



Hosted by the Department of Mathematics and
Applied Mathematics, University of Pretoria

27-29 March 2019

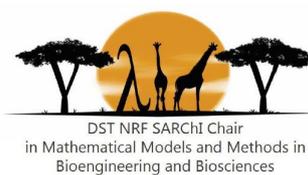
<http://sanum.github.io>



Welcome to SANUM 2019

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1 Welcome message from the organisers

The SANUM symposium has a long standing tradition as an event in the scientific life in South Africa. Following in this tradition, the scope of SANUM 2019 includes the topics of ordinary and partial differential equations, mathematical modelling, numerical analysis, biomathematics, image analysis, optimization and approximation theory. The conference features 2 special sessions: Biomathematics and Data Science.

We warmly welcome all participants in SANUM 2019 who come to Pretoria from other cities, provinces and countries. We are particularly grateful to the plenary speakers who have accepted the responsibility of delivering keynote talks, jointly covering a wide spectrum of the scope of the conference.

Growing a new generation of researchers is important for any area of science. A special event in the programme of this SANUM conference is the Young Mathematicians Function to take place on 28 March from 18:00. Thank you to Prof Charis Harley for accepting to be the guest speaker at the function.

The financial support of

- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS)
- DST NRF SARChI Chair in Mathematical Models and Methods in Bioengineering and Biosciences
- Opti-Num Solutions
- Pearson
- CASIO

is acknowledged with gratitude.

We thank all participants for their contribution to the success of this conference and wish you a productive time at the conference. We hope that SANUM 2019 will have a stimulating effect on your research and future career.

Organising Committee:

Dr Inger Fabris-Rotelli, inger.fabris-rotelli@up.ac.za

Dr Madelein Labuschagne, madelein.labuschagne@up.ac.za

Prof Michael Chapwanya, m.chapwanya@up.ac.za

2 General information

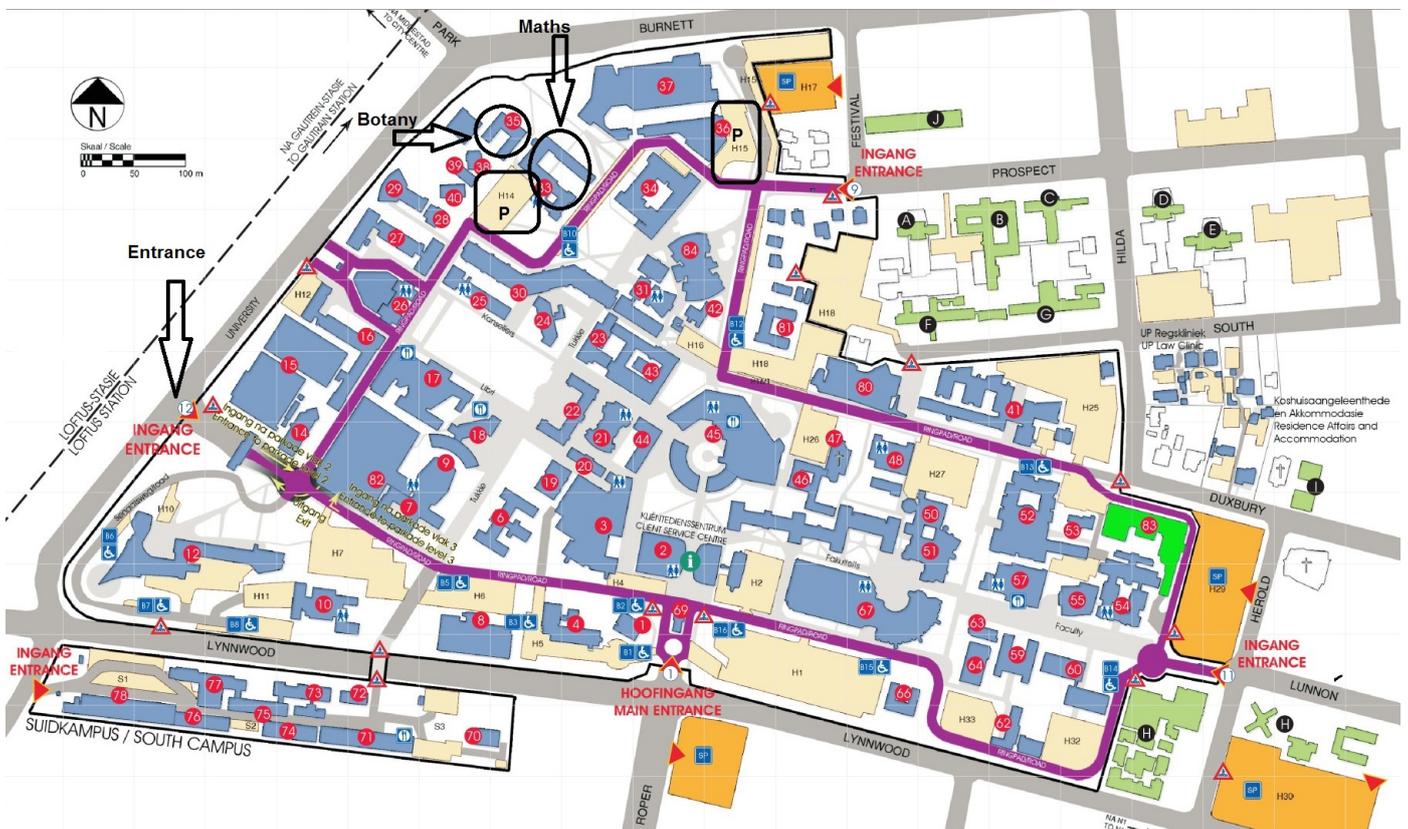
2.1 Venues

- **Registration and information:** Foyer in front of Botany 2-20.
- **Talks:** All parallel sessions will take place in Botany 2-23 or Maths 2-1. Plenary talks will be in Maths 2-1.
- **Tea:** In the atrium of the Botany building.
- **Lunch:** In the atrium and Botany 2-26.
- **Functions:**

Welcome function: Skyline bar;

Young mathematicians function: Botany 2-26;

Gala lunch: Blu Saffron. (More information under “Social events”.)



The **parking** lots are indicated on the map. Please note that the undercover parking is reserved.

2.2 Social events

Welcome evening: Wednesday at 17:30 at Skyline Pretoria.

Skyline bar is in the Protea hotel, Loftus park (416 Kirkness Street). If you need a lift to the venue, please talk to Nadege Shipley (082 770 0801) at the registration desk. We will leave from the parking lot H14 next to the Maths building at 17:15. If you have a car and extra space, please also be there at 17:40 so that we can help those without transportation.

Young mathematicians function: Thursday at 17:30 in the Botany building.

A special function for all the students and delegates who recently obtained their PhD's. Prof Charis Harley will give a guest talk.

Conference lunch: Friday at 12:30 at Blu Saffron.

Blu Saffron is located at Pretoria Country Club, 241 Sidney St, Waterkloof. If you need a lift to the venue, please talk to Nadege (082 770 0801) at the registration desk. We will leave from the parking lot H14 next to the Maths building at 12:10. If you have a car and extra space, please also be there at 12:10 so that we can help those without transportation.

2.3 Other

- The **registration desk** will be active in Botany 2-20. Nadege Shipley (conference organizer) from Savanna Skills will help you. Any additional information needed can be found there, via email (sanum@savannaskills.co.za) or ask any of the LOC members.
- There is **Wi-fi** available on campus. Log in to Tuks Guest with the following credentials: Username: sanum@up.ac.za; password: 3829

3 Programme

Link to programme on the website:

<http://sanum.github.io/programme.html>

Preliminary programme

Wednesday 27 March 2019	
8:00-8:45	<i>Registration, Tea and Coffee (Botany 2-20 & Atrium)</i>
8:45-9:00	Opening (Maths 2-1)
9:00-10:00	(Maths 2-1) Plenary Talk: Prof N Sauer <i>How sound is our accoustics?</i> Chair: M Labuschagne
10:00-10:30	<i>Tea Break (Atrium)</i>
	(Maths 2-1) Parallel Sessions (Botany 2-23)
	Chair: M Chapwanya Special session: Biomath
	Chair: I Fabris-Rotelli
10:30-11:00	IV Yatat
11:00-11:30	AS Hassan
11:30-12:00	Y Dumont
12:00-12:30	SI Oke*
12:30-13:30	<i>Lunch (Atrium & Botany 2-26)</i>
13:30-14:30	(Maths 2-1) Plenary Talk: Prof M Freitag <i>Balanced truncation model order reduction for stochastically controlled linear systems</i> Chair: I Fabris-Rotelli
	(Maths 2-1) Parallel Sessions (Botany 2-23)
	Chair: I Fabris-Rotelli Special session: Data Science
	Chair: Y Dumont
14:30-15:00	TM Kunene*
15:00-15:30	I Kyomugisha*
15:30-16:00	<i>Tea Break</i>
	(Maths 2-1) Parallel Sessions (Botany 2-23)
	Chair: I Fabris-Rotelli Special session: Data Science
	Chair: M Chapwanya
16:00-16:30	MS De Lancey*
16:30-17:00	T Alakoya*
17:30 –	Welcome Function (Skyline bar)

Thursday 28 March 2019		
8:00	Tea and Coffee	
	(Maths 2-1)	(Botany 2-23)
	Chair: N Hale Special session: Data science	Chair: M Labuschagne
8:30-9:00	DN Wilke	MD Mhlongo & BF Shozi*
9:00-9:30	R Kirsten*	MMK Mothiba*
9:30-10:30	(Maths 2-1)	
	Plenary Talk: Prof F Atay <i>Network Dynamics and Delay Equations</i> Chair: J Banasiak	
10:30-11:00	Tea Break	
	(Maths 2-1)	(Botany 2-23)
	Chair: B Bah Special session: Data Science	Chair: N van Rensburg
11:00-11:30	W Zvarevashe*	NFJ van Rensburg
11:30-12:00	JM Modiba*	M Labuschagne
12:00-12:30	J Stander*	R du Toit*
12:30-13:30	Lunch	
13:30-14:30	(Maths 2-1)	
	Plenary Talk: Prof B Bah <i>Restricted isometry constants of Gaussian matrices</i> Chair: I Fabris-Rotelli	
	(Maths 2-1)	(Botany 2-23)
	Chair: J Banasiak Special session: Biomath	Chair: I Fabris-Rotelli
14:30-15:00	R Ouifki	Opti-Num (Matlab)
15:00-15:30	J Malinzi	Pearson
15:30-16:00	Tea Break	
	(Maths 2-1)	(Botany 2-23)
	Chair: J Banasiak Special session: Biomath	Chair: M Chapwanya
16:00-16:30	CD Tikane*	SN Mungwe*
16:30-17:00	M Ayano	KM Agbavon*
17:30	Young Mathematician's Function (Botany 2-26) Guest speaker: Prof Charis Harley	

Friday 29 March 2019		
08:00	Tea and Coffee	
	(Maths 2-1)	
	Sessions	
	Chair: M Labuschagne	
8:30-9:00	N Hale	
9:00-9:30	JAC Weideman	
9:30-10:30	(Maths 2-1)	
	Plenary Talk: Prof R Deane <i>Physics with the meerkAT and square kilometre array radio telescopes</i> Chair: I Fabris-Rotelli	
10:30-11:00	Tea Break	
11:00-12:00	(Maths 2-1)	
	Plenary Talk: Prof N Trefethen <i>New Laplace and Helmholtz solvers based on rational functions</i> Chair: A Weideman	
12:30-15:30	Conference Lunch, Prizes, Closing (Blu Saffron)	

4 Abstracts - Plenary speakers

4.1 Prof Atay abstract

NETWORK DYNAMICS AND DELAY EQUATIONS

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We will consider a class of dynamical systems on networks where information transmitted between the nodes is subject to time delays. The main theme is finding the relations between geometry, given by the network structure, and the dynamical properties such as stability and convergence speed. This requires combining ideas from graph theory and delay differential equations. In addition, appropriate numerical techniques are needed to deal with the infinite-dimensional nature of the problem. We will discuss both theoretical and numerical aspects.

4.2 Prof Bah abstract

RESTRICTED ISOMETRY CONSTANTS OF GAUSSIAN MATRICES

Bubacarr Bah

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Compressed sensing is a new framework in signal processing where we measure a signal through a non-adaptive linear mapping with far fewer measurements than the ambient dimension of the signal; and are able to fairly accurately reconstruct the signal by exploiting the inherent structure in the signal. For instance, a first order structure in signals in this framework is *sparsity*. Applications of compressed sensing include the single-pixel camera, medical imaging, radar and sonar, error correction, low-rank matrix recovery, machine learning, and many more. Equivalent to condition numbers of submatrices, restricted isometry constants (RICs) of the linear operator (measurement matrix) are the most widely used tool for the analysis of compressed sensing recovery algorithms.

The RIC of a matrix \mathbf{A} measures how close to an isometry is the action of \mathbf{A} on k -sparse vectors, i.e. vectors with k nonzero entries. RICs provide a mechanism by which standard eigen-analysis can be applied to topics relying on sparsity. Specifically, the upper and lower RIC of a matrix \mathbf{A} of size $n \times N$ is the maximum and the minimum deviation from unity (one) of the largest and smallest, respectively, square of singular values of all $\binom{N}{k}$ matrices formed by taking k columns from \mathbf{A} . Calculation of the RIC is intractable for most matrices due to its combinatorial nature; however, many random matrices typically have bounded RIC in some range of problem sizes (k, n, N) . We derived tight asymptotic bounds on the RIC for Gaussian matrices. Our results are a significant improvement over prior bounds by Blanchard, Cartis, and Tanner in 2011, with improvements achieved by grouping submatrices that share a substantial number of columns.

The first set of results mentioned above are buried in very complicated expressions, necessitating the provision of simple explicit formulae for these RIC bounds of $n \times N$ Gaussian matrices with sparsity k . In this new work we focused on the three most important settings: a) n/N fixed and k/n approaching zero, b) k/n fixed and n/N approaching zero, and c) n/N approaching zero with k/n decaying inverse logarithmically in N/n ; in these three settings the RICs a) decay to zero, b) become unbounded (or approach inherent bounds), and c) approach a non-zero constant.

4.3 Prof Deane abstract

PHYSICS WITH THE MEERKAT AND SQUARE KILOMETRE ARRAY RADIO TELESCOPES

Roger Deane
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Radio astronomy is in what many have described as a second golden age with a large number of next-generation telescopes on the horizon. This is driven in large part by the combination of new scientific frontiers and the decreasing effective cost of high-performance computational power. Radio interferometers such as MeerKAT the SKA measure Fourier components of the sky brightness distribution, followed by image reconstruction. This indirect process is a major cause of this computational demand, however, it also provides significant advantages in some cases. I will provide an overview of these radio interferometers and how they are designed to address their key science cases, which include a better understanding of the so-called dark energy and dark matter energy-density components of the Universe.

4.4 Prof Freitag abstract

BALANCED TRUNCATION MODEL ORDER REDUCTION FOR STOCHASTICALLY CONTROLLED LINEAR SYSTEMS

Melina Freitag
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When solving linear stochastic differential equations numerically, usually a high order spatial discretisation is used. Balanced truncation (BT) is a well-known projection technique in the deterministic framework which reduces the order of a control system and hence reduces computational complexity. In this talk we give an introduction to model order reduction by balanced truncation and then consider a differential equation where the control is replaced by a noise term. We provide theoretical tools such as stochastic concepts for reachability and observability, which are necessary for balancing related model order reduction of linear stochastic differential equations with additive Levy noise. Moreover, we derive error bounds for BT and provide numerical results for a specific example which support the theory. This is joined work with Martin Redmann (WIAS Berlin, Germany).

4.5 Prof Sauer abstract

HOW SOUND IS OUR ACOUSTICS?

Niko Sauer
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This talk is about a fairly recent development of the theory of acoustics in ideal, isentropic gases. The framework is in terms of material coordinates (Lagrangian — introduced by Euler), and the physical principles are *balance of linear momentum* and *conservation of mass*. To these is added a *constitutive assumption* which extends notions from classical thermodynamics to a continuum.

The system of nonlinear first order partial differential equations so obtained defies reduction to a reasonable wave equation. Moreover, physical considerations lead to an inequality constraint which ensures that the system is hyperbolic so that one may talk sensibly about wave phenomena.

For spatially one-dimensional motions the system of equations is amenable to analysis by the method of characteristics, but this has difficulties of its own. By introducing the notion of *inverse characteristics* and from there a *warped time*, analysis of the system becomes less complicated and numerical calculations by most elementary means can be carried out. This will be the main thrust of the presentation.

4.6 Prof Trefethen

NEW LAPLACE AND HELMHOLTZ SOLVERS BASED ON RATIONAL FUNCTIONS

Nick Trefethen
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Consider the Laplace equation with Dirichlet boundary data in a polygon or other domain with corners. One could compute the solution u with finite elements, based on a two-dimensional representation of the solution, or integral equations, based on a one-dimensional representation. We propose instead a “zero-dimensional” representation: u is the real part of a rational function with poles exponentially clustered outside each corner. Thanks to an effect first identified by Donald Newman in 1964, the convergence is root-exponential as a function of the number of degrees of freedom, i.e. of the form $\exp(-C * \sqrt{N})$ with $C > 1$. In practice, with 40 lines of Matlab code, we can solve problems with 3 – 8 vertices in a second of laptop time, with 8-digit accuracy all the way up to the singularities in the corners. Evaluation of the solution takes around 20 microseconds per point, and everything generalizes to other kinds of boundary conditions and to the Helmholtz equation. This is joint work with Abi Gopal.

5 Abstracts - Contributed talks

Special sessions

Biomathematics

Mekonnen Ayano	16
Yves Dumont	18
Adamu Shitu Hassan	21
Joseph Malinzi	30
Segun Oke	37
Rachid Ouifki	38
Dipuo Tikane	42
Ivric Valaire Yatat Djeumen .	47

Data Science

Koffi Messan Agbavon	14
Timilehin Alakoya	15
Mark Stephen De Lancey	17
Rene Kirsten	24
Thembinkosi Kunene	25
Irene Kyomugisha	26
Jacob Modiba	34
Jean-Pierre Stander	40
Daniel Nico Wilke	45
Willard Zvarevashe	48

Other topics

PDE's, modelling, associated numerical analysis

Rudi du Toit	19
Madelein Labuschagne	27
Gerald Marewo	32
Mfanafikile Don Mhlongo	33
Makoba Mothiba	35
Franklin Shozi	33
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André Weideman	44

Other

Nick Hale	20
Chinedu Izuchukwu	22
Lateef Olakunle Jolaoso	23
Vusi Magagula	28
Kabelo Mahloromela	29
Eben Mare	31
S'yanda Mungwe	36
Krupa Prag	39
Renate Thiede	41

Construction and analysis of some nonstandard finite difference methods for the FitzHugh-Nagumo equation

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Topic: Numerical Analysis

The FitzHugh-Nagumo equation has diverse applications in the domains of flame propagation, population growth, neurophysiology, autocatalytic chemical reaction and nuclear theory [1, 2]. In this work, we construct five non-standard finite difference schemes approximate the solution of the FitzHugh-Nagumo equation with specified initial and boundary conditions under three different regimes. The properties of the methods such as positive definiteness and boundedness are investigated. The numerical experiments chosen are quite challenging due to the formation of boundary layers in the limit of some rate parameter. The performances of the schemes are compared by computing, L_1 and L_∞ errors, convergence rate with respect to time and CPU time.

References

- [1] A.M. Wazwaz and A. Gorguis, *An analytic study of Fisher's equation by using Adomian decomposition method*, Applied Mathematics and Computation, 609-620 (2004).
- [2] AH. Bhrawy, *A Jacobi-Gauss-Lobatto collocation method for solving generalized Fitzhugh-Nagumo equation with time-dependent coefficients*, Applied Mathematics and Computation, 255-264 (2013).

Strong convergence and bounded perturbation resilience of a modified forward-backward splitting algorithm and its application

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Topic: Optimization

In this paper, we introduce a new modified forward-backward splitting algorithm with perturbations for approximating a common solution of infinite families of inclusion problems and fixed point of demicontractive mappings in Banach space. First, we establish some relationship between the set of fixed points of demicontractive mappings and variational inequality. Also, we prove the strong convergence of our algorithm, its superiorization and further obtain some consequent results from our main theorems. Finally, we discuss the application of our result to approximation of solution of one-kind integro-differential equation with generalized p -Lapacian operator. Our results in this paper, extend, improve and complement many recent results in the literature.

Numerical investigation for MHD mixed convective
couple stress channel flow in a porous medium
considering chemical reaction effect

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Topic: Other (Computational fluid dynamics)

The combined effect of a transverse magnetic field, radiative and chemically reacting heat and mass transfer on steady flow of an electrically conducting couple stress fluid between vertical channels filled with porous medium is studied. The numerical solutions are obtained for the problem making use of spectral quasi linearisation and results are verified with bvp4c techniques to validate. The effects of the chemical reaction and the magnetic field parameters on velocity profile and shear stress for different values of the couple stress parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed.

Feature extraction via Discrete Pulse Transform and CLARA clustering.

MS de Lancey*, I. Fabris-Rotelli
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Topic: Image analysis

The CLARA algorithm is specifically designed for partitioning large sets of data into meaningful groups through a sampling approach. The Discrete Pulse Transform (DPT) makes use of LULU (Lower Upper, Lower Upper) smoothing to decompose a signal (such as a window or sample from an image) into block pulses. Both algorithms have important applications in artificial intelligence and pattern recognition. However, CLARA is a highly stochastic algorithm that relies on sampling for its speed and efficiency at the cost of some accuracy and consistency, while the DPT is a slow but deterministic algorithm. This research will examine how the CLARA algorithm can be used symbiotically in conjunction with the DPT to find block pulses in big data, particularly images. Several methods for sampling from images will be used and tested during the sampling stage of the CLARA algorithm. These will include window sampling, sampling of features after preprocessing using an edge detector and sampling of superpixels after superpixel clustering as a preprocessor.

About cassava mosaic disease: modeling, analysis and simulations

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Topic: Biomathematics

The aim of this talk is to present recent results on the modeling, analysis and simulations of the cassava mosaic disease (CMD). Cassava has become an important staple food for over 500 millions of people in Africa, that is why protecting this crop from pest and diseases is very important. It can grow on soils with a low nutrient capacity and is tolerant to drought. Despite this, CMD is one of the most important and damaging disease. It is caused by virus that has two routes of infection: through vectors and also through infected crops. In the field, the main tool to control CMV spreading is roguing.

We have developed and studied a compartmental temporal model, taking into account the crop growth and the vector dynamics [1]. A brief qualitative analysis of the model is provided, i.e., existence and uniqueness of a solution, existence of a disease-free equilibrium and an endemic equilibrium. We also provide conditions for local (global) asymptotic stability and show that a Hopf Bifurcation may occur, for instance, when diseased plants are removed. Numerical simulations are provided to illustrate all possible behaviors.

Reference

[1] M. Chapwanya, Y. Dumont, 2019. Application of Mathematical Epidemiology to crop vector-borne diseases. The cassava mosaic virus disease case. submitted.

Numerical simulation and analysis of a cantilever Timoshenko beam with a tip body

R du Toit

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Topic: Partial differential equations, modelling, associated numerical analysis

From simple repetitive tasks on a production line to the assembly of space stations, robotic arms are important contributors to modern development. In its simplest form, a cantilever beam with a tip load can be used to model the first limb of a robotic arm. Using the Mixed Finite Element Method (MFEM), various experiments can be conducted to investigate the behaviour of the model. These experiments include using both rigid and elastic endpoints, changing some of the dynamics of the beam or tip body and varying parameters. For each experiment, the system of ODE's acquired is presented and graphs are obtained that visually describe the motion and behaviour of the model for different parameters.

Numerical Aspects of Quadratic Padé Approximation

Nick Hale

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Topic: Numerical Analysis, Approximation theory

A classical (linear) Padé approximant is a rational approximation, $F(x) = p(x)/q(x)$, of a given function, $f(x)$, chosen so the Taylor series of $F(x)$ matches that of $f(x)$ to as many terms as possible. If $f(x)$ is meromorphic, then $F(x)$ often provides a good approximation of $f(x)$ in the complex plane beyond the radius of convergence of the original Taylor series. A generalisation of this idea is quadratic Padé approximation, where now polynomials $p(x)$, $q(x)$, and $r(x)$ are chosen so that $p(x) + q(x)f(x) + r(x)f^2(x) = O(x^{max})$. The approximant, $F(x)$, can then be found by solving $p(x) + q(x)F(x) + r(x)F^2(x) = 0$ by, for example, the quadratic formula. Since $F(x)$ now contains branch cuts, it typically provides better approximations than linear Padé approximants when $f(x)$ is multi-sheeted, and may be used to estimate branch point locations as well as poles and roots of $f(x)$.

In this talk we focus not on approximation properties of Padé approximants, but rather on numerical aspects of their computation. In the linear case things are well-understood. For example, it is well-known that the ill-conditioning in the linear system satisfied by $p(x)$ and $q(x)$ means that these are computed with poor relative error, but that in practice, $F(x)$ itself still has good relative accuracy. Luke (1980) formalises this for linear Padé approximants, and we extend his analysis to the quadratic case. A second issue in the computation of Padé approximants is the presence of Froissart doublets, which are nearby root-pole pairs that cancel in exact arithmetic, but are problematic in numerical computations. For linear Padé approximants, Gonnet *et al.* (2013) propose a robust numerical algorithm based on the SVD which limits these doublets from appearing. We show that this algorithm can be adapted to the quadratic Padé case, and compare it to an alternative approach of Sergeev (1986). Finally, once $p(x)$, $q(x)$ and $r(x)$ have been computed, one needs to evaluate $F(x)$. In the case of quadratic Padé this is complicated by the quadratic equation, and in particular the choice of branch. We discuss various strategies for branch selection as well as some other details regarding accurate evaluation of the quadratic formula.

Mathematical global dynamics and control strategies on
Echinococcus multilocularis

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Topic: Biomathematics

Echinococcus multilocularis, a major cause of echinococcosis in human, is a parasitic sylvatic disease between two major hosts in a predator–prey relation. A new model for the transmission dynamics of *echinococcus multilocularis* with control strategies in the population of red foxes and voles with environment as a source of infection is formulated and rigorously analyzed. The control reproduction number is computed, and is proved that the parasite will die out in the community when this threshold quantity is less than unity and persists otherwise. Global dynamics for two equilibria are proved and elasticity indices of parameters computed. Numerical experiments indicates that administering disinfection of environment only induce more positive impact than applying treatment only on red foxes. Furthermore, adequate values for the two control strategies for echinococcus eradication are estimated.

Proximal-Type Algorithms for Split Minimization Problem in P-Uniformly Convex Metric Spaces

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⁴ Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Saudi Arabia

⁵ Department of Mathematics, Government College University, Pakistan

⁶ Department of Mathematics and Applied Mathematics, University of Pretoria

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Topic: Optimization

In this paper, we study strong convergence of some proximal-type algorithms to a solution of split minimization problem in complete p -uniformly convex metric spaces. We also analyse asymptotic behaviour of the sequence generated by Halpern-type proximal point algorithm and extend it to approximate a common solution of a finite family of minimization problems in the setting of complete p -uniformly convex metric spaces. Furthermore, numerical experiments of our algorithms in comparison with other algorithms are given to show the applicability of our results. Our numerical experiment shows that our algorithm converges faster than that proposed by Choi and Ji [1], Ugwunnadi *et al.* [2] and Ugwunnadi *et al.* [3].

References

- [1] B. J. Choi and U. C. Ji, The proximal point algorithm in uniformly convex metric spaces, *Commun. Korean Math. Soc.*, **31** (2016), 845-855.
- [2] G. C. Ugwunnadi C. Izuchukwu, O. T. Mewomo, Strong convergence theorem for monotone inclusion problem in CAT(0) spaces *Afr. Mat.*, (2018), <https://doi.org/10.1007/s13370-018-0633-x>.
- [3] G. C. Ugwunnadi, A. R. Khan and M. Abbas, A hybrid proximal point algorithm for finding minimizers and fixed points in CAT(0) spaces, *J. Fixed Point Theory Appl.*, (2018), DOI 10.1007/s11784-018-0555-0.

A parallel combination extragradient method with
Armijo line searching for finding common solution of
finite families of equilibrium and fixed point problems

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Topic: Other (Operator Theory)

In this paper, we introduce a new parallel combination extragradient method for solving finite family of pseudo-monotone equilibrium problems and common fixed point of finite family of demicontractive mappings in Hilbert space. The algorithm is designed such that at each iteration a single strongly convex program is solved and the stepsize is determine via an Armijo line searching technique. Also, the algorithm make a single projection onto a sub-level set which is constructed by the convex combination of finite convex functions. Under certain mild-conditions, we state and prove a strong convergence theorem for approximating a common solution of finite equilibrium problems with pseudo-monotone bifunctions and finite demicontractive mappings. Finally, we present numerical examples to illustrate the applicability of the algorithm proposed. This method improves many of the existing methods in the literature.

An improved test for the similarity between two spatial point patterns

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Topic: Other (Spatial Statistics)

A spatial point pattern is where a data set, that has a location attached to each value, is plotted on any two-, three- or higher dimensional space. When we have more than one spatial point pattern, we may be interested in how similar they are to each other. The similarity of spatial point patterns is useful when researchers are looking at factors at risk in certain areas.

A simulation study is conducted to test Andresen's method. The aim of the simulation study was to conclude when this method is most effective for detecting the similarity of spatial point patterns and also determine improved thresholds for when the spatial point patterns can be said to be similar.

The cross- K function for marked spatial point patterns

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Topic: Data Science

The spatial relationship of marked point patterns is often studied. In this discussion we investigate the cross- K function of `spatstat` which is a very useful tool in the analysis of clustering of a point pattern around another point pattern. For real-life data we need to calculate an empirical estimate of the cross- K function. Estimates are often subject to variability so we need a way to quantify this variability and develop confidence intervals for the estimates. Currently in `spatstat` the `varblock` function is the only one available for calculating and plotting confidence intervals for the empirical estimate of the cross- K function. We shall present the theory behind both this method and an alternative method called Loh's bootstrap. Loh's bootstrap is currently only available for unmarked data so having investigated the theory we shall briefly explain how to use it to develop code for calculating and plotting confidence intervals for the cross- K function. Finally, we shall also briefly discuss the study of spatial dependency between a linear network and a point pattern, which is a useful extension of the cross- K function.

Machine Learning for Genomics

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Topic: Data Science

Recent advances in sequencing technology have significantly contributed to shaping the area of genomics and enabled the identification of genetic variants associated with complex traits. The evolution and advancement of high-throughput technology and computational scanning approaches in genomic research have led to the availability of enormous amounts of publicly available clinical and biological datasets.

Conventional methods for genome analysis are constantly becoming ineffective in addressing genomic queries due to the increasing complexity of data emanating from high-throughput sequencing techniques. There is need for data driven learning algorithms and models that enable the selection and prioritization of relevant genetic variants associated with diseases, in post-genomic medicine. Machine learning models can be designed to replicate the interactions between key cell molecules and DNA, cell-to-cell interactions, etc, thereby leading to identification of possible effective drug therapy targets for disease management and eradication.

Machine learning in genomic studies aims to create a better understanding of complex genomic datasets through incorporating biological knowledge and data into learning algorithms that are able to interpret the genome more efficiently and effectively. We highlight current machine learning tools and techniques used for addressing queries in genomic studies.

Cantilever Timoshenko beam and tip body with elastic behaviour at the interfaces

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Topic: Partial differential equations, modelling, associated numerical analysis

Successful manipulation of objects by robotic arms depends on good mathematical models. A simple model for a robotic arm is a cantilever beam with a tip body. We consider the boundary and interface conditions for a hybrid model of a Timoshenko beam with tip body. In previous articles the interface conditions for a Timoshenko beam differ from those for an Euler-Bernoulli beam. We propose that both alternatives can be questioned and an elastic interface must be considered. At the so called clamped end an elastic interface is also introduced. We prove an existence result for solutions, provide error estimates for the finite element approximation and present some numerical results.

Modeling the spread of a virus in a computer network

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Topic: Ordinary differential equations, modelling, associated numerical analysis

In this talk, we present a model for understanding the dynamics of a malicious code (virus) in a network of computers. The compartments are divided into four, namely Susceptible, Exposed, Infected and Removed (SEIR). Differential equations modeling the spread of the malicious code are developed. These equations are solved numerically using spectral relaxation method and Runge-Kutta methods for verification. Results showing the interaction between the various compartments are presented as graphs.

Nonconvex window selection for spatial point pattern data

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Topic: Other (Spatial Statistics)

In the field of spatial statistics, window selection, for point pattern data, is a complex process. The common approaches used are the smallest rectangular bounding window and convex windows due to the obvious use of the Euclidean distance. The chosen window must however cover the true domain of the sampled point pattern data. Choosing a window too large results in estimation in areas which are empty of observed data, but for which it has not been confirmed that observations could have occurred there. Here we present a method for selecting the domain without the restriction of convexity, allowing for a better fit to the true domain, and based on covariate information. As a covariate we use the elevation for spatial point pattern data specific to village house locations in a rural setting.

A moving boundary problem for tumour-immune interactions

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Topic: Biomathematics

A moving boundary problem is developed and analysed to study cancer dormancy. Analysis of the model is carried out for both temporal and spatio-temporal cases. Stability analysis and numerical simulations of the temporal model replicate experimental observations of immune-induced tumour dormancy. Travelling wave solutions are determined using the hyperbolic tangent method and minimum wave speeds of invasion are calculated. Travelling wave analysis depicts that cell invasion dynamics are mainly influenced by their motion and growth rates. Stability analysis of the spatio-temporal model shows a possibility of dynamical stabilization of the tumour-free steady state. Simulation results reveal that the tumour swells to a quiescent state although there is a possibility of regrowth after sometime.

The recovery theorem with application to risk management

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Topic: Risk Analysis and Quantitative Finance

The forward looking nature of option prices provides an appealing way to extract risk measures. In this paper, we extracted forecast densities from option prices that can be used in risk forecasting. More specifically, we extracted the real-world return density forecast, implied from option prices, using the recovery theorem. In addition, we backtested and compared the predictive power of the real-world return density forecast with the risk neutral return density forecast, implied from option prices and a simple historical simulation approach. In an empirical study, using the South African Top40 index, we found that the extracted real-world density forecasts, using the recovery theorem, yielded satisfying value at risk measures.

A numerical solver for systems of nonlinear reaction-diffusion equations

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Topic: Partial differential equations, modelling, associated numerical analysis

In this study, we consider a initial-boundary value problem for a system of nonlinear reaction-diffusion equations. Applications of such equations include population dynamics, image restoration and pattern dynamics to mention a few. Since the equations are nonlinear we seek a numerical solution. Discretisation in space is done using a Newton-Galerkin method. For discretisation in time, we make use of a compact finite difference scheme. Numerical experiments are performed to investigate the efficiency of the numerical method.

Transient response of longitudinal fin of triangular profile to step change in base temperature and in base heat flow conditions

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Topic: Partial differential equations, modelling, associated numerical analysis

In this paper, solutions for model describing heat transfer in longitudinal fins of triangular profile are constructed. The heat coefficients and thermal conductivity are assumed to be power law temperature dependent. The boundary conditions are the step change in base flow conditions and in base temperature. Both local and nonlocal symmetry techniques are employed to analyze the problem at hand. Nonlocal symmetries are admitted when some arbitrary constants appearing in the governing equations are specified. The effects of the thermo-geometric fin parameter and the power law exponent on temperature distribution are studied.

Multiscale Spatio-Temporal Analysis of Crime Data

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Topic: Image analysis, Spatial Statistics

The idea of automatic scale selection was developed in the book *Scale-Space Theory in Computer vision* by Lindeberg, where he uses Gaussian filters to smooth data. However, images are stored as a collection of discrete pixels, thus a discrete approximation to the Gaussian kernel was developed, the DPT. The DPT is a naturally discrete scale-space and possesses many strong structural properties such as edge preservation. Its use is thus preferable over the Gaussian scale-space. Thus this research aims to develop an automatic scale selection for the DPT scale-space and follow on to apply spatio-temporal modelling of the DPT pulses at the selected salient scales on temporal remote sensing images for road detection as well as change detection of the roads over time. With the development of a way to automatically select a salient scale using the Discrete Pulse Transform the complexity is reduced. The idea will be used to identify roads over time in a satellite image of Khayelitsha. The idea will then be used to analyse crime data from Khayelitsha. It is crucial to identify roads over time since in an informal area new roads may appear and old ones can disappear.

Comparison of the Charpit and Adomian Decomposition Method

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Topic: Partial differential equations, modelling, associated numerical analysis

There are various methods for solving linear and nonlinear partial differential equations. In this talk, we present a comparative study between two methods of solving first order nonlinear equations involving two independent variables, namely, the Charpit and the Adomian decomposition method. We show that the Adomian decomposition method has more advantage or it is easier than the Charpit Method when dealing with different types of partial differential equations.

Spectral deferred correction

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Topic: Ordinary differential equations, modelling, associated numerical analysis

Ordinary and partial differential equations, and in particular initial value problems and initial-boundary value problems, are ubiquitous in applications from science and engineering. Numerical approximations are typically required, usually with some kind of method of lines approach to separate the time and space variable discretisations. If high accuracy is required, then spectral methods can be used to discretise the spatial part. There are many techniques advance the time variable (known as time-stepping), such as multistep methods, Runge-Kutta methods, and exponential integrators, but these are typically low accuracy in time. Greengard et al. introduced in 1998 the idea of spectral deferred correction (SDC), which allows an efficient way to get spectral accuracy (geometric convergence) in time. The method achieves the spectral accuracy in time through the correction function developed using Picard integral equation, repeatedly correcting errors from a low order method to obtain high order accuracy. In this talk we introduce SDC as derived by Greengard et al., and discuss some of its advantages and drawbacks. We show some modifications of the approach, such as the implications of the choice of quadrature nodes on the stability and the convergence, as well as some other recently proposed advances.

Global Stability Analysis for Breast Cancer Model with Treatment Strategies

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Topic: Biomathematics

We propose and analyse breast cancer model with combined ketogenic diet and anti-cancer drug. The stable Treatment Tumor-Free Equilibrium (TTFE) of the model coexists with a stable endemic equilibrium when the treatment induced invasion reproduction number, R_i^* is less than unity. However, Lyapunov function was constructed to prove that TTFE is globally asymptotically stable for a special case and sensitivity index was calculated. Hence, we apply optimal control theory to investigate how the combination of anticancer drugs and ketogenic-diet was implemented, for a certain period, in order to inhibit the tumor growth and eliminate cancer cells. Furthermore, we establish the existence of the optimality system and use Pontryagin's Maximum Principle to characterize the optimal levels of the two treatment measures. In addition, the simulation results show that the combination of anticancer drugs and ketogenic diet is the most cost-effective strategy for treating breast cancer.

Mathematical analysis of a malaria model with two
Gamma distributed delays, logistic growth and
density-dependent biting rate

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Topic: Biomathematics

We present and analyse a vector-borne disease model with two gamma distributed delays representing the incubation periods of the disease in the vector and hosts. The model assumes a logistic growth for both the host and vector populations and includes a density-dependent biting rate.

Conditions for the occurrence of backward bifurcation are derived in terms of the delays and the (nonlinear) biting rate.

Computational Logistics of the Vehicle Routing Problem with Time Windows

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Topic: Optimisation

For the efficiency of operational logistics in today's inter-connected and globalised world, the study of the Vehicle Routing Problem with Time Windows (VRPTW) is imperative. The objective of the VRPTW is to minimise both the number of dispatched vehicles and their respective travelled distances, for delivery operations. The research reviews the application of various deterministic and non-deterministic optimisation techniques to the VRPTW. Specifically, the comparative study considers the application of two metaheuristic techniques: Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO) algorithm. These two techniques are applied and compared for experiments conducted under standardised experimental conditions. Routes, distances and CPU time of each produced solution are recorded. These are studied to draw conclusions and highlight correlations among data set types, applied solution techniques and evaluation metrics.

Stochastic gradient descent in deep learning

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Topic: Optimization

The topic being presented is an optimization algorithm called stochastic gradient descent. The performance of different gradient descent (stochastic and non-stochastic) algorithms are compared and problems that may be encountered are discussed. Examples are included to show which type of optimization works best for different scenarios. The possibility of using GANs together with LULU operators to detect texture in images for possible object detection is also discussed.

Statistical accuracy of a linear object extraction algorithm for greyscale images

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Topic: Image analysis

Informal unpaved roads in developing countries arise naturally through human movement and informal housing setups without government authorities being informed. These roads are not authorised nor maintained by council, and mapping them from satellite images is a common problem. Information on informal roads is critical for sustainable city growth and maintenance, and may be gleaned from spatial big data. Attempts to obtain the required information are sparse, and no automatic or semi-automatic approach has yet been employed. In this presentation, we will consider the challenges associated with the extraction of informal roads. We aim to detect these roads in greyscale images using LULU smoothing operators and the Discrete Pulse Transform (DPT). The DPT is obtained via recursive application of the LULU operators. We will develop an algorithm for the extraction of linear features based on fundamental morphological base shapes. These fundamental shapes are obtained by considering the possible ways in which pixels can connect using 4-connectivity. More complex shapes are formed by connecting these base shapes according to specific rules, which take into account shape characteristics such as compactness. Possible challenges for road extraction are discussed by studying areas in Gauteng Province and North West Province, South Africa exhibiting a variety of environmental and road characteristics. Sources of uncertainty are the indefinite boundaries, surface type heterogeneity and the presence of stationary objects such as trees. We find that the conceptualisation of informal road boundaries, and hence the definition of an informal road, may be adapted to address local challenges. These include the existence of clear boundaries, the visibility of road edges and whether or not it is desirable to use only the central part of the road for transport. A rigorous image analysis technique based on image pulses will be developed and applied to detect a variety of informal roads in various contexts.

Beam models for tap root systems in plants

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Topic: Partial differential equations, modelling, associated numerical analysis

The following experimental results are reported in *The Mechanics of Root Anchorage*, A.R Ennos, *Advances in Botanical Research vol 33, 2000*. “As the plant is pushed over the tap root is bent and rotates about a point some way below the soil surface, the top moving leeward and the bottom to the windward. These movements are resisted by two components of anchorage; the compressive resistance of the soil to lateral motion; and the bending resistance of the tap root itself. . . . the exact mode of failure depends on the soil properties. Soft, wet soil fails readily in compression and the plant rotates underground pushing the soil sideways and eventually leaning over permanently. In stronger, drier soil, in contrast the tap root or even stem are more likely to fail”. In this talk, we present a mathematical beam model to study the reaction of a plant, with a tap root, to lateral static and dynamic loads. The theoretical results are compared to the experimental findings.

Linear approximation for a nonlinear eigenvalue problem

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Topic: Partial differential equations, modelling, associated numerical analysis

A well-known example of a nonlinear eigenvalue problem and its linear counterpart is the pair of boundary value problems for the buckling of a beam. Both problems admit only the zero solution for small values of the relevant parameter. For critical values of the parameter, the linear eigenvalue problem has infinitely many solutions but only the zero solution for all other values. On the other hand the nonlinear problem has exactly three solutions when the parameter exceeds the first critical value provided that it does not become too large.

In general one may ask what is the “linear approximation” for a nonlinear eigenvalue problem and what useful information can be obtained from it. In this presentation only examples will be considered. An elementary nonlinear eigenvalue problem for a system of (ordinary) equations is briefly discussed since the linear approximation is rigorously defined. Finally, the main example is a nonlinear eigenvalue problem for a system of differential equations. The problem also arise from a beam application but of interest is the different boundary value problems.

Complex Singularities in Nonlinear Wave Equations

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Topic: Partial differential equations, modelling, associated numerical analysis

Solutions to nonlinear wave PDEs exhibit a wide range of interesting phenomena such as shocks, solitons, recurrence, and blow-up. As an aid to understanding some of these features, the solutions can be viewed as analytic functions of a complex space variable. The dynamics of poles and branch-point singularities in this complex plane can often be associated with nonlinear properties of the solution. For example, shock formation and multi-valuedness in the inviscid Burgers equation can be understood as a conjugate pair of branch-point singularities that travel in the complex plane and meet on the real axis at the particular instant the shock is formed. In this talk we explore such behaviour through analysis, numerics, and visualization. Model PDEs include the Burgers, Korteweg-de Vries, and nonlinear Schrödinger equations.

Gradient-only optimization by identification of non-negative gradient projection points

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Topic: Optimization

In practice numerical minimization is an iterative process for finding the design vector \mathbf{x}^* which yields the minimum function value over all feasible designs. To find the minimum there are different definitions of optimality that can be used, depending on the underlying assumptions regarding the nature of the merit and constraint functions describing the design problem. In this study we restrict ourselves to unconstrained multivariate real parameter non-linear minimization, i.e. we seek the minimizer $\mathbf{x}^* \in \operatorname{argmin}_{\mathbf{x}} f(\mathbf{x})$ with $\mathbf{x} \in \mathcal{R}^n$ of the merit function $f : \mathcal{R}^n \rightarrow \mathcal{R}$. Here no smoothness nor continuity assumptions regarding f is made.

In general, finding \mathbf{x}^* precisely and economically may be computationally very difficult, hence candidate minimizers are searched for instead. A candidate minimizer or local minimum, $\tilde{\mathbf{x}}^*$, is defined as any point for which you can find an open ball B such that $f(\tilde{\mathbf{x}}^*) < f(\mathbf{x})$, $\forall \mathbf{x} \in (B \cap \mathcal{R}^n)$. Restriction on f is however made, that excludes any constant function over \mathcal{R}^n .

Gradient-based optimization for determining the candidate minimizers, has evolved from the additional assumption that f is at least twice continuously differentiable. This allows for the candidates to be obtained from the necessary and sufficiency conditions, i.e. from the so-called optimality criteria. A candidate minimizer $\tilde{\mathbf{x}}^*$ is thus defined by necessary condition $\nabla f(\tilde{\mathbf{x}}^*) = \mathbf{0}$ and that the Hessian matrix at $\tilde{\mathbf{x}}^*$ satisfies $\mathbf{x}^T \mathbf{H}(\tilde{\mathbf{x}}^*) \mathbf{x} > 0$ in the neighbourhood of $\tilde{\mathbf{x}}^*$.

The main concern of this paper are recently gradient-only optimization techniques, that have been developed as an alternative for determining candidate minimizers. This approach relaxes the smoothness and continuity assumptions of classical gradient-based optimization to define candidate solutions as non-negative gradient projection points (NN-GPP), \mathbf{x}_g^* as any point such that for every $\mathbf{u} \in \{\mathbf{y} \in \mathcal{R}^n \mid \|\mathbf{y}\| = 1\}$ there exists a real number $r_u > 0$ for which, $\nabla_A^T f(\mathbf{x}_g^* + \lambda \mathbf{u}) \mathbf{u} \geq 0$, $\forall \lambda \in (0, r_u)$, holds. Here, ∇_A denotes the associated gradient [1]. The only assumption regarding f is that the

associated gradient is everywhere defined [1].

It is shown that the latter gradient-only formulation has profound implications in design optimization problems where the objective function is computed from the solution of models involving systems of differential or mixed algebraic and partial differential equations. Here the computed objective function may often be effectively discontinuous. The construction of surrogate based optimization using only gradient information for discontinuous functions then becomes necessary. This application has indeed been successfully demonstrated [1]. The numerical computation of associated gradients for discontinuous functions using the complex-step method is also discussed.

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Mathematical modelling for practical application of Sterile Insect Technique

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Topic: Ordinary differential equations, modelling, associated numerical analysis

Vector control, like sterile insect technique (SIT), is essential to reduce the risk of vector-borne diseases [3]. SIT control generally consists of massive releases of sterile insects in the targeted area in order to reach elimination or to lower the pest population under a certain threshold.

The model presented here is minimalistic with respect to the number of parameters and variables. However, it yields the same qualitative behavior of the insect population with SIT control, as more complicated models like in [1, 2, 3]. More precisely, as in mentioned models, our minimalistic SIT model has two stable equilibria.

Practically, massive releases of sterile males are only possible for a short period of time. That is why, using the bistability property, we develop a new strategy to maintain the wild population under a certain threshold, for a sustainable low level of SIT control.

We illustrate our theoretical results with numerical simulations, in the case of SIT mosquito control.

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Time Series Analysis of Rainfall Variability in South Africa Using Ensemble Empirical Mode Decomposition

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Topic: Other (Data Science and Statistics)

Quantitative and qualitative understanding of the multi-scale shifts in the rainfall is very crucial for better preparedness and management. A data adaptive method ensemble empirical mode decomposition (EEMD) is used to decompose the rainfall data into different time scale called intrinsic mode function (IMF). Using phase synchronisation the IMFs are linked to different climate variables such as quasi-biennial oscillation and El Nino. An optimal amplitude of noise to be used in EEMD is also proposed.

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