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MaSS

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UNIVERSITY OF JOHANNESBURG

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Welcome Address

The Department of Mathematics and Applied Mathematics in the Faculty of Science at the University of Johannesburg is tremendously proud to host and welcome all the SANUM delegates to the 44th SANUM conference.

After postponements and cancellations due to the COVID pandemic, we had hoped to have our first in-person, independent conference in 2023 at UJ for the first time. Unfortunately, do to the uncertainty of unrest on campus we were forced to follow an online format for SANUM again this year. Nevertheless, SANUM continues to remain relevant in the post-COVID age, perhaps even more so. Applied Mathematics and all tangential fields are becoming increasingly aligned with data-driven science and 4IR.

We hope that your online SANUM experience is productive, introduces some new ideas and hopefully sparks new conversations and collaborations.

Byron Jacobs
On behalf of the SANUM LOC

Logistics

The Conference

- **Access links:** Here are the MS Teams links to access the sessions. These links will be valid for the duration of that particular session, so please organize your schedule beforehand, to avoid missing a session.

  Welcome Address

  Plenary Day 1: Jan Nordström

  Day 1 Session 1

  Day 1 Session 2

  Plenary Day 2: Sandile Motsa

  Day 2 Session 1

  Day 2 Session 2

  Plenary Day 3: Clare Dunning

  Day 3 Session 1

  Day 3 Session 2

  Please do not share these links with anyone.

- **Plenary lectures:** Plenary Lectures are 60 minutes long (55 min talk + 5 min questions).

- **Contributed talks:** If you are presenting your work, you will be given the permission to start your camera, microphone and to share your screen. Speakers are required to enter in the room at least 10 minutes before the session starts. Please send a private message to the host (via chat) to receive the permission to share your screen. Contributed talks will be kept strictly to 30 minutes (25 min talk + 5 min questions).

  We must keep the time schedule.

- **Q&A:** Questions and answers will be managed by the chair of the session. Participants can write questions on the chat or "raise hand" for a direct question using the microphone. If time is not sufficient, some questions may be deferred to the end of the day.

- **Tea and lunch breaks:** Please see the conference program to organize your tea and lunch breaks.

Conference Organising Committee

If you have any questions, comments or concerns please feel free to contact any of the conference organising committee members:

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Programme

Link to programme on the website: https://sanum.github.io/programme.html

Monday, 03 April	
Time	**Event**
09:00 – 09:15	**Welcome**
09:15 – 10:15	**Plenary: Jan Nordstrom** **Title: Nonlinear Boundary Conditions for Energy and Entropy Stable Initial Boundary Value Problems in Computational Fluid Dynamics** *Chair: Byron Jacobs*
10:15 – 10:30	**Tea Break**
SESSION 1 **Chair: Byron Jacobs**	
10:30 – 11:00	**Nick Hale** *A spectral collocation method for functional and delay differential equations*
11:00 – 11:30	**Carel Olivier** *Pseudospectral methods with odd discretizations revisited*
11:30 – 12:00	**Emma Nel** *Spectral Domain Decomposition*
12:00 – 12:30	**André Weideman** *Beyond blow-up: A numerical and asymptotic study*
12:30 – 13:30	**Lunch**
SESSION 2 **Chair: Simphiwe Simelane**	
13:30 – 14:00	**Claude Moutsinga** *A time multidomain spectral method for valuing affine stochastic volatility and jump diffusion models*
14:00 – 14:30	**Molwantwa Shapa Kanyane** *A Mathematics Model for Pricing Bond under Affine Process*
14:30 – 15:00	**Walter Mudzimbabwe** *A policy iteration approximation for stochastic optimal control with random coefficients*
15:00 – 15:30	**Katlego Joshwin Mabena** *Numerical and analytical methods for the valuation of financial derivatives*

Tuesday, 04 April	
Time	**Event**
09:00 – 10:00	**Plenary: Sandile Motsa** **Title: Overlapping Grids and Rationalized Hybrid Block Methods for Efficient Numerical Solutions of Stiff IVPs** *Chair: Phumlani Dlamini*
10:00 – 10:15	**Tea Break**
SESSION 3 **Chair: Phumlani Dlamini**	
10:15 – 10:45	**Prince Nchupang** *A provably stable and high-order accurate finite difference approximation for the incompressible boundary layer equations*
10:45 – 11:15	**Uthman Olamide Rufai** *A One-Step Multi-Derivative Hybrid Block Method with Modified-Picard Iteration for the Solution of Second Order IVPs*
11:15 – 11:45	**James Malele** *Highly accurate compact finite difference schemes for two-point boundary value problems with Robin boundary conditions*
11:45 – 12:15	**Felix Damilola Ajibade** *Investigation of the f^* algorithm on strong pseudo-contractive mappings and its application*
12:15 – 13:30	**Lunch**
SESSION 4 **Chair: Katlego Sebogodi**	
13:00 – 13:30	**Inger Fabris-Rotelli** *Linear hotspot detection for a point pattern in the vicinity of a linear network*
13:30 – 14:00	**Rene Stander** *Multiscale decomposition of spatio-temporal lattice data for hotspot trend predictions*
14:00 – 14:30	**Kabelo Mahloromela** *Window selection in spatial point pattern analysis*
14:30 – 15:00	**Renate Thiede** *Measuring homogeneity of spatial line patterns*
15:00 – 15:30	**Alta Jooste** *On the zeros of Meixner polynomials*

Wednesday, 05 April	
Time	**Event**
09:00 – 10:00	**Plenary: Clare Dunning** **Title: Rational solutions of the Painleve equations and partition combinatorics** *Chair: Farai Chirove*
10:00 – 10:15	**Tea Break**
SESSION 5 **Chair: Farai Chirove**	
10:15 – 10:45	**Kirstin Hohls** *Existence and properties of solutions of a semi-linear Timoshenko rod model with axial force*
10:45 – 11:15	**Nic van Rensburg** *Spectral theory for linear rod models*
11:15 – 11:45	**Sonja du Toit** *A nonlinear model for a local linear elastic rod*
11:45 – 12:15	**Madelein Labuschagne** *Finite element application to earthquake induced oscillations in vertical structures*
12:15 – 12:45	**Belinda Stapelberg** *Modal analysis of a new model for earthquake induced oscillations of high-rise vertical structures*
12:45 – 13:30	**Lunch**
SESSION 6 **Chair: Vivien Visaya**	
13:30 – 14:00	**Thasheel Singh Jagutpal** *A stable and high-order continuous Galerkin method for linear hyperbolic equations*
14:00 – 14:30	**Praise-God Madueme** *Understanding the transmission pathways of Lassa Fever: a mathematical modeling approach*
14:30 – 15:00	**Gerald Marewo** *Numerical solution of the minimal surface problem*
15:00 – 15:30	**Sinenhlanhla Mtshali** *On the validation of a fractional order model for pharmacokinetics using clinical data*
15:30 – 15:45	**Closing**

Plenary Abstracts

Nonlinear Boundary Conditions for Energy and Entropy Stable Initial Boundary Value Problems in Computational Fluid Dynamics

Jan Nordström

Linköping University

We derive new boundary conditions and implementation procedures for nonlinear initial boundary value problems that lead to energy and entropy bounded solutions. A step-by-step procedure for general nonlinear hyperbolic problems on skew-symmetric form is presented. That procedure is subsequently applied to the three most important equations in computational fluid dynamics: the shallow water equations and the incompressible and compressible Euler equations. Both strong and weak imposition of the nonlinear boundary conditions are discussed. Based on the continuous analysis, we show that the new nonlinear boundary procedure lead to energy and entropy stable discrete approximations if the scheme is formulated on summation-by-parts form in combination with a weak implementation of the boundary conditions.

Overlapping Grids and Rationalized Hybrid Block Methods for Efficient Numerical Solutions of Stiff IVPs

Sandile Motsa

University of Eswatini

This study presents a novel method for developing optimal single-step hybrid block methods for solving first order initial value problems (IVPs) with high accuracy. To maximize the order of accuracy, the method involves collocating at specific grid points derived from polynomials that minimize local truncation error. The resulting method is of general order and can be improved by including more intra-step points, with efficiency improved by a variable integration step-size that is adaptive. To implement hybrid block methods on non-linear IVPs, an iterative approach based on linear partitioning of the differential equation is proposed. To validate the accuracy and performance of the proposed rationalized optimal hybrid methods, numerical experiments are performed on examples of stiff IVPs with chemical reactions applications, and the results are compared to existing methods. The overlapping hybrid block technique is also introduced in the study, which reduces local truncation error by at least one order of integration step size.

Rational solutions of the Painleve equations and partition combinatorics

Clare Dunning

University of Kent

Rational solutions of the Painlev\E9 equations and their higher-order analogues are often written in the form of a logarithmic derivative of a ratio of special polynomials. These polynomials have a wide range of applications, including in classical and quantum integrable systems, random matrix theory, supersymmetric quantum mechanics, orthogonal polynomials. The polynomials may be expressed as certain Wronskian determinants labelled by partitions. Combinatorial aspects of the partitions turn out to play intriguing roles in various aspects of the polynomials including their coefficients, distribution of zeros in the complex plane and the discriminant.

Conference Abstracts

A spectral collocation method for functional and delay differential equations

Nick Hale
Stellenbosch University

A simple but effective method for the numerical solution of functional and delay differential equations (DDEs) via spectral collocation is described. The method is demonstrated by means of several examples of linear and nonlinear DDEs with various delay types, including discrete, proportional, continuous, and state-dependent delay. The approach is a natural extension of standard spectral methods based on polynomial interpolation and can be readily incorporated in existing spectral discretisations, for example, the Chebfun/Chebop framework for the automated solution of differential equations.

A time multidomain spectral method for valuing affine stochastic volatility and jump diffusion models

Claude Moutsinga[1], Edson Pindza[2], Eben Maré[2]
[1]Sefako Makgatho Health Sciences University
[2]University of Pretoria

The general form of existing multivariate are based on stochastic volatility and jump diffusion models. These models are widely used in financial engineering for pricing derivatives, risk management, and asset allocation. The way to handle them typically lead to the need of solving systems of stiff Riccati differential equations. In this talk, we propose a time spectral domain decomposition method for solving systems of stiff Riccati differential equations. The technique is applied to solving stiff diffusion model problems found in oil pricing, interest rate and electricity models. Numerical methods show that the present approach is efficient, highly accurate and a good alternative to the existing numerical methods.

A Mathematics Model for Pricing Bond under Affine Process

Molwantwa Shapa Kanyane

Sefako Makgatho Health Sciences University

In this dissertation, the bond pricing problem is discussed. The problem usually results in solving a general parabolic partial differential equation. In the case of affine settings such as Vasicek or CIR model, the pde yields a system of linear ODE for which an exact solution is derived. However, in the general context numerical method come to the rescue.

In the one-dimensional case we especially focus on the Crank Nicolson finite difference method, a method that has gained so much popularity in financial mathematics. Among the three finite difference scheme commonly known, that is the Explicit, Implicit and Crank-Nicolson, the later proves to perform better than the other two. For the multidimensional case the attention is given to the Alternating Direction Implicit (ADI) method. The numerical solutions are plotted versus the analytical solutions in order to evaluate the error. Finite difference method shows to be simple and reliable for bond pricing problem, as the error decays rapidly with the number of points in the discretization.

Understanding the transmission pathways of Lassa Fever: a mathematical modeling approach

Praise-God Madueme

University of Johannesburg

The spread of Lassa fever infection is increasing in West Africa over the last decade. The impact of this can better be understood when considering the various possible transmission routes. We designed a mathematical model for the epidemiology of Lassa Fever using a system of nonlinear ordinary differential equations to determine the effect of transmission pathways toward the infection progression in humans and rodents including those usually neglected such as the environmental surface and aerosol routes. We analyzed the model and carried out numerical simulations to determine the impact of each transmission routes. Our results showed that the burden of Lassa fever infection is increased when all the transmission routes are incorporated and most single transmission routes are less harmful, but when in combination with other transmission routes, they increase the Lassa fever burden. It is therefore important to consider multiple transmission routes to better estimate the Lassa fever burden optimally and in turn determine control strategies targeted at the transmission pathways.

Investigation of the F^* algorithm on strong pseudo-contractive mappings and its application

Felix Damilola Ajibade
Federal University Oye Ekiti, Nigeria

This paper is centered on investigating the convergence of the F^* algorithm to the fixed point of strongly psuedo-contractive mapping in uniformly convex Banach spaces with application. These were to investigate the convergence of the F^* algorithm for a class of psuedo-contractive mapping in the framework of uniformly convex Banach space. To achieve these objectives, certain existing inequalities in convex space were utilized following the assumption of the existence of a fixed point, other neccessary conditions were obtained for the convergence of the sequence generated by the F^* algorithm to the fixed point of strong psuedo-contractive mapping. The results showed that the sequence generated by the F^* algorithm converged to the fixed point of the class of psuedo-contractive mapping investigated.

A One-Step Multi-Derivative Hybrid Block Method with Modified-Picard Iteration for the Solution of Second Order IVPs

Uthman Olamide Rufai, Precious Sibanda, Sicelo Goqo
University of KwaZulu-Natal, South Africa

This study presents a one-step multi-derivative hybrid block method (OSMDHBM) with order 10, which incorporates third derivatives for the solution of linear and non-linear second order initial value problems (IVPs). The derivation incorporates a multi-step collocation and interpolation method using an approximated power series as the basis function. The intra-step points are obtained from the derivative of a shifted Legendre polynomial of degree four. The accuracy, consistency and stability properties of the method are analyzed. The non-linear IVPs is linearized using the modified-Picard iteration method. In order to demonstrate the superiority of the method, numerical experiments are presented. Comparisons are made between the numerical results obtained and results from other methods and similar schemes in the literature.

Numerical and analytical methods for the valuation of financial derivatives

Katlego Joshwin Mabena

Sefako Makgatho Health Sciences University

The understanding and pricing of a barrier options, are of great importance in the financial world. It becomes more complex when we need to project the future price of barrier options because the main feature of barrier options is that the contracts becomes activated only if the price of the underlying asset reaches a predetermined level. In this talk we will focus on the performance of the Black-Shole model, Trinomial method and the standard finite difference method in pricing the barrier option under different conditions.

Existence and properties of solutions of a semi-linear Timoshenko rod model with axial force

Kirstin Hohls, Nicolaas Janse van Rensburg

University of Pretoria

For the semi-linear Timoshenko rod model with axial force, the nonlinear system of partial differential equations considered is

$$\begin{aligned}
\partial_t^2 w &= \partial_x \left(\partial_x w - \phi\right) + \left(\frac{D}{\gamma} + \frac{1}{2\gamma}\int_0^1 (\partial_x w)^2\right)\partial_x^2 w, \\
\frac{1}{\alpha}\partial_t^2 \phi &= \partial_x w - \phi + \partial_x \left(\frac{1}{\beta}\partial_x \phi\right).
\end{aligned}$$

This mechanical system, suggested by M.H. Sapir and E.L. Reiss [Dynamic Buckling of a nonlinear Timoshenko beam, *SIAM J. Appl. Math.* **37** (1979), 290-301], is a special case of a problem of the form

$$u'' = Au + f(u)$$

on some Hilbert space, where A is a linear operator and f a nonlinear mapping.

In the presentation, an outline of the existence theory of the above model will be given. In addition to this, investigations of other properties of solutions will be shown. These investigations include the possibility of a non-trivial equilibrium as well as dynamic buckling. In order to aid in the investigations, results from the spectral theory of the related linear Timoshenko rod model are also considered.

Finite element application to earthquake induced oscillations in vertical structures

Madelein Labuschagne, Sonja du Toit, Belinda Stapelberg

University of Pretoria

In this presentation the focus is on the numerical investigation of a new model for earthquake induced oscillations in a vertical structure. The foundation of the structure is considered as a rigid body attached to an adapted Timoshenko beam model and the effect of ground motion is modelled as forces acting on the foundation.

Spectral theory for linear rod models

Nic van Rensburg, Kirstin Hohls

University of Pretoria

This presentation concerns spectral properties of differential operators with the emphasis on application to vibration models and wave propagation. The main application considered is a linear rod model based on Timoshenko's beam theory. The spectral theory for the standard case is improved and new results for the axially loaded beam are presented. We show briefly how these results are useful for dynamic models (vibration) and a nonlinear model. The spectral theory for the model mentioned above is also useful to demonstrate aspects of "the" general theory. Finally, it is interesting to see how the general theory relates to Fourier series.

A policy iteration approximation for stochastic optimal control with random coefficients

Walter Mudzimbabwe

University of the Witwatersrand

In this talk, we develop a numerical method to solve an optimal control problem driven by stochastic differential equations (SDEs) with random coefficients. In particular, we solve the associated Hamilton-Jacobi-Bellman equation (HJB) which takes the form of a second order stochastic partial differential equation (SPDE) by deriving a policy iteration method. We also investigate convergence properties of the approximation and its relation to Newton method. We illustrate the usage of the resulting algorithm with example problems from utility maximisation and finance.

Highly accurate compact finite difference schemes for two-point boundary value problems with Robin boundary conditions

James Hloniphani Malele, Phumlani Dlamini, Simphiwe Simelane

University of Johannesburg

In this study, a high-order compact finite difference method is used to solve boundary value problems with Robin boundary conditions. The norm is to use a first-order finite difference scheme to approximate Neumann and Robin boundary conditions, but that compromises the accuracy of the entire scheme. As a result, new higher-order finite difference schemes for approximating Robin boundary conditions are developed in this work. Six examples for testing the applicability and performance of the method are considered. Convergence analysis is provided, and it is consistent with the numerical results. The results are compared with the exact solutions and published results from other methods. The method produces highly accurate results, which are displayed in tables and graphs.

Pseudospectral methods with odd discretizations revisited

Carel Olivier[1], N.V. Alexeeva[2]

[1]North-West University

[2]University of Cape Town

It is common for applications that employ the fast Fourier transform (FFT) [1] to choose an even number of discretized points in order to optimize the efficiency of the FFT. However, this results in an asymmetry that leads to complications in the treatment of the highest frequency mode. More specifically, for pseudospectral treatments of real solutions, no complex conjugate term exists that automatically preserves the real nature of the solution. In this paper, we investigate the effects on spectral differentiation, and consider the alternative, namely the use of grids with an odd number of grid points. The heat equation and the Korteweg-de Vries equation are used to illustrate the findings obtained from a aliasing error analysis.

References

1. J. W. Cooley and J. W. Tukey, An Algorithm for the Machine Calculation of Complex Fourier Series, Math. Comput. 19 (1966), 297–301.

A provably stable and high-order accurate finite difference approximation for the incompressible boundary layer equations

Prince Nchupang[1], Arnaud Malan[1], Fredik Laurén[2], Jan Nordström[2,3]

[1]University of Cape Town
[2]Linköping University
[3]University of Johannesburg

Energy stability of the laminar incompressible boundary layer equations is considered. We first derive continuous energy estimates, and then proceed to the discrete setting. We formulate the discrete approximation using high-order finite difference methods on summation-by-parts form and implement the boundary conditions weakly using the simultaneous approximation term method. By applying the discrete energy method and imitating the continuous analysis, the discrete estimate that resembles the continuous counterpart is obtained proving stability. We also show that these newly derived boundary conditions removes the singularities associated with the null-space of the nonlinear discrete spatial operator. Numerical experiments that verifies the high-order accuracy of the scheme and coincides with the theoretical results are presented. The numerical results are compared with the well-known Blasius similarity solution. Furthermore, on the same setting, we make comparison with the incompressible Navier-Stokes equations.

A nonlinear model for a local linear elastic rod

Sonja du Toit[1], Madelein Labuschagne[1], Alna van der Merwe[2]

[1]University of Pretoria
[2]Auckland University of Technology

A nonlinear model for the planar motion of a rod that undergoes flexure, shear and extension, was recently published. For this model, an algorithm based on the mixed finite element method was developed as part of my doctoral study and later improved in collaboration with others. The algorithm is used for an investigation into elastic waves propagated in the rod. Interesting properties of the rod include the increased propagation speed of elastic waves when compared to the linear Timoshenko beam, and the appearance of buckled states or equilibrium solutions for compressed beams. It is also shown that the rod is applicable to a wide range of slender elastic objects; from beams to highly slender flexible rods.

Modal analysis of a new model for earthquake induced oscillations of high-rise vertical structures

Belinda Stapelberg, Sonja du Toit, Madelein Labuschagne
University of Pretoria

A new model for earthquake induced oscillations of high-rise vertical structures is introduced, which is modelled by an adapted Timoshenko beam model. The adaptation leads to a new and more realistic model for soil-structure interaction, where the foundation of the structure is modelled as a separate entity to the structure. Using the variational form, it will be shown how the work of [1] can be extended to do modal analysis on this model. An error estimate for the modal solution will also be derived.

References

1. D. Civin, N.F.J. van Rensburg and A.J. van der Merwe, Using energy methods to compare linear vibration models, *Applied Mathematics and Computation*, 321, pp. 602-613, 2018.

A numerical solver for the minimal surface problem

Gerald Marewo
North-West University

In this study, we consider a boundary value problem for the minimal surface equation. A real life application of this problem is the analysis and design of semiconductor devices. The nonlinearity of the minimal surface equation motivates using a numerical method of solution. To this end, we couple the Newton method with a Galerkin finite element method. The overall method is implemented on the computer using `deal.ii`. Numerical experiments are used to investigate the efficiency of the method.

References

1. S. C. Brenner, R. Scott, The mathematical theory of the finite element method, Springer, 2008.

2. L. Evans, Partial differential equations, AMS, 2010.

3. D. Arndt, W. Bangerth, B. Blais, M. Fehling, R. Gassmöller, T. Heister, L. Heltai, U. Köcher, M. Kronbichler, M. Maier, P. Munch, J. Pelteret, S. Proell, K. Simon, B. Turcksin, D. Wells, J. Zhang, The deal.II library, Version 9.3 Journal of Numerical Mathematics, vol. 29, no. 3, pages 171-186, 2021, DOI: 10.1515/jnma-2021-0081.

A stable and high-order continuous Galerkin method for linear hyperbolic equations

Thasheel Singh Jagutpal[1], Arnaud Malan[1], Jan Nordström[2]

[1]University of Cape Town

[2]Linköping University

We present a continuous Galerkin finite element method (CGFEM) with weak imposition of boundary data via Simultaneous Approximation Terms (SAT) for linear hyperbolic partial differential equations. The discrete derivative operator is shown to have a diagonal norm and to fulfill Summation-By-Parts (SBP) characteristics. The SBP-SAT semi-discrete formulation allows for the formal proof of energy stability. The Cartesian framework of the finite elements is extendable to an arbitrary number of orthogonal spatial dimensions d, and orders of polynomial p. We provide numerical results using $1 \leq p \leq 8$ for the advection of a Gaussian pulse. We consider smooth fields on structured and orthogonal meshes only.

Beyond blow-up: A numerical and asymptotic study

André Weideman

Stellenbosch University

The topic of this talk is a nonlinear PDE, namely the heat equation with a quadratic forcing term. It exhibits a point-blow-up in finite time. In a recent paper in Physica D (with co-authors Marco Fasondini and John King) it was shown that the solution can be continued numerically through the singularity. In this talk numerical strategies for doing this are discussed. Previous perturbation methods for describing the blow-up profile are extended to predict also the post-blow-up behaviour.

On the validation of a fractional order model for pharmacokinetics using clinical data

Sinenhlanhla Mtshali, Byron Jacobs

University of Johannesburg

This study aims to validate the hypothesis that the pharmacokinetics of certain drug regimes are better captured by fractional order differential equations rather than ordinary differential equations. To support this research, two numerical methods the Grunwald-Letnikov and the L1 approximation, were implemented for the two-compartment model with Michaelis-Menten clearance kinetics for oral and intravenous administration of the drug. The efficacy of the numerical methods is verified through the use of the method of manufactured solutions due to the absence of an analytic solution to the proposed model. The model is derived from a phenomenological process leading to a dimensionally consistent and physically meaningful model. Using clinical data, the model is validated and shown that the optimal model parameters select a fractional order for the clearance dynamic for certain drug regimes. These findings support the hypothesis that fractional differential equations better describe some pharmacokinetics.

Spectral Domain Decomposition

Emma Nel, Nick Hale
Stellenbosch University

In this project we investigate techniques for solving second-order elliptic ordinary and partial differential equations with variable coefficients on one- and two-dimensional domains. We explore spectral collocation methods, ultraspherical element methods and domain decomposition strategies for this task, and discuss a Hierarchical Poincaré–Steklov (HPS) based approach, similar to that introduced by Gillman and Martinsson [1]. The HPS method is a recursive domain decomposition. It merges solution and Dirichlet-to-Neumann operators between subdomains, enforcing continuity of the solution and its derivative across domain boundaries. The result is a spectrally accurate discretization with an explicit fast direct solve that can be applied to problems with smooth solutions. A major advantage of the HPS strategy is that after the solution and Dirichlet-to-Neumann operators have been constructed, applying the boundary conditions and solving the problem is computationally inexpensive, meaning the scheme is ideal for solving the same problem with different boundary data. An additional benefit of the approach is that it can be parallelised across multiple processors to minimize computation time.

References

1. A. Gillman and P.G. Martinsson, A direct solver with *O(N)* complexity for variable coefficient elliptic PDEs discretized via a high-order composite spectral collocation method, *SIAM Journal on Scientific Computing* **36** (2014), A2023-2046.

Linear hotspot detection for a point pattern in the vicinity of a linear network

Inger Fabris-Rotelli[1], J. Modiba[1], A. Stein[2], G. Breetzke[1]
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The analysis of point patterns on linear networks is receiving current attention in spatial statistics. This refers to the analysis of points in a spatial domain that coincide with a linear network like a road network. The linear network is modelled as a set of lines that are connected at their ends or are intersecting, that is, modelled as mathematical graphs. Limited research so far has been conducted on spatial points that fall on the Euclidean space containing the linear network. This study addresses new steps by exploring points in the vicinity of the network that do not necessarily fall on the linear network. We present a novel method that is motivated by crime locations amongst a road network. The aim is to detect spatial hotspots around a linear network, where crime locations are considered as a point pattern lying in the vicinity of the linear road network. A new connectivity measure is also introduced to define the line segment neighbours of a line segment. The methodology is applied to crime data in Khayelitsha, South Africa. We detect a pattern of crime locations within the network that can be well interpreted. We conclude that our method is well applicable and could potentially help governmental organisations to allocate measures to reduce criminality.

Multiscale decomposition of spatio-temporal lattice data for hotspot trend predictions

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The identification of hotspots in spatial data has become an important part of spatial analysis. In some applications such as crime analysis and disease mapping the prediction of location of a hotspot becomes important for local governments to implement preventative measures. In this work, we propose the use of the Discrete Pulse Transform (DPT) on spatial lattice data along with the multiscale Ht-index and the Spatial Scan Statistic as a measure of saliency on the extracted pulses for feature detection at each timestep. The overall trend at each location allows for classification into emerging hotspot classes from which prediction can be made.

Window selection in spatial point pattern analysis

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The analysis of spatial point pattern data is typically done to expand the basic understanding of the first and second order properties of the point process that generated the data. First and second order properties of spatial point patterns are estimated using density and distance based measures. These measures rely implicitly or explicitly on the specification of the window domain. Thus, the correct specification of a window domain and the use of an appropriate distance metric to quantify proximity on the chosen window has an important role in the analysis of spatial point pattern data. Herein, we consider the influence of window choice on the analysis of first and second order properties of spatial point patterns.

Measuring homogeneity of spatial line patterns

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A spatial line pattern is a collections of lines in geographic space, and can be used to model physical phenomena such as road networks. Homogeneous spatial line patterns exhibit consistent characteristics such as length and density across geographic space, while the characteristics of inhomogeneous line patterns vary. Although there exists a variety of tests for the homogeneity of point patterns, no equivalent statistical tests currently exist to quantify the homogeneity of line patterns. This research shows that existing tests for the homogeneity of point patterns can be extended to define tests for homogeneity of line patterns. Line patterns are represented by point patterns, with points as the midpoints of each line. We investigate the power of the proposed tests and show that the tests are successfully able to identify homogeneity and heterogeneity of line patterns.

On the zeros of Meixner polynomials

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In his 1934 paper, Josef Meixner classifies five classes of orthogonal polynomials, p_n, with generating function $f(t)e^{xu(t)} = \sum_{n=0}^{\infty} c_n p_n(x) t^n$, which include the so-called Meixner polynomials. The Meixner polynomials lie on the ${}_2F_1$ plane of the Askey scheme of hypergeometric orthogonal polynomials and are orthogonal on the positive real line, with respect to a discrete weight function. We discuss results on the zeros of these polynomials that were published in three different papers during the past 10 years, which include results on the location of the first few zeros of the quasi-orthogonal order 1 and quasi-orthogonal order 2 Meixner polynomials.

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