The 42nd Annual South African Symposium on Numerical & Applied Mathematics (SANUM)

4th to 6th April 2018

Department of Mathematical Sciences Stellenbosch University, South Africa

Local organising committee:
Nick Hale, Andie de Villiers & André Weideman

The organising committee acknowledges sponsorship from the Centre of Excellence - Mathematical and Statistical Sciences

Programme:

		WEDNESDAY 4th April		THURSDAY 5th April		FRIDAY 6th April	
From	To	Room 2002	Room 3001	Room 2002	Room 3001	Room 2002	Room 3001
		Tea, coffee, and registration: 8:00 - 8:45		Tea and coffee: 8:30 - 9:00		Tea and coffee: 8:30 - 9:00	
8:45	9:00	Welcome: Weideman					
9:00	9:50	Plenary: Chapman		Plenary: Simoncini		Plenary: Milewski	
9:50	10:40	Plenary: Jordaan		Plenary: Rahut		Plenary: Patidar	
10:40	11:10	Tea and coffee in room 1005		Tea and coffee in room 1005		Tea and coffee in room 1005	
			Industrial mathematics		*Data Science*		*FEM*
11:10	11:35	Jooste	Hutchinson	Kirsten	Bouchot	Magan	Marewo
11:35	12:00	Gagandeep	Mason	Maritz	Fabris-Rotelli	Tladi	Weldegiyorgis
12:00	12:25	van Nieuwkerk	Ali	Falcone	Tahiridimbisoa	Weideman	de Villiers
12:25	13:45	Lunch @ Katjiepiering restaurant		Lunch @ Katjiepiering restaurant		Lunch in Ind. Psych. building	
13:45	14:35	Plenary: Huybrechs		Plenary: Dorey			
			Industrial mathematics		*Data Science*		
14:35	15:00	Webb	Amikiya	Alexeeva	Sonono		
15:00	15:25	Samuel	Attipoe	Barashenkov	Steyn		
15:25	15:50	Mungwe	Koffi	Kara	Zvarevashe		
15:50	16:20	Tea and coffee		Tea and coffee			
			Industrial mathematics	*Data Science*			
16:20	16:45	van Rensburg	Woolway	Zimmerman & Herbst			
16:45	17:10		Chapman				
17:10	17:35	Conference photograph @ 17:10					
		Reception: 17:30 @ Katjiepiering					
19:00				Conference dinner: 19:00 @ Neethlingshof			

Key	Mins
Plenary talk	50
Contributed talk	25
Special session	25
Tea/coffee	30
Lunch time	80

For changes to the timetable, see http://sanum.github.io/programme.html

Presentation Details:

- Please load your talks on the available machines **before** the session in which you are speaking.
- If you are using your own laptop for the presentation, please check the connections in **advance**.
- A laser pointer and a clicker will be available in each room.
- Contributed and special session talks will be kept **strictly** to 25 (20+5) minutes.
- We ask that the first speaker in each contributed session (counting the 14:35–17:35 block as one session) will chair, with the second speaker acting as chair for the first talk.
- Chairs for the special sessions will be arranged by the sessions organiser(s).

Welcome to SANUM! The SANUM meeting came into existence in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York), happened to visit South Africa at the same time. A number of local researchers used the opportunity to organise a meeting at the (then) University of Natal, at which a total of ten papers were presented. From here the meeting grew in strength, with numerous famous numerical and applied mathematicians passing through. Aside from missing a single year (1996) and on one occasion (1997) being a mere special session of a larger conference, the meeting has been held annually, which makes 2018 the 42nd occurrence of this major event in the calendar of South African numerical and applied mathematics.

Conference Venue: The 2018 conference is held on the first two floors of the Mathematical Sciences building (building number 67 on the campus map – next page). Registration takes place on 4th April from 08:00 to 08:45 in the foyer on the ground floor.

Wifi details: Will be made available at conference registration on the first day.

Tea & Coffee: Tea & Coffee will be in room 1006 on the ground floor. Early morning coffee will available 08:00–08:45 on Wednesday and 08:30–09:00 on Thursday and Friday.

Water restrictions: Significantly lower than average rainfalls over the past few years mean that there is currently a severe water shortage in much of the Western Cape, and the Cape Town area is currently under level 6b water restrictions. This means that each person may use no more that 50 litres of water per day. We ask that delegates please adhere to this and keep all water usage to a minimum.

Lunch Venue: Lunch will be at the Katjiepiering Restaurant, the Stellenbosch Botanical Gardens, approximately a 5-10 minute walk from the conference venue. On the first day, please assemble in the foyer so that we can walk to the gardens en masse. *Please ensure your name badge is worn and visible.*

Conference Photo: Please assemble in the foyer after the final talk on the Wednesday (17:10) so that we can take a photo of all the attendees!

Conference Reception: A drinks reception with light snacks will be held at 17:30 on the 4th April in the Katjiepiering Restaurant. *Please ensure your name badge is worn and visible.*

Conference Dinner: The conference dinner will take place at Neethlingshof restaurant, (approximately a 10 minute drive from the conference venue) at 19:00 on the 5th April. Transportation will be organised during the conference. Dress code is smart / casual. *(No shorts or sandals, please!)*

Luggage: On the final day (only) luggage may be stored in room 1006A. (Not room 1006!)

In and around Stellenbosch: The Neeslie Student Centre on campus has a variety of food outlets and a small supermarket (SPAR). The larger Eikestad Mall is located at the intersection of Victoria and Andringa Streets (see attached map) and includes all the amenities you are likely to require, including a large supermarket (Checkers).

The majority of the bars and restaurants in Stellenbosch can be found in around the area bordered by Plein, Drosdy, Dorp, and Andringa Streets.

For those wishing to enjoy a short walk - the Jan Marais Nature Reserve is nearby (indicated by "Marais" on the right-hand side of the map) and offers a wonderful view of the nearby mountains and Jonkershoek valley.

The green dashed line on the map is the "Green Route" which is actively patrolled by campus security in the late evenings and at night. However, although Stellenbosch is a quiet and relatively safe town, it is not entirely free of crime. We therefore advise delegates to take normal safety precautions at night time, such as traveling in groups, avoiding dark alleys, and keeping valuables out of sight.

There are literally dozens of wine farms in the surrounding area. Feel free to explore and find one on your own, or to ask a local for advice.

Emergency contacts: Campus Security, +27 (0)21 808 2333, Nick Hale, +27 (0)79 434 9011.

STELLENBOSCH CAMPUS MAP INDEX

SUPPORT SERVICES
1 Administration, Block A
2 Administration, Block B
3 Administration, Block C
4 JS Gericke Library
5 Neelsie Student Centre
6 Campus Security
7 Centre for Student Recruitment
8 Centre for Student Counselling and Development: Reception (CSCD)
9 CSCD: Office for Students with Special Learning Needs (Disabilities); Den Bosch
10 CSCD: Unit for Psychotherapeutic and Support Services
11 Centre for Teaching and Learning
12-14 Division for Student Affairs
15 Language Centre: Reading Lab and Language Enrichment Courses
16 Campus Health Services
17 Maties Sport
18 Coetzenburg Centre
19 Old Conservatoire
20 SU Art Galery
21 University Museum
22 Centre for Learning Technology
23 Language Centre (Writing Lab)
24 WAT, 115 Banghoek Road
25 Matie Community Services, Lückhoff School
26 SU Vehicle Fleet
27 Legal Aid Clinic
28 IT Hub (Help Centre)
30 Purchasing and Provision Services
31 Facilities Management
32 Information Technology

SPORTS FACILITIES
33 Danie Craven Stadium
34 Coetzenburg Athletics Stadium
35 Coetzenburg Tennis Courts
36 PSO Club House and Hockey Fields
37 Netball Courts
38 Swimming Pool
39 SU Gymnasium
40 Tennis Courts (Residences)
41 Old Mutual Sports Centre (squash courts)
42 Tennis Courts
43 Heidehof Rugby Fields

ACADEMIC BUILDINGS
44 Food Science
45 PO Sauer
46 Electrical/Electronic Engineering
47 Civil Engineering
48 Mechanical/Mechatronic/Industrial Engineering
49 Process Engineering
50 Knowledge Centre
51 Engineering, General
52 Africa Centre for HIV and AIDS Management
53 JC Smuts – Biological Sciences
54 De Beers – Chemistry
55 Mike de Vries
56 Chemistry – first-years
57 Inorganic Chemistry
58 CGW Schumann
59 Van der Sterr
60 Accounting and Statistics
61 JS Marais
62 Polymer Science
63 Visual Arts
64 JH Neethling
65 AI Perold
66 Merensky
67 Mathematical Sciences and Industrial Psychology
68 Nursery
69 Natural Sciences
70 Chamber of Mines
71 RW Wilcocks
72 Old Main Building
73 HB Thom Theatre
74 CL Marais Library
75 Journalism
76 GG Cillié
77 Arts and Social Sciences
78 Lombardi
79 Konservatorium (University Choir)
80 Theological Seminary
81 Paul van der Bijl Laboratories
82 Agronomy
83 Welgevallen Experimental Farm
84 Sport Science

WOMEN'S RESIDENCES
85 Monica
86 Harmonie
87 Heemstede
88 Huis ten Bosch
89 Lydia
90 Minerva
91 Nerina
92 Erica
93 Nemesia
94 Serruria
95 Tinie Louw Dining Hall; ▲ amaMaties hub
96 Sonop (Huis van Niekerk)
97 Irene

MEN'S RESIDENCES
98 Eendrag
99 Helshoogte
100 Simonsberg
101 Huis Visser
102 Huis Marais
103 Dagbreek
104 Majuba
105 Wilgenhof
106 Helderberg

MEN'S AND WOMEN'S RESIDENCES
107 Huis Neethling
108 Metanoia
109 Russel Botman House; ▲ Wimbledon hub
110 Academia
111 Concordia
112 Goldfields
113 Huis McDonald

UNIVERSITY FLATS AND HOUSES
114 Lobelia
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116 Huis de Villiers
117 Waldenhof

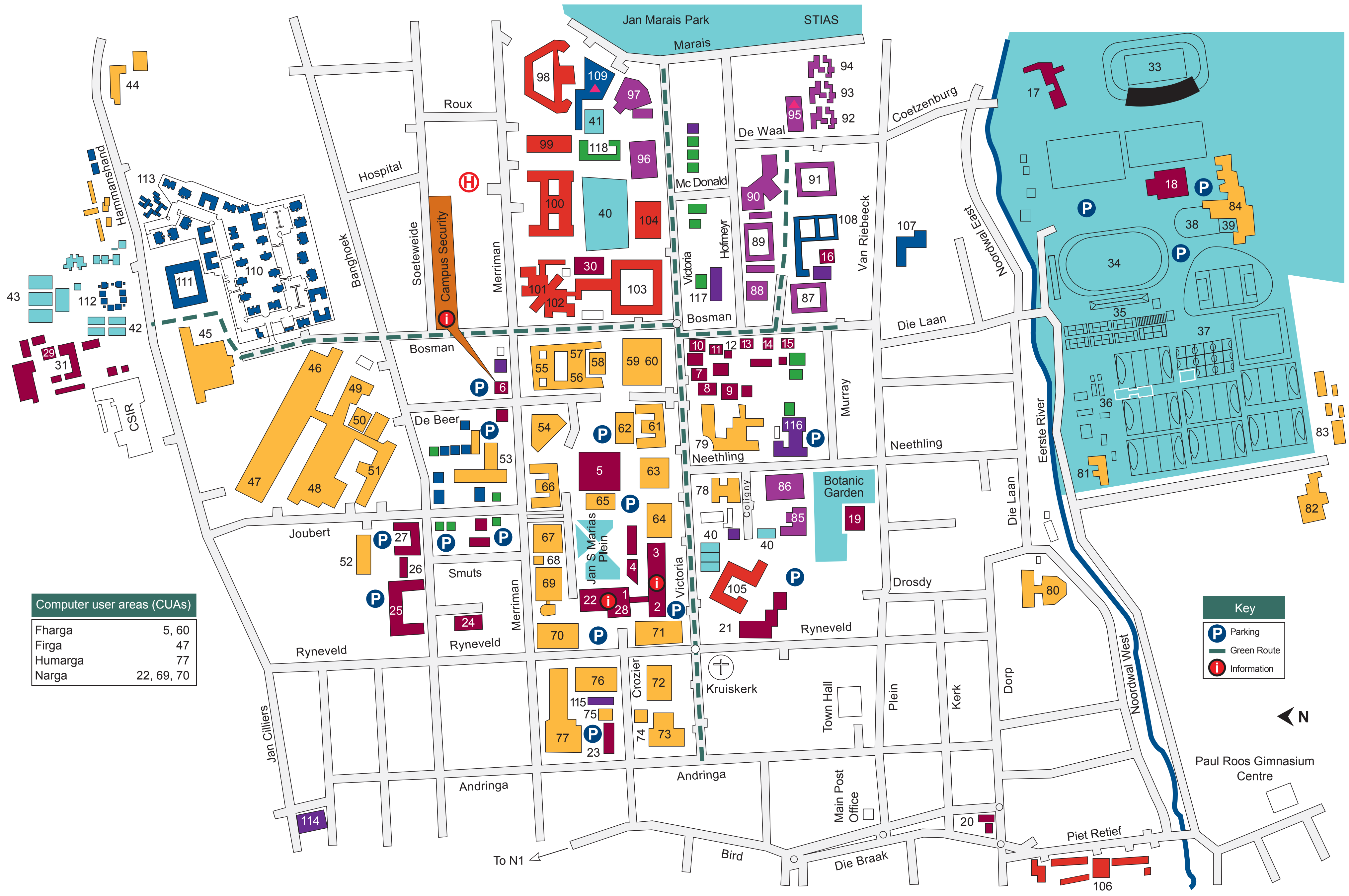
LISTEN, LIVE AND LEARN
118 LLL village
■ LLL houses

CLUSTER HUBS
▲ amaMaties 95
▲ Wimbledon 108

FACULTIES (IN COLOUR) AND DEPARTMENTS

Ⓟ Parking

STELLENBOSCH CAMPUS MAP



Computer user areas (CUAs)	
Fharga	5, 60
Firga	47
Humarga	77
Narga	22, 69, 70

Plenary abstracts

The making of an Escher: conformal maps and the Droste effect

Jon Chapman (University of Oxford)

I will explain the mathematics behind the Escher print “Picture Gallery”, and show how it can be used to create new Escher-style pictures.

Breaking integrability at the boundary

Patrick Dorey (University of Durham)

This talk will describe some work on the bouncing of particle-like (kink) solutions to a nonlinear wave equation, called the sine-Gordon equation, against a fixed boundary. Away from the boundary, this equation has a property known as integrability, making the scattering of the kinks particularly simple. However, if this integrability is broken at the boundary, then the scattering becomes surprisingly complicated, in ways that will be outlined in the talk with the help of some movies.

The benefits and pitfalls of redundancy in the approximation of functions

Daan Huybrechs (KU Leuven)

Continuous functions, for example solutions to partial differential equations, are usually represented in a basis. This is convenient once a basis is available, but unfortunately bases are inflexible and often hard to construct. What if the domain geometry is complicated? What if the function has singularities? What if you know properties about the function, that do not match well with the basis you’re using? Such questions do have accepted answers, for example the use of meshes, and adaptive refinement. In this talk we present a simple alternative that enables a lot of flexibility: redundancy. With some redundancy, it becomes simple—in fact, as we will show, nearly trivial—to represent functions on domains of arbitrary shape, or to add features to an approximation space. On the other hand, redundancy rapidly leads to ill-conditioning in algorithms. Surprisingly, with the right approach and with suitable restrictions, high accuracy can be achieved in a numerically stable manner. Moreover, efficient algorithms can be devised for many cases. In this talk we explore the introduction of redundancy in approximations, and we give practical guidelines about what is the ‘right approach’ and what are the ‘suitable restrictions’.

Properties of orthogonal polynomials characterized by structural relations

Kerstin Jordaan (UNISA)

In this talk we consider orthogonal polynomials that are characterized either by a structural relation of type

$$\pi(x)SP_n(x) = \sum_{k=-r}^{s} a_{n,n+k}P_{n+k}(x), \tag{1}$$

where $\pi(x)$ is a polynomial and S is a linear operator that maps a polynomial of precise degree n to a polynomial of degree $n-1$ or by

$$\Pi(x)TP_n(x) = \sum_{k=-p}^{q} a_{n,n+k}P_{n+k}(x), \tag{2}$$

where $\Pi(x)$ is a polynomial and T is a linear operator that maps a polynomial of precise degree n to a polynomial of degree $n-2$. It is understood that S annihilates constants while T annihilates polynomials of degree 1. When $S = \frac{d}{dx}$, the structure relation (1) characterizes semiclassical orthogonal polynomials. We discuss properties of certain classes of semiclassical orthogonal polynomials. We characterize Askey-Wilson polynomials and their special or limiting cases as the only monic orthogonal polynomial solutions of (2) when $T = \mathcal{D}_q^2$ where $\mathcal{D}_q$ is the Askey-Wilson divided difference operator and $\Pi(x)$ is a polynomial of degree at most 4. We use the structure relation (2) to derive bounds for the extreme zeros of Askey-Wilson polynomials.

Understanding the complex dynamics of Faraday pilot waves

Paul Milewski (University of Bath)

Faraday pilot waves are a newly discovered hydrodynamic object that consists a bouncing droplet which creates, and is propelled by, a Faraday wave. These pilot waves can behave in extremely complex ways and result in dynamics mimicking quantum mechanics. I will show some of this fascinating behaviour and will present a surface wave-droplet fluid model that captures many of the features observed in experiments, particularly focusing on the emergence, under chaotic dynamics, of statistical of states with surprising structure.

Modelling and robust simulations of slow-fast dynamical systems

Kailash Patidar (University of the Western Cape)

Deterministic models are often indispensable tools for studying biological and ecological systems. Many of these models are nonlinear and highly complex systems of differential equations for which analytical studies are limited to describing the underlying dynamics semi-qualitatively. In recent years, we have studied a class of slow-fast dynamical systems through various types of differential equation models. These include the mathematical models of co-infections of HIV-TB and HIV-Malaria. In this talk, we will consider a class of problems arising in mathematical ecology. Construction of numerical schemes for such problems exhibiting multiple time scales within a system has always been a challenging task. To this end, using geometric singular perturbation theory, we will discuss appropriate decoupling of full system into slow and fast sub-systems. This will be followed by construction of sub-algorithms for these sub-systems. The algorithm for the full problem will then be obtained by utilizing a higher-order product method by merging the sub-algorithms at each time-step.

Compressive sensing and its use for the numerical solution of parametric PDEs

Holger Rauhut (RWTH Aachen University)

After giving a short introduction to this field, we move to an application to the numerical solution of parametric operator equations where the parameter domain is high-dimensional. In fact, one can show that the solution of certain parametric operator equations (parametric PDEs) is analytic in the parameters which can be exploited to show convergence rates for nonlinear (sparse) approximation. Building on this fact, we show that methods from compressive sensing can be used to compute approximations from samples (snapshots) of the parametric operator equation for randomly chosen parameters, which in turn can be computed by standard techniques including Petrov-Galerkin methods. We provide theoretical approximation rates for this scheme. Moreover, we then pass to a multilevel version of this scheme, similarly to multilevel Monte Carlo methods, and show that the overall complexity of computing parametric solutions can be further reduced.

On the numerical solution of large-scale linear matrix equations

Valeria Simoncini (Università di Bologna)

Linear matrix equations such as the Lyapunov and Sylvester equations and their generalizations have classically played an important role in the analysis of dynamical systems, in control theory and in eigenvalue computation. More recently, matrix equations have emerged as a natural linear algebra framework for the discretized version of (systems of) deterministic and stochastic partial differential equations ((S)PDEs), and new challenges have arisen.

In this talk we will review some of the key methodologies for solving large scale linear matrix equations. Emphasis will be put on rank-structured and sparsity-structured problems, as they occur in applications. If time allows, we will also discuss recent strategies for the numerical solution of advanced linear matrix equations, such as multiterm equations and bilinear systems of equations, which are currently attracting great interest due to their occurrence in new application models associated with (S)PDEs.

Contributed abstracts

Call center resource leveling optimization

Montaz Ali and Adham Stolt (University of the Witwatersrand)

This work is a result of the necessity to resolve a real world problem: a call centre capacity problem. The result of the capacity challenge is a high cost of operations and large number of disgruntled customers. The research comprises two components: developing a new mathematical model as an extension to existing scheduling problems and then also the industrial engineering component of implementing this new scheduling model into the call centre operation in which the problem was identified.

he problem is shown to share many features with the class of problems known as Scheduling Problems. Particular attention is paid to the sub classes of Resource Constrained Project Scheduling (RCPSP) and Resource Levelling Problems (RLP). The special and unique features of this problem are then shown to be: the need to perform scheduling on a batch basis every day, without knowledge of what will occur the following day and also the requirement that resources (call centre capacity) are fully utilized and not exceeded. The result of the research is that a new mathematical model called the Critical Time Window Resource Levelling Problem (CTWRLP) with the Continual Rescheduling method is developed and proposed as a method for solving the real world scheduling problem. This method addresses all the requirements of the real world problem.

An approach to solving the model in the practical environment is also presented. This involves additional pre-processing required to prepare all inputs for the scheduling model, namely creating sub-models for calculating resource consumption and resource availability. The global optimization technique of Simulated Annealing is used to solve the optimization component of the CTWRLP.

Solitons in $\mathcal{PT}$-symmetric ladders of coupled optical waveguides

Nora Alexeeva (UCT), Igor Barashenkov (UCT), and Yuri Kivshar (ANU)

We consider a ladder-shaped optical array consisting of a chain of lossy waveguides coupled to a parallel chain of waveguides with gain. All waveguides have the focusing Kerr nonlinearity. The array supports two co-existing solitons, a high- and a low-frequency one, and each of these can be centred either on a lattice site or midway between two neighbouring sites. Both bond-centred solitons are found to be unstable regardless of their amplitudes or parameters of the model. The high-frequency site-centred soliton is stable when its amplitude lies below an analytically expressible threshold that depends on the gain-loss coefficient. The threshold is the lowest when the coupling between the neighbouring dimers of gaining and lossy waveguides is close to the gain-to loss coupling within the dimer; reducing the inter-dimer coupling to small values or, conversely, raising it to the continuum limit expands the stability domain. Likewise, the low-frequency site-centred soliton has a stability domain bounded from above by a critical amplitude. In addition, the low-frequency soliton stabilizes when its amplitude becomes sufficiently large. Both small- and large-amplitude stability domains expand as the “in-chain” coupling is reduced.

A stoichiometric method for reducing simulation cost of chemical kinetic models

Emmanuel A. Amikiya and Mapundi K. Banda (University of Pretoria)

Mathematical models for chemically reacting systems have high degrees of freedom (very large) and are computationally expensive to analyse. In this discussion, we present and analyse a model reduction method that is based on stoichiometry and mass balances. This method can significantly reduce the high degrees of freedom of such systems. Numerical simulations are undertaken to validate and establish efficiency of the method. A practical example of acid mine drainage is used as a test case to demonstrate the efficacy of the procedure. Analytical results show that the stoichiometrically-reduced model is consistent with the original large model, and numerical simulations demonstrate that the method can accelerate convergence of the numerical schemes in some cases.

Fitted-Mimetic finite difference method for pricing American put options in higher dimensions

David Sena Attipoe (UCT/AIMS) and
Antoine Tambue (UCT / Western Norway University of Applied Sciences)

We consider developing an efficient and accurate numerical method for pricing two assets American put options. American put options are known to be governed by Linear Complementary Problems (LCP's), which are based on the Black-Scholes partial differential equation (PDE). However, these LCP's do not have analytical solutions and are generally challenging to solve numerically. We consider higher dimensional (two assets) put options, and we formulate a corresponding power penalty approach with its associated well-posedness and convergence proofs. The resulting penalized model is discretized in space using Mimetic finite difference method which is special spatial discretization technique that preserves and conserves general properties of the continuum process in the discrete case. With the time discretization, we consider the standard Euler scheme and Rosenbrock-type methods. We present some numerical simulations to show the efficiency and accuracy of the methods presented. In this talk we will present the Mimetic finite difference method and our Novel approach that deals with the degeneracy around the boundary. We shall show some results in comparison with already existing methods (Fitted-Finite Volume Method and Mimetic Finite difference Method). For simplicity, we will present the 1D case for European Put Options.

$\mathcal{PT}$-symmetry: jamming anomaly and necklaces with gain and loss ombre

Igor Barashenkov (University of Cape Town)

A product of the nonhermitian quantum mechanics, the $\mathcal{PT}$-symmetry is one of the latest trends in optics. In this talk, I discuss two $\mathcal{PT}$-symmetric optical systems — a continuous and a discrete one.

My first system is the Schrödinger equation with a bicentric $\mathcal{PT}$-symmetric potential. At low gain-loss rates γ, raising the rate results in the energy flux from the active to the leaky part of the system being boosted. I expose the anomalous behaviour occurring for larger γ, where the increase of the rate produces a drop of the total gain and loss in the system. This anomaly has the same mechanism as the traffic congestion in a road network.

The second $\mathcal{PT}$-symmetric structure consists of a ring-like necklace of $2N$ coupled optical waveguides with the clockwise gain-loss variation. I show that unlike necklaces with alternating or clustered fixed-value gain and loss, the symmetry-breaking threshold γ_c in this system does not tend to zero as $N \to \infty$. Despite the complex structure of the ombré necklace, the system is found to be exactly solvable and the value of $\gamma_c(N)$ is obtained in closed form.

Compressed sensing, distributed networks, and fusion frames

Jean-Luc Bouchot (RWTH Aachen), Roza Aceska (Ball State University),
and Shidong Li (San Francisco State University)

We extend the classical results of compressed sensing to the case where the signals to recover are assumed to be acquired in a distributed fashion. Local information are sampled in a local frame basis and recovered by means of compressed sensing. We show that by applying a fusion process of these frames coefficients, it is possible to break the usual low complexity expected by traditional single sensor compressed sensing. We also show that increasing linearly the number of sensors allows to decrease the number of measures required at each location. Some numerical results will also be presented, illustrating the feasibility of our approach.

Effective transport properties of lattices

Jon Chapman (University of Oxford)

The method of multiple scales is used to calculate the effective macroscopic transport properties of a random walk on a periodic lattice with arbitrary, spatially-dependent transition rates. In contrast to the standard multiple-scales approach for continuous media, the fast scale here is discrete, and only the slow scale is continuous. The solution is found as the discrete probability distribution of finding the particle at a particular node in the unit cell, modulated by a continuous slow function of the position of the unit cell in the macroscopic material. This last function represents the macroscopic drift-diffusion of the particle, with drift and diffusion coefficients calculated in terms of the individual transition probabilities.

The method is then applied to a multi-phase flow problem on a network of pipes, a model commonly used to model oil extraction from rock. The homogenised model is similar to the Buckley-Leverett equation, with the (usually empirically determined) fractional flow function given explicitly in terms of the network parameters.

Stabilization methods to ensure a robust numerical framework in a FSI model for vascular access in haemodialysis

Andie de Villiers (Stellenbosch University), Daya Reddy (University of Cape Town), and Andrew McBride (University of Glasgow)

The flow rate inside arteriovenous fistulas can be up to 30 times higher than physiological flow and is accompanied by high wall shear stress resulting in low patency rates. A computational model that aims to improve the detailed understanding of the unique haemodynamics and vascular mechanics in these fistulas are described. The problem is approached with a monolithic solution scheme using the finite element software library deal.II. The nonlinear system is solved by a Newton-like method. The emphasis of this presentation will be on the instabilities present in the model as well as the stabilization techniques used to ensure a robust numerical framework. Lastly, flow results from patient-specific geometries are compared to 4D MRI data.

The LULU-based connected median filter to remove noise from images

Inger Fabris-Rotelli (University of Pretoria), Magnus van Niekerk (University of Twente), and Alfred Stein (University of Twente, University of Pretoria)

In this talk we present a new median smoothing filter for images. This median filter combines the idea of level sets in the LULU filters in images with an algorithm similar to that of the adaptive median filter. An algorithm is given and we compare the new filter to existing methods. In a simulation study we applied the Normal distribution, the Gumbel distribution and salt and pepper noise at different levels. The filter is further applied on images from the Labeled Faces in the Wild database and a sample of remote sensing images. The new median filter compares well with the adaptive median, and outperforms it in most cases, particularly with regard to edge preservation.

From sand piles to dunes

Maurizio Falcone and Stefano Finzi Vita (Università di Roma “La Sapienza”)

The mathematical modeling of granular materials is a source of challenging problems with many interesting applications which range from the microscopic to the macroscopic scale. We study from a numerical point of view some differential models in this area. The starting model describes the growth of a sandpile on a flat bounded table under the action of a vertical source and is based on a two-layers formulation. The model is made by a nonlinear system of PDEs where an eikonal equation for the standing layer of the pile is coupled with an advection equation for the rolling layer. We will discuss some properties of the solution and present two approximation schemes for that system. The two-layers model can be extended to deal with other deposition problems for granular materials and to describe the creation and motion of sand dunes under wind action. We will describe some of the recent developments presenting experiments in one and two dimensions.

Convergence analysis of a fourth-order iterative method in Banach spaces using recurrence relations and its dynamics

Gagandeep (Gujral Punjab Technical University, India)

Numerical methods are extremely powerful problem-solving tools. Newton method and its variants are the most commonly used iterative methods to solve nonlinear equations. The convergence of Newton's method in Banach spaces was established by Kantorovich. The convergence of the sequence obtained by the iterative expression is derived from the convergence of majorizing sequences. Rall has suggested a new approach for convergence of these methods by using recurrence relations. In semilocal convergence, starting from initial vectors, for which certain conditions are satisfied, convergence to some solutions is guaranteed. In this paper, we study the semilocal convergence of a fourth-order method for solving nonlinear equations in Banach spaces. The semilocal convergence of this method is established by using recurrence relations giving the existence-uniqueness theorem that gives priori error bounds. Numerical application on nonlinear integral equation is given to show our approach.

Classification using tensorflow

Ben Herbst (Praelexis and Stellenbosch University)

This is a tutorial-style talk about tensorflow. One may well ask whether it is necessary to come to grips with tensorflow if a high-level packages such as keras are readily available, that no-doubt makes life very easy for us. However, if you are serious about deep learning there will come a time when you will have a need to try out your own ideas. And in that case, a lower-level system such as tensorflow is essential. I'll use the classification problem to illustrate the basic ideas behind tensorflow. I'll hopefully be able to convince you that tensorflow is not that intimidating!

Tensile fracture analysis of a thin Euler-Bernoulli beam and the transition to the voussoir model

Ashleigh Hutchinson (University of the Witwatersrand)

The structure of roof rock in underground mines, which is usually comprised of horizontal stratified layers, has an important effect on the stability of the rock. Underground strata separate upon deflection and each beam transfers its own weight to its supports. The sequence of fractures in a mine roof rock beam leading to the formation of a voussoir beam is considered. The analysis of Obert and Duvall (1966) of the sequence of fractures of an Euler-Bernoulli beam acted on by its own weight is extended to an Euler-Bernoulli beam also acted on by a horizontal axial compressive force at each end. A single beam initially clamped at each end is considered. Once the tensile strength of the clamped beam has been exceeded, which first occurs at the end points when the beam number exceeds a critical value B_m, crushing at the abutments results in a change in boundary conditions from clamped to hinged. The first mode of the clamped beam is considered with beam number in the range 0 to 2π. It is shown that for an inherited beam number B_m in the range $0 \leq B_m < \pi$ the hinged beam will immediately undergo tensile fracturing centred around its mid-point and the percentage of the length of the beam which could fracture increases from 57.74% for $B_m = 0$ to 100% for $B_m = \pi$. For $B_m = \pi$ the beam could undergo snap-through failure. For $\pi < B_m < 2\pi$, fracturing centred around the mid-point of the hinged beam will occur only for $\pi < B_m < 4.43$ and the percentage of the length of the beam which could fracture reduces from 100% for B_m = to 0 for $B_m = 4.43$. For inherited beam number in the range $4.43 < B_m < 2\pi$ the hinged beam will not undergo tensile fracture and a voussoir beam will not form.

On q-orthogonal polynomials: something old and something new

Alta Jooste (University of Pretoria)

Let $0 < q < 1$. Orthogonal and q-orthogonal polynomials have hypergeometric and q-hypergeometric (or basic hypergeometric) representations, respectively. The hypergeometric function was studied by Gauss in 1813 and the (more general) basic hypergeometric function was introduced by Heine in 1847. In 1948 Hahn introduced a class of q-orthogonal polynomials that satisfy q-difference equations; these are known as the Hahn class of orthogonal polynomials. I look at the background of the basic hypergeometric orthogonal polynomials that appear in the q-analogue of the Askey scheme [KLS, p.413] and I discuss some recently published results on the behaviour of the zeros of these polynomials.

[KLS] R. Koekoek, P.A. Lesky and R.F. Swarttouw, Hypergeometric Orthogonal Polynomials and Their q-Analogues, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2010.

Symmetries and conservation laws of some Schrödinger equations

Abdul Hamid Kara (University of the Witwatersrand)

In this presentation we construct multipliers and conservation laws of a Cubic Schrödinger partial differential equation. Further, we show that if a symmetry is associated with a conservation law, we can use the fundamental theorem of reduction to obtain exact solutions.

Comparison of direct methods for Sylvester matrix equations

Gerhard Kirsten (Stellenbosch University)

In this paper a comparative analysis is done for the direct solution of the Sylvester equation $AX + XB = C$, with $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times m}$ of relatively small dimension. Several algorithms for the direct solving of the Sylvester equation in Sylvester form are described and compared to one another by means of algebraic operation counts, as well as computational time. The efficiency of these algorithms is also compared to the direct solving of the system expressed as a linear equation $\tilde{A}\boldsymbol{x} = \boldsymbol{b}$, with $\tilde{A} \in \mathbb{R}^{nm\times nm}$. Some numerical experiments allow us to conclude which algorithms will be best utilised in which context. Dense and sparse coefficient matrices are investigated separately for all algorithms.

A fitted multi-point flux approximation method for pricing options

Rock Stephane Koffi (UCT/AIMS) and
Antoine Tambue (UCT/Western Norway University of Applied Sciences)

In this paper, we present a novel numerical method based on the Multi-Point Flux Approximation (MPFA) method to solve partial differential equation (PDE) arising from pricing multi-asset option. The MPFA is applied through a fitted finite volume to handle the degeneracy of the PDE . For the spatial discretization, in one hand, we combine the MPFA method with the upwind methods method (first and second order). In the other hand, we introduce the fitted MPFA method and combine it also to the upwind methods. The -method is used for the time discretization. Numerical simulations are performed to show the accuracy of the method for pricing multi-assets option.

Two-dimensional and axial solitary stress waves described by a subclass of implicit constitutive equations

Avnish Magan (University of the Witwatersrand)

The propagation of stress waves for a subclass of implicit constitutive equations in a rectangular slab and circular cylinder is discussed. The general class of implicit constitutive equations contain Cauchy elastic and hyperelastic materials as subclasses. We consider a special subclass of implicit constitutive equations where the strain is prescribed in terms of a non-invertible function of the stress. Two constitutive equations are studied. The first constitutive equation is called the power-law constitutive equation due its analogy with the constitutive equation for a power-law fluid. This constitutive equation can describe elastic response where the stress and linearised strain are nonlinearly related and such a phenomenon cannot be captured in the context of Cauchy elasticity and hyperelasticity. The second constitutive equation is called the strain-limiting constitutive equation and can describe materials that exhibit limiting stretch. To derive the mathematical models we assume a special semi-inverse solution where specific forms for both the displacement and stress are sought. This assumption leads to a system of partial differential equations which can be reduced to a single nonlinear hyperbolic partial differential which describes the propagation of solitary stress waves. We find expressions for the speed of the solitary stress wave for both constitutive equations in both the rectangular slab and circular cylinder. The solitary stress wave develops a shock at the front of the wave for the strain-limiting constitutive equation and at the back of the wave for the power-law constitutive equation. Estimates for the times at which these shocks will occur are also derived.

Numerical solution of some nonlinear Poisson equations

Gerald Marewo (North-West University)

In this study, we consider a boundary value problem for nonlinear Poisson-type equations. Such equations have many real life applications including the analysis and design of semiconductor devices. The nonlinearity of the equations motivates using a numerical method of solution. To this end, we couple the Newton method with a Galerkin finite element method. The overall method is implemented on the computer using MATLAB. Numerical experiments are used to investigate the efficiency of the method.

Solving linear systems with a defective right hand side

Milton Maritz (Stellenbosch University)

The problem discussed in this talk is the solution of the linear system of equations $A\boldsymbol{x} = \boldsymbol{b}$, where some components of $\boldsymbol{b}$ are unknown, but in order to compensate for this "loss", an equal number of components of $\boldsymbol{x}$ are known.

The practical application of this solution arises in the problem of deconvolution of images in order to correct for either defocus blur or motion blur or both. Standard non-iterative deconvolution methods are based on obtaining the Fast Fourier transform of the image and applying suitable filters (such as for example the Wiener filter), before transforming back. However, all these methods treat the image as a two-dimensional periodic function, and since this is never true for natural images, there are usually artifacts in the deconvolved image due to the incorrect periodicity assumption.

The talk will discuss using the aforementioned solution method to correct for the non-periodicity of images.

Axisymmetric turbulent far wake of a wind turbine

David Mason (University of the Witwatersrand)

We investigate the far downstream axisymmetric turbulent wake of a wind turbine with non-rotating rotor disk. A non-rotating disk model was used, for example, by Betz to obtain the maximum theoretical efficiency of a wind turbine. The fluid flow in the far wake is modeled using boundary layer theory. Prandtl's mixing length model is used for the eddy viscosity. It is shown how the conserved quantity for the wake can be derived from the elementary conservation law. The Lie point symmetry associated with the elementary conserved vector is calculated and used to reduce the boundary layer equation for the stream function to an ordinary differential equation. An exact analytical solution is derived by neglecting the kinematic viscosity. In this solution the wake is bounded in the radial direction. A numerical solution using a shooting method is obtained for non-zero kinematic viscosity. The wake is unbounded in the radial direction although very weak beyond the theoretical boundary for vanishing kinematic viscosity.

Using Spectral Methods on HIV Infection with Tat and Ssu72 Activation

S'yanda Mungwe (Stellenbosch University)

HIV dynamics within the host are complex especially when a reservoir of latently infected $CD4^+$ T cells are present. The failure of the immune system and antiviral therapy to suppress the virus has been suggested to be enhanced by the latently infected $CD4^+$ T cells which are responsible for persistence of HIV within the host. Cells remaining in latent state have been shown to lack sufficient levels of Tat and associated activation-dependent host factor that are necessary for processive transcription of the virus. Tat is a protein that is capable of activating the latently infected $CD4^+$ T cells. Recently, as a protein, Ssu72 was found to be responsible for activation and replication of the virus. Ssu72 enhances the effects of Tat activation in a mutualistic interactive manner. The interaction of Tat and Ssu72 thus, enhances the activation of the latently infected $CD4^+$ T cells which may in turn expose the virus for possible attack by the immune system reaction. In the current study, we modify a constant virus HIV model to incorporate the effects of Tat and Ssu72 on latently infected $CD4^+$ T cells. We analyze the models using both analytic and spectral methods (multistage spectral relaxation method) techniques. Important threshold were derived and model analysis carried out. The incorporation of Tat and Ssu72 proteins on the HIV-1 model with a constant virus shown that, with time the uninfected and infectious classes decrease to zero for a threshold value of 30-40 copies of each protein..

A new multidomain spectral collocation method for solving parabolic partial differential equations

Mutua Samuel (UKZN) and Sandile Motsa (University of Swaziland)

In this article a new multidomain spectral collocation method is introduced and applied to approximate solutions of parabolic partial differential equations (PDEs) that are defined on large intervals. There exist multidomain approaches that have been applied to either space or time variable but not both. These approaches are based on the assumption that the variable that overlooked spans over a small interval. In this work, the domain of approximation is decomposed into smaller equal overlapping and non-overlapping subintervals in space and time directions, respectively. The study is aimed at demonstrating that decomposing both large spatial and time computational domain simultaneously is equally advantageous. We observe that this approach guarantees improved accuracy and reduced computational time as opposed to the increment of the number of grid points in a single domain. In the solution process, the PDE is discretized on each subinterval using spectral collocation at Chebyshev-Gauss-Lobatto points and the approximate solution is obtained by matching solutions at different subintervals along the common boundaries. The accuracy and effectiveness of the proposed method is evinced by presenting numerical results of well known parabolic PDEs that have been reported literature. The current method can easily be extended to solve systems of parabolic PDEs that model real-life problems.

Imbalanced data problems in machine learning

Energy Sonono (North-West University)

Many "real-world" problems are characterized by imbalanced learning data, where at least one class is under-represented relative to the other. Examples include (but are not limited to): fraud detection, identifying potential customers, detecting network intrusions, predicting failures of technical equipment *et al.* In such situations, most of the classifiers are biased towards the major classes. This makes it so difficult to estimate the minor class. Although such problems related to imbalanced data are of interest, the problems still remain open and are in fact often encountered, especially in real word applications. The focus of this work is on describing the various approaches for enhancing solving of imbalanced data problems using various machine learning classifiers as well as evaluating the performance of the classifiers. A real world application dataset will be used to explore the problem at hand.

A data-driven approach to short-term stream flow forecasting and downstream gap infilling

Melise Steyn (CSIR)

Accurate stream flow forecasting in a river is used in the study of various hydro-environmental aspects, and may assist in reducing the consequences of droughts or floods. The utility of time series records might be dependent on continuous, uninterrupted observations, but gaps in the data are often unavoidable. This study proposes the application of machine learning techniques to address these challenges. The first part focuses on short-term stream flow forecasting at a single river station. Support vector regression and neural network models are trained on historical stream flow and precipitation data, to forecast with a lead time of up to 7 days. The second part investigates the ability of our models to infill incomplete stream flow records, using data from upstream stations and rain gauges. Results illuminate the promising role of machine learning in the modelling of stream flow in rivers.

The use of S_N symmetry in analyzing the Axelrode model for dissemination of culture

Nirina Maurice Hasina Tahiridimbisoa (AIMS & Stellenbosch University)
and Yabebal Tadesse (AIMS)

The Axelrode model for dissemination of culture is revisited. We propose the quantity $\bar{s}(t)$ to study the evolution of the system over time. This is used to measure the departure of the culture contents in the population from being uniform. We argue the permutation group S_N is a symmetry of this observable. Numerical simulations are provided and give some plots of the generic evolution of the system. We also provide analyses of the culture contents of the system at each time and toward to the equilibrium configuration.

Geometric analysis of nonlinear evolutionary equations

Maleafisha Tladi (University of Limpopo)

Most differential equations occurring in multiscale modelling of physical and biological systems cannot be solved analytically. Numerical integrations do not lead to a desired result without qualitative analysis of the behavior of the equations solutions. The authors study the quasigeostrophic and rotating Boussinesq equations describing the motion of a viscous incompressible stratified fluid in a rotating system which is relevant, e.g., a Lagrangian and Eulerian analysis of a geophysical fluid flow. These equations consist of the Navier-Stokes equations with buoyancy-term and Coriolis-term in beta-plane approximation, the divergence-constraint, and a diffusion-type equation for the density variation. They are considered in a plane layer with periodic boundary conditions in the horizontal directions and stress-free conditions at the bottom and the top of the layer. Additionally, the authors consider this model with Reynolds stress, which adds hyper-diffusivity terms of order 6 to the equations. This course focuses primarily on deriving the quasigeostrophic and rotating Boussinesq equations for geophysical fluid dynamics, showing existence and uniqueness of solutions, and outlining how Lyapunov functions can be used to assess energy stability. The main emphasis of the course is on Faedo-Galerkin approximations as well as LaSalle invariance principle. The authors also addresses singular problems for which the equation has parabolic structure (rotating Boussinesq equations) and the singular limit is hyperbolic (quasigeostrophic equations) in the asymptotic limit of small Rossby number. In particular, this approach gives as a corollary a constructive proof of the well-posedness of the problem of quasigeostrophic equations governing modons or Rossby solitons. New understanding of quasigeostrophic turbulence called mesoscale eddies and vortex rings of the Gulf Stream and the Agulhas Current Retroflection could be helpful in creating better ocean and climate models.

Singular-value decomposition: the MATLAB origins and voyage

Adri van Nieuwkerk (Opti-Num)

I will deliver a brief talk about the singular-value decomposition (SVD) and how the SVD led Cleve Moler to share his mathematical programming language with the stars of Star Trek. The talk will include a comprehensive overview of MATLAB - what it is, who teaches with it, who learns using it, and the latest improvements.

Second order hyperbolic type problems with applications

Nic van Rensburg (University of Pretoria)

Linear models for the vibration of elastic bodies and systems of elastic bodies have the same abstract form as the multi-dimensional wave equation. We refer to these models as second order hyperbolic type problems and the abstract form as the general linear second order hyperbolic problem.

In the presentation we focus on the importance of the theory for numerical approximations and real world applications. A brief review of existence theory is given, including recent results. Two recent applications are discussed in some detail.

Numerical solution of differential equations by Fourier extension frames

Marcus Webb (KU Leuven)

Spectral methods are the state of the art for boundary value problems with smooth solutions on simple domains such as intervals, tori, squares, circles, spheres, and cylinders. One reason spectral methods are confined to such particular problems is that you need a "spectral" basis for both the differential equation and the domain – something not always achievable. An alternative is to use a frame instead of a basis, which may not be linearly independent. With a Fourier extension frame (a.k.a. Fourier continuation) it is possible to represent functions in a "spectral" way on arbitrarily shaped domains, but the redundancy of a frame means finding such representations is an extremely ill-conditioned problem. Fast and stable algorithms overcoming ill-conditioning for the Fourier extension approximation problem exist (in papers by Lyon and Matthysen-Huybrechs), which we generalise to solve differential equations. While this talk will only contain work and numerical examples for the ODE problem, we indicate how we intend to naturally extend the algorithms to PDEs on complicated domains.

Numerical Boundary feedback stabilization of non-uniform linear hyperbolic systems of balance laws with additive disturbances

Gediyon Weldegiyorgis (University of Pretoria)

The focus of this work is an analysis of the numerical boundary stabilization of a non-uniform linear hyperbolic system of balance laws with additive disturbances. As a preamble, a review of analytical stability analysis of the system will be presented. For the numerical discretization of the balance laws, a first order explicit upwind scheme is used for the spatial discretization. For the temporal discretization a splitting technique is applied. A discrete input-to-state stability (ISS)-Lyapunov function is employed to investigate conditions for the stability of the system. After constructing discrete numerical ISS-Lyapunov functionals, we prove an asymptotic exponential stability result. Hence, the convergence of the solution to its equilibrium is proved. Further, application of the approach to practical problems through concrete examples is presented together with suggestions for numerical implementation. The numerical computations also demonstrate the stability of the numerical scheme with parameters chosen to satisfy the stability requirements.

Gauss-Hermite vs trapezium rule

André Weideman (Stellenbosch University)

For integrals defined on the real line, there are two popular quadrature rules: the trapezium rule and Gauss-Hermite quadrature. Which one converges faster? The answer is not so simple because for each of these rules it is possible to map the infinite line to itself with a simple linear transformation, and that introduces free parameters. Only after optimal values of these scaling parameters have been established can one compare the two methods fairly. We investigate this question for integrals with Gaussian decay in the case when the integrand can be extended to a complex analytic function in the neighbourhood of the real line.

Scheduling of multipurpose batch plants

Matthew Woolway (University of the Witwatersrand)

This talk will describe several techniques utilised in the scheduling of multipurpose batch plants. The majority of the literature regarding these problems make use of mathematical programming methods. Modelling batch plants in this manner often leads to computational intractability due to the sheer number of binary variables required. More recently, promising non-deterministic approaches have been proposed in the attempt to reduce computational times. A comparison of these techniques applied to some well known literature examples will be shown.

Neural networks: differences between classical optimization and learning

Hans Georg Zimmermann (Fraunhofer Society)

The task of classical optimization is to find as fast as possible a well defined optimum. In system identification with neural networks we have to face two different challenges: first the problem of local minima in nonlinear models and second the problem of over-parameterization. The overall challenge is model building and not high speed optimization. To solve these tasks new numerical methods and new interpretations of known methods have to be combined.

Analysis of summer rainfall and temperature data using ensemble empirical mode decomposition

Willard Zvarevashe (University of Zululand), Syamala Khrishnannair (University of Zululand), and Venkataraman Sivakumar (University of KwaZulu-Natal)

Climatic variables such as rainfall and temperature have nonlinear and non-stationary characteristic such that analyzing them using linear methods inconclusive results are found. Ensemble empirical mode decomposition (EEMD) is a data adaptive method which has conventionally used for data with nonlinear and non-stationary characteristic. In the present study, the monthly mean rainfall and temperature data for a selected region in South Africa is decomposed into successive intrinsic mode functions (IMFs) at different time scales. The selected IMFs time scale illustrate the variability from inter-annual to inter-decadal variability. The influence of climatic oscillations such as El-Niño Southern Oscillation (ENSO), quasi-biennial oscillation (QBO) and quasi-triennial oscillation (3-4 years) is identified. The influence of temperature variability on rainfall variability is also shown at different time scales. Based on the results obtained, the EEMD method is found to be suitable to identify different oscillations in the rainfall and temperature data over South Africa.

Participants

- Nora Alexeeva (University of Cape Town)
- Montaz Ali (University of the Witwatersrand)
- Adoliwine Emmanuel Amikiya (University of Pretoria)
- David Attipoe (AIMS / UCT)
- Bubacarr Bah (AIMS South Africa)
- Igor Barashenkov (University of Cape Town)
- Bruce Bassett (AIMS / SAAO / SKA / UCT)
- Jean-Luc Bouchot (RWTH Aachen University, Germany)
- Jon Chapman (University of Oxford, UK)
- Kevin Colville (Centre for High Performance Computing (CHPC))
- Patrick Dorey (Durham University, UK)
- Inger Fabris-Rotelli (University of Pretoria)
- Maurizio Falcone (Università di Roma “La Sapienza”, Italy)
- Aeeman Fatima (North-West University)
- Gagandeep (Gujral Punjab Technical University, India)
- Nick Hale (Stellenbosch University)
- Ben Herbst (Praelexis / Stellenbosch University)
- Ashleigh Hutchinson (University of the Witwatersrand)
- Daan Huybrechs (KU Leuven, Belgium)
- Alta Jooste (University of Pretoria)
- Kerstin Jordaan (University of South Africa (UNISA))
- Abdul Hamid Kara (University of the Witwatersrand)
- Gerhard Kirsten (Stellenbosch University)
- Dirk Laurie (Stellenbosch University, AIMS, LitNet)
- Rock Stephane Koffi (AIMS / UCT)
- Avnish Magan (University of the Witwatersrand)
- Gerald Tendayi Marewo (North-West University)
- Milton Maritz (Stellenbosch University)
- David P Mason (University of the Witwatersrand)
- Paul Milewski (University of Bath, UK)
- S’yanda Mungwe (Stellenbosch University)
- Adri van Nieuwkerk (Opti-Num Solutions)
- Kailash Patidar (University of the Western Cape)
- Jessica Phalafala (University of Stellenbosch)
- Holger Rauhut (RWTH Aachen, Germany)
- Nic van Rensburg (University of Pretoria)
- Mutua Samuel (University of KwaZulu Natal)
- Valeria Simoncini (University of Bologna, Italy)
- Energy Sonono (North-West University)
- Melise Steyn (CSIR)
- Nirina Maurice Hasina Tahiridimbisoa (AIMS South Africa)
- Maleafisha Tladi (University of Limpopo)
- Andie de Villiers (Stellenbosch University)
- Marcus Webb (KU Leuven, Belgium)
- André Weideman (Stellenbosch University)
- Gediyon Weldegiyorgis (University of Pretoria)
- Matthew Woolway (University of the Witwatersrand)
- Hans Georg Zimmermann (Fraunhofer Society, Germany)
- Willard Zvarevashe (University of Zululand)


equations
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