “An algorithm for the convolution of Legendre series”, SISC 2014
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Flow Reversal in an Expanding Channel

Charis Harley¹, Ebrahim Momoniat¹ and Kumbakonam R. Rajagopal²

¹University of the Witwatersrand, ²Texas A&M University

The flow of an incompressible fluid through convergent-divergent channels is considered where the choice of the viscosity is such that the stress tensor is not degenerate in nature. We observe the appearance of boundary layers for the non-Newtonian fluid, even in the case of divergent flow. Sharp and pronounced boundary layers develop adjacent to the solid boundaries, even at zero Reynolds number. Furthermore, for increasing values of the angle, we detect flow reversal; i.e. different flow regimes are observed, namely inflow and outflow. We are also able to gauge the impact of introducing a traction boundary condition on the behaviour of the fluid, and observe asymmetric solutions as a consequence.

The Invariance and Conservation Laws of Discrete/Difference Equations

Abdul Hamid Kara, Mensah Folly-Gbetoula

University of the Witwatersrand

The theory and reasoning behind the construction of symmetries for differential equations (DEs) is now well established and documented. Moreover, the application of these in the analysis of DEs, in particular, for finding exact solutions, is widely used in a variety of areas from relativity to fluid mechanics. Secondly, the relationship between symmetries and conservation laws has been a subject of interest since Noether’s celebrated work for variational DEs. The extension of this relationship to DEs which may not be variational has been done more recently. The first consequence of this interplay has led to the double reduction of DEs. In this talk, we apply these concepts to some classes of Difference Equations.

Multiple-beam model for a building subjected to earthquake induced oscillations

Madelein Labuschagne
University of Pretoria

Damage to non-structural components and building contents due to earthquakes can result in billions of dollars of losses in a single year. To investigate accelerations for this purpose, buildings are usually modelled as a single beam. Research indicates however that it is necessary to use a model where there are rigid bodies that link the flexible parts of a building. In this presentation we consider such a multiple-beam model where we link Timoshenko beams with rigid bodies representing the floors. A special version of the Finite Element Galerkin approximation is used to handle the interface conditions. Simulations of earthquake induced oscillations using simple basis functions will also be shown.

Asymptotic solutions of nonlinear wave equations described by a new class of constitutive equations

Avnish Magan, Charis Harley
University of the Witwatersrand

The propagation of displacement waves and stress waves for implicit constitutive equations are investigated in this paper. The new class constitutive equations contain Cauchy elastic and hyperelastic bodies as subclasses. In this talk we consider a particular subclass where the strain is expressed in terms of a non-invertible function of the stress. These types of constitutive equations describe elastic responses where the stress and linearised strain are nonlinearly related. Such a phenomenon cannot be captured in the classical theory. We will study two different constitutive equations which describe a nonlinear relationship between the stress and linearised strain. The first constitutive equation is analogous to the constitutive equation for a power-law fluid and the second constitutive equation can describe elastic bodies which exhibit limiting strain. The special semi-inverse solution gives a system of nonlinear hyperbolic equations. These systems can be written as a single partial differential equation. We find that the implicit asymptotic solutions for this partial differential equation describe shear stress waves. The system of equations is solved by considering straightforward perturbation expansions for the stress and displacement. Two different approaches are considered. Firstly, the equations at each order are reduced to canonical form and the canonical equations are solved to give travelling wave solutions. In the second approach we find separable solutions which also describe travelling wave solutions. The parameter n influences the shape and speed of the wave and it is found that as the value of n increases the distortion of the waves becomes more apparent and the speed of the wave decreases.

Fracture development in rock based on the extension strain criterion

David Mason¹, Colin Please², Neville Fowkes³, Keegan Anderson⁴, Jonathan Esteves¹ and Richard Stacey¹

¹University of the Witwatersrand, ²University of Oxford, ³University of Western Australia, ⁴University of Johannesburg

The presentation is based on work done on a problem submitted by Professor Richard Stacey to the Mathematics in Industry Study Group meeting held at the African Institute for Mathematical Sciences in Cape Town in January 2017. The complex behaviour of rock masses has presented paradoxes, one of which is the fracturing of seemingly strong rock under low stress conditions. These conditions often occur near excavation boundaries where the confining stress is low. The vertical compressive stress at a depth must be sufficient to support the weight of the overburden rock mass and the horizontal compressive stress is obtained from Heime's rule. In order to understand rock fracture and failure around excavations the stress around an excavation in the form of a cylindrical tunnel was calculated using the plain strain theory of elasticity. It was found that there could be extension in the radial direction even although the three normal principal stresses are compressive. This will lead to rock failure and fracture if the extension strain in the rock exceeds the critical value of extension strain.

Introduction to Adaptive Dynamics: An Application to Interacting Species

Mozzamil Mohammed
Stellenbosch University

Adaptive dynamics theory is a general mathematical framework to study the long-term evolutionary dynamics of coevolving systems. The theory investigates the evolution of phenotypic traits when a sufficiently rare mutant infrequently appears in a large resident population and determines the fate of such a mutant. It studies the ecological systems in the evolutionary time scale (slower time scale relative to the ecological time scale), and links the population dynamics with the evolutionary dynamics. We present the general framework of adaptive dynamics and the classifications of the singular strategy at which the selection gradient vanishes including the Evolutionary Stable Strategy (ESS), Convergence Stable Strategy, Mutual invisibility and Evolutionary Branching Conditions. An application to the theory in ecological interaction will be presented.

Hydraulic fracture with Darcy flow in a porous medium

Mathibele Nchabeleng, Adewunmi Gideon Fareo
University of the Witwatersrand

This research is concerned with the analysis of a two-dimensional Newtonian fluid-driven fracture. The flow of fluid in the fracture is laminar and the fracture is driven by the injection of an incompressible Newtonian fluid into it. The fluid leak-off into the rock-mass is modeled using Darcys law. With the aid of lubrication theory and the Perkins-Kern-Nordgren approximation, a system of nonlinear partial differential equations for the fracture half-width and the extent of leak-off are derived. The theory of Lie group analysis of differential equations is used to solve the nonlinear coupled system of partial differential equations to obtain group invariant solutions for the fracture half-width, leak-off depth and length of the fracture.

Weak convergence for a stochastic exponential integrator and finite element discretization of stochastic partial differential equations with multiplicative & additive noise

Jean Medard T Ngnotchouye¹ and Antoine Tambue²

¹University of KwaZulu-Natal, ²African Institute for Mathematical Science

We consider a finite element approximation of a general semi-linear stochastic partial differential equation (SPDE) driven by space-time multiplicative and additive noise. We examine the full weak convergence rate of the exponential Euler scheme when the linear operator is self adjoint and provide preliminaries results toward the full weak convergence rate for non-self-adjoint linear operator. A key part of the proof does not rely on Malliavin calculus. Depending on the regularity of the noise and the initial solution, we found that in some cases the rate of weak convergence is twice the rate of the strong convergence. Our convergence rate is in agreement with some numerical results in two dimensions.

On the relationship between M/M/1 and M/G/1 queue analysis

Olumuyiwa Olupitan¹ and Taye Faniran²

¹ Achievers University Owo, ²Lead-City University Ibadan

In this work, analysis of an M/G/1 queueing system is treated. An M/G/1 system is a queueing system having exponentially distributed interarrival time with parameter λ ; generally distributed service times which is arbitrary; single server; and infinite capacity. With the M/M/1 queueing system having a stochastic process such that arrivals occur at rate λ according to a Poisson process and move the process from state i to $i + 1$. Service times have an exponential distribution with rate parameter μ the M/M/1 queue, where $1/\mu$ is the mean service time. This is the same continuous time Markov chain as in a birthdeath process. In order to make proper decision on

either to wait on a queue or not, it is proper to generate a model and technique which helps to analyze the effect of investment on waiting time. Condition of stability in the system, steady state, average number of customers and response time are determined which afford a relationship between the M/M/1 and M/G/1 system.

Can Agent-Based Models Probe Market Microstructure?

Donovan Platt and Tim Gebbie
University of the Witwatersrand

We extend prior evidence that naively using intraday agent-based models that involve realistic order-matching processes for modeling continuous-time double auction markets seems to fail to be able to provide a robust link between data and many model parameters, even when these models are able to reproduce a number of well-known stylized facts of return time series. We demonstrate that while the parameters of intraday agent-based models rooted in market microstructure can be meaningfully calibrated, those exclusively related to agent behaviors and incentives remain problematic. This could simply be a failure of the calibration techniques used but we argue that the observed parameter degeneracies are most likely a consequence of the realistic matching processes employed in these models. This suggests that alternative approaches to linking data, phenomenology and market structure may be necessary and that the stylized fact-centric validation of intraday agent-based models is insufficient, and warns that increased mechanistic complexity of agent-based market models may lead to flawed insights.

Existence and uniqueness of solutions of linear second order hyperbolic problems

Belinda Stapelberg
University of Pretoria

We consider a general linear second order hyperbolic problem in variational form. Models for the vibration of systems of elastic bodies can be written in this form. We show how different types of damping influence the theory. Results in previous publications and limitations to the applicability of these results are discussed. We also present improvements to the theory. Examples are provided.

Existence and Stability of Shock-Wave Solutions to a Class of Riemann Problems to Systems of Conservation Laws

Maleafisha Tladi

University of Limpopo

Conservation laws with singular shocks are emerging as a powerful tool for modelling challenging multiscale phenomena including overlapping microscopic and macroscopic scales, anomalous transport, and long range time memory or spatial interactions. However, these problems raise new theoretical, computational, and experimental aspects of dynamical systems that have not been encountered in the context of classical conservation laws. Viscous singular shock profile problems for conservation laws are coherent structures in nonlinear differential equations that are sought by reducing the underlying partial differential equation to the travelling wave ordinary differential equation that involve the construction of specific orbits such as homoclinic, heteroclinic, or periodic orbits. Geometric singular perturbation (GSP) theory and Melnikov techniques are utilized to address the question of the existence and stability of shock waves to a class of Riemann problems to systems of conservation laws. Singular shock waves provide one of the principal tools for solving Riemann problems which cannot be solved by classical shocks and rarefactions. We investigate a general mechanism, utilizing nonclassical shock waves, for nonuniqueness of solutions of Riemann initial-value problems for systems of two conservation laws. This nonuniqueness occurs whenever there exists a pair of viscous shock waves forming a 2-cycle. We prove that a 2-cycle gives rise to an open region of Riemann data for which there exist multiple solutions of the Riemann problem, and we determine all solutions within a certain class. We also present results from associated numerical analysis that illustrate how these solutions arise in the time-asymptotic limit of solutions of the conservation laws, as augmented by viscosity terms. Our results shed light on the behavior of conservation laws with singular shocks and advance the mathematical theory of singular shocks.

Locally linear rod or beam models

Nic van Rensburg

University of Pretoria

In applications beam models are often encountered. Engineers and (applied) mathematicians use various linear and nonlinear beam models and are often rather vague about assumptions such as “thick”, “thin”, “small vibrations”, “large motion”, etc. Comparisons of different models for the same situation are rare.

In this presentation, we derive a model for planar motion of a beam where the material is linearly elastic. The model allows for bending, shear and elongation. Since large motion is possible, the model problem is nonlinear. Simpler models, even linear models, can be derived in a systematic way by making additional assumptions.

A Gauss-Hermite Quadrature Method for the Inversion of the Laplace Transform

André Weideman
Stellenbosch University

Many numerical inversion methods for the Laplace transform approximate the complex contour integral formula known as the Bromwich integral. This approach has two key ingredients: (a) a choice of quadrature rule, and (b) a choice of contour. The latter choice is a freedom offered by Cauchy's theorem, which allows one to deform the original Bromwich line (parallel to the imaginary axis) to a contour that is better suited for numerical computation.

One of the most popular methods in this family is Talbot's method. It uses (a) the trapezoidal rule, on (b) a cotangent contour. The contour begins and ends in the left half-plane, the idea being that on such a contour the exponential factor in the Bromwich integral produces a rapidly decaying integrand. This means that the integral can be approximated efficiently by the trapezoidal rule.

In this talk we present a new idea in which we use (a) Gauss-Hermite quadrature on (b) a parabolic contour. On such a contour, the exponential factor in the Bromwich integral produces a decay of Gaussian type. This means that the integral can be approximated efficiently by Hermite quadrature.

In both the Talbot method and the Hermite method there is a number of parameters that can be tuned for optimal accuracy. In the case of Talbot's method this has been discussed by the speaker and co-workers in a series of papers published in the last decade. In this talk it is demonstrated how it can be done for the Hermite method, specifically for the important special case where the singularities of the transform are located on the negative real axis.

Assuming such an optimal choice of parameters, we present several test examples which show that the Hermite method converges faster than the Talbot method. The improvement is not spectacular but in a typical problem the Hermite method could save around three or four function evaluations. This can be significant when the transform is expensive to evaluate, such as when PDEs are solved.

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