SANUM 2024

The 45th Annual South African Symposium on Numerical & Applied Mathematics (SANUM)

03 to 05 April 2024

Department of Mathematical Sciences
Stellenbosch University, South Africa

Stellenbosch
UNIVERSITY
IYUNIVESITHI
UNIVERSITEIT

forward together
sonke siya phambili
saam vorentoe

Local organising committee:
Nick Hale, Andie de Villiers, Prince Nchupang, & André Weideman

The organising committee acknowledges generous sponsorship from the DSI-NRF Centre of Excellence - Mathematical and Statistical Sciences

MaSS
DSI-NRF CENTRE OF EXCELLENCE IN MATHEMATICAL & STATISTICAL SCIENCES

Programme:

		Wednesday 03 April		Thursday 04 April		Friday 05 April	
Start	**To**	**Room 2002**	**Room 3001**	**Room 2002**	**Room 3001**	**Room 2002**	**Room 3001**
9:00	10:00	Julia workshop		Plenary: Lubuma		Plenary: Harley	
10:00	10:30			Orakwelu		Nordstrom	
10:30	11:00			Tea and coffee in room 1006		Tea and coffee in room 1006	
11:00	11:30			Plenary: Cuyt		Nchupang	de Villiers
11:30	12:00					Olivier	Stadler
12:00	12:30			Weideman		Tshivhi	Goedhals
12:30	14:00			Lunch		Close	
14:00	14:30	Welcome: Moyo		Plenary: Olver			
14:30	15:00	Plenary: Bindel					
15:00	15:30			Nel		**Key**	**Mins**
15:30	16:00	Tea and coffee in room 1006		Tea and coffee in room 1006		Plenary talk	60
16:00	16:30	Jafari	Van Rensburg	Barashenkov	Mudzimbabwe	Contributed talk	30
16:30	17:00	Nkomo	Hohls	Dika	Adetona	Tea/coffee	30
17:00	17:30	Hale	Fareo	Alexeeva	Nchabeleng	Lunch time	90
						Programme change	
17:30		Conference photograph @ 17:30					
17:10		Reception / posters: 17:35 @ Maths					
19:00				Conference dinner: 18:30 @ Middelvlei			

For changes to the timetable, see https://sanum.github.io/programme.html

Presentation details:

- Please load your presentations on the available machines **before** the session in which you are speaking.
- If you are using your own laptop for the presentation, please check the connections in **advance**.
- A laser pointer and a clicker will be available in each room.
- Contributed and special session talks will be kept **strictly** to 30 (25+5) minutes.
- The first speaker in each contributed session will chair, with the second speaker acting as chair for the first talk.
- Chairs for the special sessions will be arranged by the sessions organiser(s).

Welcome to SANUM! The SANUM meeting came into existence in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York), happened to visit South Africa at the same time. A number of local researchers used the opportunity to organise a meeting at the (then) University of Natal, at which a total of ten papers were presented. From here the meeting grew in strength, with numerous famous numerical and applied mathematicians passing through. With a only few exceptions, SANUM has taken place every year since, and the 2024 marks the 45th anniversary of the meeting.

Conference Venue: The 2024 conference is held on the first two floors of the Mathematical Sciences building (building number 67 on the campus map – next page). On-site registration and namebadge collection takes place on 03 April from 12:30 to 13:50 in the foyer on the ground floor.

Wifi details: eduroam is available throughout the venue.

Tea & Coffee: Tea & Coffee will be available at the times indicated in the schedulec in room 1006.

Lunch Venue: In your registration pack, you will find vouchers for use in the Neelsie Student centre, located on the Rooiplein approximately 100m from the conference venue. Each voucher is loaded with R100 and may be used at any of the following stalls: Break Bites; Buzz Café; DCM; Jack's Bagel; Jeffs Place; Mariam's Kitchen; My Brew Kiosk; My Brew Coffee Roastery; Nca'kos; Romans Pizza; Sausage Saloon; Sweetbeet; TaMaties; Toast; Vida-E.

Conference Reception: A Canapé welcome reception will be held at 5:30PM in the foyer of the Mathematical Sciences building.

Conference Dinner: The conference dinner will take place at Middelvlei wine farm, (approximately a 10 minute drive from the conference venue) at 18:30 on Thursday. Transportation will be organised during the conference.

Luggage: On the final day (only) luggage may be stored in room 1006A. (Not room 1006!)

Safety: The green dashed line on the map is the "Green Route" which is actively patrolled by campus security in the late evenings and at night. However, although Stellenbosch is a quiet and relatively safe town, it is not entirely free of crime. We therefore advise delegates to take normal safety precautions at night time, such as traveling in groups, avoiding dark alleys, and keeping valuables out of sight.

Emergency contacts: Campus Security, +27 (0)21 808 2333, Emergency services, 112, Police, 10111.

45 SANUM meetings:

1. Durban, 10–11 April 1975
2. Durban, 8–9 April 1976
3. Durban, 6–7 April 1977
4. Durban, 18–20 July 1978
5. Durban, 18–20 July 1979
6. Durban, 21–23 July 1980
7. Durban, 20–22 July 1981
8. Durban, 19–21 July 1982
9. Durban, 18–20 July 1983
10. Ballito, 2–4 July 1984
11. Umhlanga Rocks, 8–10 July 1985
12. Umhlanga Rocks, 14–16 July 1986
13. Umhlanga Rocks, 13–15 July 1987
14. Umhlanga Rocks, 11–13 July 1988
15. Umhlanga Rocks, 17–19 July 1989
16. San Lameer, 9–11 July 1990
17. Umhlanga Rocks, 15–17 July 1991
18. Durban, 13–15 July 1992
19. San Lameer, 12–14 July 1993
20. Umhlanga Rocks, 4–6 July 1994
21. Scottburgh, 10–12 July 1995
22. Cape Town, 15–17 April 1998
23. Stellenbosch, 29–31 March 1999
24. Stellenbosch, 3–5 April 2000
25. Stellenbosch, 9–11 April 2001
26. Stellenbosch, 3–5 April 2002
27. Stellenbosch, 31 March–2 April 2003
28. Stellenbosch, 5–7 April 2004
29. Stellenbosch, 30 March–1 April 2005
30. Stellenbosch, 3–5 April 2006
31. Stellenbosch, 2–4 April 2007
32. Stellenbosch, 2–4 April 2008
33. Stellenbosch, 6–8 April 2009
34. Stellenbosch, 15–17 April 2010
35. Stellenbosch, 23–25 April 2011
36. WITS, 2–4 April 2012
37. Stellenbosch, 3–5 April 2013
38. WITS, 14–16 April 2014
39. Pretoria, 30 March–1 April 2015
40. Stellenbosch, 22–24 March 2016
41. WITS, 28–30 March 2017
42. Stellenbosch, 04–06 April 2018
43. Pretoria, 27–29 March 2019
44. University of Johannesburg, 3–5 April 2023
45. **Stellenbosch, 2–5 April 2024**

STELLENBOSCH CAMPUS MAP INDEX

SUPPORT SERVICES
1 Administration, Block A
2 Administration, Block B
3 Administration, Block C
4 JS Gericke Library
5 Neelsie Student Centre
6 Campus Security
7 Centre for Student Recruitment
8 Centre for Student Counselling and Development: Reception (CSCD)
9 CSCD: Office for Students with Special Learning Needs (Disabilities); Den Bosch
10 CSCD: Unit for Psychotherapeutic and Support Services
11 Centre for Teaching and Learning
12-14 Division for Student Affairs
15 Language Centre: Reading Lab and Language Enrichment Courses
16 Campus Health Services
17 Maties Sport
18 Coetzenburg Centre
19 Old Conservatoire
20 SU Art Galery
21 University Museum
22 Centre for Learning Technology
23 Language Centre (Writing Lab)
24 WAT, 115 Banghoek Road
25 Matie Community Services, Lückhoff School
26 SU Vehicle Fleet
27 Legal Aid Clinic
28 IT Hub (Help Centre)
30 Purchasing and Provision Services
31 Facilities Management
32 Information Technology

SPORTS FACILITIES
33 Danie Craven Stadium
34 Coetzenburg Athletics Stadium
35 Coetzenburg Tennis Courts
36 PSO Club House and Hockey Fields
37 Netball Courts
38 Swimming Pool
39 SU Gymnasium
40 Tennis Courts (Residences)
41 Old Mutual Sports Centre (squash courts)
42 Tennis Courts
43 Heidehof Rugby Fields

ACADEMIC BUILDINGS
44 Food Science
45 PO Sauer
46 Electrical/Electronic Engineering
47 Civil Engineering
48 Mechanical/Mechatronic/Industrial Engineering
49 Process Engineering
50 Knowledge Centre
51 Engineering, General
52 Africa Centre for HIV and AIDS Management
53 JC Smuts – Biological Sciences
54 De Beers – Chemistry
55 Mike de Vries
56 Chemistry – first-years
57 Inorganic Chemistry
58 CGW Schumann
59 Van der Sterr
60 Accounting and Statistics
61 JS Marais
62 Polymer Science
63 Visual Arts
64 JH Neethling
65 AI Perold
66 Merensky
67 Mathematical Sciences and Industrial Psychology
68 Nursery
69 Natural Sciences
70 Chamber of Mines
71 RW Wilcocks
72 Old Main Building
73 HB Thom Theatre
74 CL Marais Library
75 Journalism
76 GG Cillié
77 Arts and Social Sciences
78 Lombardi
79 Konservatorium (University Choir)
80 Theological Seminary
81 Paul van der Bijl Laboratories
82 Agronomy
83 Welgevallen Experimental Farm
84 Sport Science

WOMEN'S RESIDENCES
85 Monica
86 Harmonie
87 Heemstede
88 Huis ten Bosch
89 Lydia
90 Minerva
91 Nerina
92 Erica
93 Nemesia
94 Serruria
95 Tinie Louw Dining Hall; ▲ amaMaties hub
96 Sonop (Huis van Niekerk)
97 Irene

MEN'S RESIDENCES
98 Eendrag
99 Helshoogte
100 Simonsberg
101 Huis Visser
102 Huis Marais
103 Dagbreek
104 Majuba
105 Wilgenhof
106 Helderberg

MEN'S AND WOMEN'S RESIDENCES
107 Huis Neethling
108 Metanoia
109 Russel Botman House; ▲ Wimbledon hub
110 Academia
111 Concordia
112 Goldfields
113 Huis McDonald

UNIVERSITY FLATS AND HOUSES
114 Lobelia
115 Crozierhof
116 Huis de Villiers
117 Waldenhof

LISTEN, LIVE AND LEARN
118 LLL village
■ LLL houses

CLUSTER HUBS
▲ amaMaties 95
▲ Wimbledon 108

FACULTIES (IN COLOUR) AND DEPARTMENTS

Accounting 59
African Languages 77
Afrikaans en Nederlands 77
Agricultural Economics 61

AGRISCIENCES 44, 45, 53, 55, 61, 64, 78, 81–83

Agronomy 82
Ancient Studies 77
Animal Sciences 55
Applied Mathematics 51

ARTS AND SOCIAL SCIENCES 63, 70, 71, 73, 75, 77, 79

Biochemistry 53
Business Management 58
Botany and Zoology 69
Chemistry and Polymer Science 54–57, 62
Computer Science (Math. Sc.) 51
Conservation Ecology and Entomology 61
Drama 73
Earth Sciences 70

ECONOMIC AND MANAGEMENT SCIENCES 52, 58–60, 67

EDUCATION 76, 84

ENGINEERING 47–51

English 77
Food Science 44
Forest and Wood Sciences 45
General Linguistics 77
Genetics 53
Geography and Environmental Studies 70
History 71
Horticulture 78
Industrial Psychology 67
Information Science 77
Institute for Wine Biotechnology 64
Institute for Plantbiotechnology 69
Journalism 75

LAW 72

Mathematics (Math. Sc.) 67
Microbiology 53
Modern Foreign Languages 77
Music 79
Philosophy 77
Physics 66
Physiological Sciences 55
Plant Pathology 78
Political Science 77
Psychology 71
Public Development Management 59

SCIENCE 51, 53–57, 59, 62, 65, 66, 67, 70

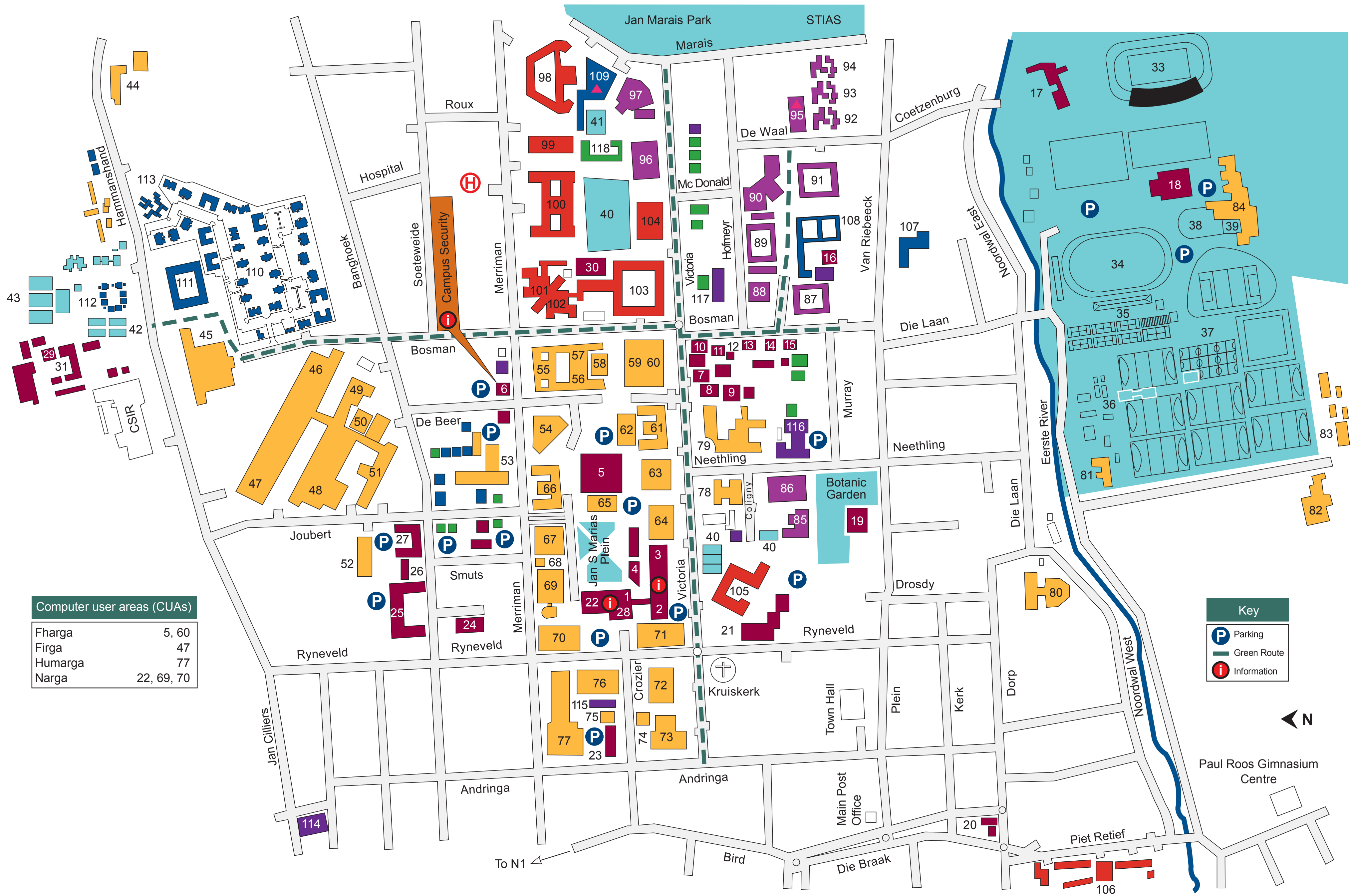
Social Work 77
Sociology and Social Anthropology 77
Soil Science 78
Sport Science 84
Statistics and Actuarial Science 60

THEOLOGY 80

Visual Arts 63
Viticulture and Oenology 64

Ⓟ Parking

STELLENBOSCH CAMPUS MAP



Computer user areas (CUAs)	
Fharga	5, 60
Firga	47
Humarga	77
Narga	22, 69, 70

Plenary abstracts

Optimizing magnetic confinement devices for fusion plasmas

David Bindel (Cornell University)

Stellarators are non-axisymmetric magnetic field configurations for confinement of fusion plasmas. In contrast to the more popular axisymmetric tokamak geometries, stellarators rely on symmetry breaking to confine particles. The Simons Collaboration on Hidden Symmetries and Fusion Energy is a team dedicated to advancing the mathematical and computational state of the art in stellarator design, and producing modern stellarator optimization codes and new underlying theory. In this talk, I describe some of the challenges of stellarator optimization, and give examples both of success stories and of challenges that remain.

On the cross-fertilisation of sparse interpolation and exponential analysis

Annie Cuyt (University of Antwerp) and Wen-shin Lee (University of Stirling)

The concepts of polynomial and trigonometric interpolation are well-known. The concepts sparse interpolation (used in computer algebra) and exponential analysis (from digital signal processing) are generalizations thereof, where respectively the powers or the frequencies in the interpolant are not predefined to be $0, 1, 2, \ldots$. Half of the data is then used to determine the appropriate powers or frequencies.

We discuss how sparse interpolation and exponential analysis can cross-fertilize and lead to new results. We thereby focus on the overarching inverse problem of identifying, from given values $f_k \in \mathbb{C}$, the nonlinear parameters $\phi_1, \ldots, \phi_n \in \mathbb{C}$, the linear coefficients $\alpha_1, \ldots, \alpha_n \in \mathbb{C}$ and the sparsity $n \in \mathbb{N}$ in the interpolation

$$\sum_{j=1}^{n} \alpha_j \exp(\phi_j k\Delta) = f_k, \qquad k = 0, \ldots, 2n-1, \ldots \quad \Delta \in \mathbb{R}^+. \tag{1}$$

In the literature, several cases of (1) are considered to be hard:

- When some of the ϕ_j cluster, the identification and separation of these clustered ϕ_j becomes numerically ill-conditioned. We show how the problem may be reconditioned.
- Retrieval of the correct value of n is difficult, and more so in case of clustered ϕ_j and noisy samples f_k. Here, decimation of the data offers a way to obtain a reliable estimate of n automatically.
- Such decimation allows to divide and conquer the inverse problem statement. The smaller subproblems are largely independent and can be solved in parallel, leading to an improved complexity and efficiency.
- At the same time, the sub-Nyquist[†] Prony method proves to be robust with respect to outliers in the data. Making use of some approximation theory results [6,7,8], we can also validate the computation of the ϕ_j and α_j.
- Avoiding the Nyquist constraint [9] offers so-called superresolution, or the possibility to unearth higher frequency components in the samples.

All of the above can be generalized in several ways, to the use of more functions besides the exponential on the one hand [4], and to the solution of multdimensional inverse problems on the other [2].

[†] The Nyquist constraint [9] is the digital signal processing equivalent of stating that the argument of a complex exponential $\exp(\phi\Delta)$ with $\phi \in \mathbb{C}$ and $\Delta \in \mathbb{R}^+$ can only be retrieved uniquely under the condition that $|\Im(\phi)|\Delta < \pi$. It governs signal processing since the beginning of the 20-th century. In the past two decades this constraint was first broken with the use of randomly collected signal samples [5,1] and later for use with uniform samples [3].

[1] Emmanuel J Candès, Justin Romberg, and Terence Tao. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on information theory*, 52(2):489–509, 2006.

[2] Annie Cuyt and Wen-shin Lee. Multivariate exponential analysis from the minimal number of samples. *Advances in Computational Mathematics*, 44:987–1002, 2018.

[3] Annie Cuyt and Wen-shin Lee. How to get high resolution results from sparse and coarsely sampled data. *Applied and Computational Harmonic Analysis*, 48(3):1066–1087, 2020.

[4] Annie Cuyt and Wen-shin Lee. Multiscale matrix pencils for separable reconstruction problems. *Numerical Algorithms*, 95(1):31–72, 2024.

[5] David L Donoho. Compressed sensing. *IEEE Transactions on information theory*, 52(4):1289–1306, 2006.

[6] Jacek Gilewicz and Maciej Pindor. Padé approximants and noise: a case of geometric series. *Journal of Computational and Applied Mathematics*, 87(2):199–214, 1997.

[7] Jacek Gilewicz and Maciej Pindor. Padé approximants and noise: rational functions. *Journal of computational and applied mathematics*, 105(1-2):285–297, 1999.

[8] Ferre Knaepkens and Annie Cuyt. On the robustness of exponential base terms and the padé denominator in some least squares sense. *Numerical Algorithms*, 92(1):747–766, 2023.

[9] Harry Nyquist. Certain topics in telegraph transmission theory. *Transactions of the American Institute of Electrical Engineers*, 47(2):617–644, 1928.

Stabilizing the nonlinear initial boundary value problem governing thin film flow

Charis Harley & E. Momoniat (University of Johannesburg), J. Nordström (Linköping University)

We solve the nonlinear initial boundary value problem that arises in the study of thin film flows. The generally accepted version is shown not to lead to an energy bound and stability. To deal with that, we firstly consider two modifications of the original equation. Secondly, we develop new splitting techniques for the discretisation of the nonlinear terms. Thirdly, we provide boundary conditions and an implementation procedure that finally leads to provably conservative and stable nonlinear schemes.

Modelling the transmission dynamics and control of the Ebola virus disease under war and travel challenges

Jean M-S Lubuma (University of Witwatersrand)

The 2018-2020 Ebola Virus Disease (EVD) was the first outbreak that occurred in a tumultuous, active conflict and war eastern region of the Democratic Republic of Congo (DRC) characterized by the violence, destruction of Ebola Treatment Centres and escape of patients from hospitals. Likewise, the 2014-2016 West Africa EVD came with an unprecedented challenge in that the outbreak simultaneously arose in three different countries (viz. Guinea, Liberia, and Sierra Leone) to and from which migrations and travels of people by road and air were considerable.

For the 2018-2020 outbreak, we develop a Susceptible-Infective-Recovered (SIR)-type model in which the associated disruptive events and the indirect/slow transmission through the contaminated environment are incorporated. Due to the challenge mentioned above for the 2014-2016 EVD outbreak, we construct for it a metapopulation model in each patch of which, we consider an extended Susceptible-Exposed-Infective-Recovered (SEIR) model modified by adding the disease induced deceased, the Isolated and the Quarantine compartments to account, among others, for travellers who undergo the exit screening intervention at the borders, as recommended by the World Health Organization.

For the two models, our focus is three-fold. We compute the basic reproduction numbers and explicit thresholds, thanks to which the existence and the local/global asymptotic stability (LAS/GAS) of the disease-free, endemic and boundary equilibria (DFE, EE & BE) are investigated. Next, we design nonstandard finite difference (NSFD) schemes that replicate these qualitative properties of the continuous models. Finally, we conduct a global sensitivity analysis and use the real data from the affected regions to provide numerical simulations and undertake a statistical data analytics study, which supports the theory. It is shown that the key parameters, which measure the impact of travels and war, significantly influence the increase in the numbers of infected and deaths individuals of the 2014-2016 and 2018-2020 EVD outbreaks, respectively.

Quasi-optimal hp-FEM

Sheehan Olver (Imperial College London)

We consider the efficient solution of PDEs using hp-Finite Element Methods, which are finite element methods where both the grid size (h) and the polynomial order (p) may very. A classical quasi-optimal solver for h-FEM (where the polynomial degree is fixed at 1) is The Fast Poisson Solver which achieves $O(N^2 \log N)$ operations with N degrees of freedom in each dimension. Very recently a spectral method, equivalent to p-FEM with a single element, were introduced by Fortunato and Townsend that achieves quasi-optimal complexity as the polynomial order increases using an Alternating Direction Implicit (ADI) method. In this talk, we extend their results for general hp-FEM, achieving complexity that is optimal independent of the choice of h and p, by using a sparsity respecting solver for piecewise integrated Legendre polynomials as introduced by Babuska combined with ADI. In addition to Poisson and screened Poisson equations on rectangles, we consider s olving PDEs on disks and cylinders which are divided into annular elements using non-classical multivariate orthogonal polynomials on annuli.

Contributed abstracts

B-spline solution of 2D partial differential equations using collocation on finite elements

Rasheed Adetona, Nabendra Parumasur, & Pravin Singh (University of KwaZulu-Natal)

This work implements orthogonal collocation on finite elements (OCFE) with a modified quadratic basis functions on bivariate and trivariate two-dimensional partial differential equations. The advantage of using the new basis is that the continuity conditions are automatically built into the basis functions applied in two spatial variables. Various examples are provided and the results compared favourably with previous ones in the literature.

Oscillons: numerical continuation and variational analysis

Nora Alexeeva & Igor Barashenkov (UCT) and Alla Bogolubskaya & Elena Zemlyanaya (JINR)

Oscillons are long-lived localised pulsating states in the nonlinear Klein-Gordon equations. We approximate the oscillons by periodic standing waves which are determined numerically as solutions of the boundary-value problem posed on a two-dimensional domain. The resulting energy-frequency diagram features a series of resonances where the resonant standing waves emerge by the period-doubling of solutions continuing from linearised eigenfunctions. To gain theoretical insights into the oscillon dynamics, we formulate a multiscale variational approach. The new approach is free from the blowup singularities that marred all previously proposed variational methods.

PT-symmetric necklaces: Hamiltonian structure and Liouville integrability

Igor Barashenkov and Frank Smuts (University of Cape Town)

We consider discrete $\mathcal{PT}$-symmetric nonlinear Schroedinger equations defined on finite periodic lattices. These mathematical constructs describe necklaces of coupled optical waveguides or tightly bound rings of quantum dots, with the gain and loss of energy. Despite representing open systems, some $\mathcal{PT}$-symmetric dimers (necklaces consisting just of two pearls) proved to exhibit the Hamiltonian structure and two integrals of motion in involution. In this talk, we show that the Hamiltonian formulation and Liouville integrability is not a special privilege of dimers. We present a $\mathcal{PT}$-symmetric necklace of N pearls with N Poisson-commuting integrals of motion:

$$i\dot{\psi}_n + \left(e^{i\theta_n}\psi_{n+1} + e^{-i\theta_{n-1}}\psi_{n-1}\right)(1 + |\psi_n|^2) = 0,$$

where $\psi_{n+N} = \psi_n$, and $\theta_1, \theta_2, ..., \theta_N$ is a set of arbitrary real parameters ($\theta_{n+N} = \theta_n$). Our construction makes use of the Lax representation and classical r-matrix.

Continuum-kinematics-inspired peridynamics: Transverse isotropy

Andie de Villiers (Stellenbosch University), Georges Limbert (University of Southampton), Ali Javili (Bilkent University), Andrew McBride (University of Glasgow), Paul Steinmann (University of Erlangen-Nuremberg)

Peridynamics is a non-local continuum theory. Continuum-kinematics-inspired peridynamics (CPD) is geometrically exact and utilises non-local kinematic measures that are analogous to those from classical continuum mechanics. In this presentation, the CPD framework is extended to transverse isotropy.

Kink-Breather interaction in the ϕ^4 equation

Igor Barashenkov and Alain Dika (University of Cape Town)

In the present study, we used the variational Ansatz for the breather-wobbler configuration of the ϕ^4 equation. This reduces the partial differential equation to a system of ordinary equations for the amplitudes of the breather and wobbler and the distance between the two objects. The system of three second-order equations is amenable to analytical consideration. The variational descriptions of the kink-breather interaction include the wobbling degree of freedom and, as a result, has captured the kink-breather bound state.

Hydraulic fracture with fluid leak-off: Mathematical models and solutions

Adewunmi Fareo (University of the Witwatersrand) and Mathibele Nchabeleng (University of Pretoria)

Hydraulic fracturing is a well stimulation technique used in order to improve the productivity of crude oil and natural gas from underground reservoirs. The technique involves injecting fluid at ultra-high pressure into the reservoir rock in order to propagate pre-existing fracture networks or create new fractures. A crucial goal in hydraulic fracturing is the use of the high pressure fluid to continually create and propagate new fractures in the rock formation, in order to continually liberate the crude oil and natural gas that may have been trapped. However, because of the permeable nature of the rock formations, fluid leak-off into the formations do occur, the rate of which increases as more and more fractures are created. The effect of fluid leak-off on fracture extension or propagation is significant and several research in this area has been done. In this talk, a mathematical model for the propagation of a pre-existing hydraulic fracture with fluid leak-off in a permeable rock will be presented. The empirical Darcy law is used to describe the fluid leak-off through the permeable fracture interface into the rock formation. The fluid flow in the fracture is laminar and the fracture is driven by a viscous incompressible Newtonian fluid. When lubrication theory is applied to the fracturing fluid flow in the hydraulic fracture, a system of integro-differential equations for the fracture half-width and the leak-off depth is obtained. Similarity and numerical solutions obtained for the integro-differential system are presented.

Numerical Modelling of Blood Flow During Syringing

Jaimé Goedhals (Stellenbosch University & DSI-NRF CoE-MaSS),
Andie de Villiers & Francois Smit (Stellenbosch University)

Hemolysis, defined as the breaking open of red blood cells, can occur due to a range of factors which can be physical, chemical or biological in nature. Physical damage can manifest in scenarios such as emergency rapid blood transfusions conducted through syringing, a practice which may be employed in resuscitation procedures, particularly in rural hospitals. It has been demonstrated that such syringe-based transfusions result in noteworthy hemolysis, significantly surpassing the impact of pressure bag usage. This heightened hemolysis carries potential negative implications for the recipient of the transfusion.

The core objective of this project revolves around emulating the blood flow through a sudden contraction, simulating a syringe, to emulate and quantify the underlying mechanisms driving hemolysis. Employing numerical simulations alongside a variety of hemolysis models, the goal is to identify the specific flow conditions that are most conducive to hemolysis occurrence.

The modelling process commences with the depiction of blood flow as a Newtonian fluid traversing a sudden contraction, representing the syringe's action. This initial setup is subsequently expanded to account for non-Newtonian flow behaviours. To validate the modelling approach, a two-dimensional scenario is first simulated using the finite element software deal.II. This is then repeated and compared to results from numerical simulations using ANSYS Fluent. The scope extends further to encompass a three-dimensional setup, incorporating a representative syringe and hypodermic needle. The time history of shear stress along streamlines is then extracted to facilitate hemolysis analysis.

Approximate solutions to a nonlinear functional differential equation

Nick Hale & JAC Weideman (Stellenbosch University) and Enrique Thomann (Oregon State University)

Through combination of some analytical and numerical approaches, we investigate various solutions of the α-Ricatti equation

$$u'(t) + u(t) = u^2(\alpha t), \quad \alpha > 1;$$

a modification of the well-known logistic growth equation.

Existence of and properties of solutions of a semi-linear rod model

Kirstin Hohls and Nic van Rensburg (University of Pretoria)

The nonlinear system of partial differential equations considered is

$$\partial_t^2 w = \partial_x \left(\partial_x w - \phi\right) + \left(\frac{d}{\gamma} + \frac{1}{2\gamma}\int_0^1 (\partial_x w)^2\right)\partial_x^2 w,$$
$$\frac{1}{\alpha}\partial_t^2 \phi = \partial_x w - \phi + \partial_x\left(\frac{1}{\beta}\partial_x \phi\right).$$

This mechanical system, which describes the vibration of a semi-linear rod with an axial force, was suggested by [1] and has been investigated by other authors. Notable among these investigations are the articles [2] and [3].

We will investigate the properties of solutions, such as modal solutions, non-trivial equilibria and dynamic buckling. As a part of the investigation, we will use the spectral theory of the related linear Timoshenko rod model to find a pseudo-modal solution. This results in a system of ODEs that is equivalent to the original problem. The rod model is a special case of a problem of the form

$$u'' = Au + f(u)$$

on some Hilbert space, where A is a linear operator and f a nonlinear mapping. During the presentation, we will also discuss the local and global existence theory of a solution of the aforementioned model.

[1] K. Ammari, Global Existence and Uniform Stabilization of a Nonlinear Timoshenko Beam, *Portugaliae Mathematica*, **59**:2 (2002), 125-140.

[2] A. Javili, et al. (2020). The Computational Framework for Continuum-Kinematics-Inspired Peridynamics. Computational Mechanics, 66(4), 795–824.

[3] J. Peradze and Z. Kalichava, A numerical algorithm for the nonlinear Timoshenko beam system, *Numerical Methods for Partial Differential Equations*, **36** (2020), 1318-1347.

A numerical scheme for solving a class of variable-order differential equations using operational matrices

S. Salati (University of Mazandaran) and H. Jafari (University of Mazandaran & UNISA)

In this work, we use the operational matrices (OMs) based on Hosoya polynomials (HPs) and collocation method (CM) to obtain numerical solution for a class of variable order differential equations (VO-DEs). The fractional derivatives and the VO-derivatives are in the Caputo sense. The operational matrices are computed based on the Hosoya polynomials (HPs) of simple paths. First, we assume the unknown function as a finite series by using the Hosoya polynomials as the basis functions. To obtain unknown coefficients of this approximation, we computed the operational matrices of all terms of the main equations. Then, by using the operational matrix and collocation points, the governing equations converted to a set of algebraic equations. Finally, an approximate solution is obtained by solving that algebraic equations.

Stochastic θ finite differences for a class of stochastic partial differential equations (SPDEs) arising in finance.

Walter Mudzimbabwe (University of the Witwatersrand)

In this talk, we consider a class of stochastic partial differential equations (SPDEs) that arise in finance. We seek to solve numerically the SPDEs as generalised stochastic differential equations (SDEs). We develop a numerical solution of the (SPDEs) by combining an implicit and explicit versions of the Milstein scheme for the SDE resulting in a stochastic finite difference scheme. We then consider the mean square stability conditions that the composite scheme must satisfy. Theoretic results of the stability conditions using Fourier analysis show that the composite method is stable under certain conditions that dependent on constant parameters of the SPDE. We also confirm that the stability conditions of are the expected extension of the corresponding partial differential equations (PDEs) for this particular class of SPDEs. Nu merical experiments show the stability and convergence properties of the developed scheme.

Operational Matrix for models derived using fractional operators.

Nolwazi Nkomo and H. Jafari (UNISA)

Recently, operational matrices were modified for solving several kinds of fractional differential equations (FDE). These differential equations have non-integer order derivatives. The non-integer operator presents additional complexity in fractional calculus and numerical methods are required when solving FDEs. In this study, we present a mathematical model derived from fractional operators in terms of Caputo. Bernoulli polynomials are used together with collocation method in the derivation of the operational matrix for FDEs. Lastly, we apply a technique based on the operational matrix to the fractional Burgess equation to confirms the effectiveness of the operational matrix.

An evaluation of the finite difference method for solving Cauchy singular integral equations of the first kind

Adewunmi Fareo (University of the Witwatersrand) and Mathibele Nchabeleng (University of Pretoria)

The numerical solution of Cauchy singular integral equations (CSIEs) have been studied since as far back as in the 1970's. The methods commonly employed in the solution of CSIEs are either based on the projection method or the quadrature method. Simply very little research has been done on the use of numerical techniques such as finite difference method. This work explores the viability of the finite difference method for solving Cauchy-type singular integral equations of the first kind over a finite interval. In the solution the finite difference method is used to obtain systems of linear algebraic equations and these systems are solved numerically. We found that when certain conditions are met, the numerical solution of the singular integral equation converges.

Provably stable and high-order accurate finite difference approximation for the incompressible Navier–Stokes equations: conflicting boundary conditions

Arnaud Malan (University of Cape Town), Prince Nchupang (Stellenbosch University),
and Jan Nordström (Linköping University and University of Johannesburg)

In recent years, there has been considerable interest in the numerical simulations of the incompressible flows due to their numerous industrial applications. The lid-driven cavity problem is often used to validate numerical schemes. The challenge however that arises when using this benchmark problem occurs at the boundaries where the moving and the stationary bounding walls meet, leading to singularities in the solution domain. We circumvent this by implementing boundary conditions weakly using the simultaneous approximation term (SAT) technique. We discretize the spatial derivatives using high-order finite difference methods on summation-by-parts (SBP) form. The resulting scheme is provably stable and high-order accurate.

An ultraspherical spectral element method for solving partial differential equations

Emma Nel and Nick Hale (Stellenbosch University)

We investigate the ultraspherical spectral element method for solving second-order partial differential equations in two dimensions. Moreover, a novel coordinate transformation is introduced to broaden the scope of the method, making it applicable to rectangular domains with circular holes (square donuts), as well as certain types of curved boundaries. The presented method is an integration of two approaches, namely the ultraspherical spectral method and the hierarchical Poincaré–Steklov (HPS) scheme. The ultraspherical method is a Petrov–Galerkin scheme that presents operators in the form of sparse and almost-banded matrices, enabling both stability and computational efficiency. The HPS method is a recursive domain decomposition strategy that enables fast direct solves. It merges solution operators and Dirichlet-to-Neumann operators between subdomains, enforcing continuity of the solution and its derivative across domain boundaries. The fusion of these two methods, combined with a bilinear mapping, results in an accurate discretisation with an explicit direct solve that can be applied to problems on arbitrary polygonal domains with smooth solutions. A major advantage is the reuse of precomputed solution operators facilitated by the HPS scheme, enhancing the efficiency of elliptic solves within implicit and semi-implicit time-steppers. Additionally, the approach is highly parallelisable, allowing for efficient computation time. An implementation of the method is established as a software system, ultraSEM, which employs the HPS method to solve on rectangular and polygonal domains. We extend this implementation to allow for solving on domains with circular cavities. This extension relies on a nonlinear coordinate mapping and proves to work effectively, achieving near machine level precision accuracy. On some simple test problems, we demonstrate geometric convergence for refinement of the polynomial degree and algebraic convergence for domain refinement. Furthermore, we show that execution times scale comparably to those achieved for a rectangular domain. We demonstrate the application of the method on various time-dependent and fluid dynamics examples, including contaminant transport and reaction-diffusion systems, and underscore the practical applicability of the methodology and the new domain.

Nonlinear energy stable schemes for multi-phase flow problems

Jan Nordström (Linköping University & University of Johannesburg) and Arnaud G. Malan (UCT)

We have recently shown that a specific skew-symmetric reformulation of the shallow water equations, the incompressible and compressible Euler and Navier-Stokes equations can be derived [1, 2, 3]. Following [4], we combined the skew-symmetric formulation with summation-by-parts (SBP) difference operators and weak boundary conditions using the simultaneous-approximation-term (SAT) technique and arrived at nonlinearly stable schemes. In this talk we show how to derive skew-symmetric formulations of the governing equations for multiphase flows which lead to a provable energy stable nonlinear scheme.

[1] J. Nordström, Nonlinear Boundary Conditions for Initial Boundary Value Problems with Applications in Computational Fluid Dynamics, Journal of Computational Physics, Vol. 498, 112685, 2024.

[2] J. Nordström, A skew-symmetric energy and entropy stable formulation of the compressible Euler equations, Journal of Computational Physics, Vol 470, 111573, 2022.

[3] J. Nordström, Nonlinear and Linearised Primal and Dual Initial Boundary Value Problems: When are they Bounded? How are they Connected?, Journal of Computational Physics, Vol 455, 111001, 2022.

[4] J. Nordström, A Roadmap to Well Posed and Stable Problems in Computational Physics, Journal of Scientific Computing, Volume 71, Issue 1, pp. 365-385, 2017.

A fluid simulation code for solitons in plasmas

Carel Olivier (North-West University)

A plasma fluid model consisting of two partial differential equations coupled with a Poisson-type boundary value problem are considered. A numerical scheme is constructed through a method-of-lines approach where spatial derivatives are approximated by five-point-stencil finite difference approximations, and the resulting system of ordinary differential equations are solved by means of a 4th order Runge-Kutta method. The boundary value problem is solved by means of a Newton's method. The coupling of the boundary value problem with the Runge-Kutta method is discussed in detail. In addition, some examples of soliton dynamics are considered.

Block hybrid backward differentiation formulae for differential equations

Hermane Mambili–Mamboundou and Maduabuchi Orakwelu (University of KwaZulu-Natal)

This research presents a new type of Single Step Hybrid Block Backwards Differentiation Formulae (SSBHBDF) designed to solve stiff differential equations directly. These methods use interpolation and collocation techniques at Bhaskara nodes to improve accuracy. The proposed schemes have been rigorously evaluated for zero stability, consistency, and convergence. This approach enables the generation of single-step A-stable and L_0- stable schemes. The effectiveness of these proposed methods is validated through tests on linear and nonlinear Differential Equations (ODEs), and their performance compared to other existing methodologies. The study shows that the type of intra-step points in the proposed methods significantly improves solution accuracy.

Continuum kinematics-inspired peridynamics in Julia: A computational perspective

Andie De Villiers & Johan Stadler (Stellenbosch University) and Ali Javili (Bilkent University)

Continuum kinematics-inspired peridynamics (CPD) is a peridynamic (PD) formulation that uses the same kinematic measures as classical continuum mechanics (CCM), providing a geometrically exact formulation [2]. PD is a non-local continuum formulation, wherein the behaviour of each material point is influenced by material points within a finite neighbourhood of that point [3]. By incorporating non-locality as a fundamental modelling concept, the range of interactions considered at each point is expanded, encompassing influences beyond its immediate neighbours. Simultaneously, the integral formulation simplifies the representation of spatial discontinuities by eliminating the need for explicit spatial gradient computations, making it highly suitable for modelling the intricate and heterogeneous nature of biological tissues. The implementation emphasises the complexities associated with incompressibility in material modelling.

This contribution focuses on the implementation of CPD in Julia [1]. Several key aspects render Julia an ideal choice for the implementation of CPD. Julia is renowned for its computational performance, underpinned by its just-in-time compilation that allows for the efficient execution of numerical simulations [1]. Julia's performance-oriented design is well-suited for implementing and executing complex simulations efficiently. Julia has built-in support for parallel computing which empowers researchers to harness the full potential of multi-core processors and distributed computing environments. This capability proves invaluable, especially for large-scale simulations, where parallelization can lead to substantial acceleration in computational tasks. Additionally, Julia seamlessly connects with external libraries, simplifying the integration of existing numerical tools. The implementation of CPD in Julia offers a robust and powerful tool for simulating the intricate behaviour of materials where non-local effects are important, as well as in the presence of discontinuities. This work not only contributes to advancing the capabilities of PD simulations but also demonstrates the synergy between innovative computational frameworks and high-performance programming languages like Julia in tackling complex engineering and scientific challenges.

[1] Bezanson, J., Edelman, A., Karpinski, S., & Shah, V.B. (2017). Julia: A Fresh Approach to Numerical Computing. SIAM Review, 59(1), 65-98. doi:10.1137/141000671.

[2] Javili, A., et al. (2020). The Computational Framework for Continuum-Kinematics-Inspired Peridynam-ics. Computational Mechanics, 66(4), 795–824.

[3] Silling, S.A. (2000). Reformulation of Elasticity Theory for Discontinuities and Long-Range Forces. Journal of Mechanics and Physics of Solids.

Enhancing heat transfer efficiency: A study on nanofluid dynamics in the presence of magnetic fields, heat source, and radiation

Samuel Tshehla Maashutha and Khodani Sherrif Tshivhi (Stellenbosch University)

This research investigates how the combination of heat source and radiation influences the boundary layer of magnetohydrodynamic stagnation point flow involving a copper-water nanofluid over an exponentially stretching slippery surface. The fundamental governing equations for mass, momentum, and energy are transformed into dimensionless ordinary differential equations using similarity transformations. These equations are then numerically solved as an initial value problem using the Runge-Kutta-Fehlberg technique.

The results are analyzed with a focus on parameters relevant to engineering and industrial applications, including velocity, temperature, skin friction, and Nusselt number. The analysis indicates that increasing heat source and radiation parameters result in higher temperature and Nusselt number in the copper-water nanofluid flow. Conversely, an increase in the magnetic parameter leads to a decrease in the temperature profile. To validate these findings, comparisons are made with reported results for a specific case. The outcomes are presented graphically and discussed within the context of engineering and industrial applications.

Spectrum of the axially loaded Timoshenko rod

Kirstin Hohls and Nic van Rensburg (University of Pretoria)

It is generally accepted that a structure can be understood and its behaviour predicted when the natural frequencies and modes of vibration are known. The practical side of (experimental) modal analysis is treated in engineering textbooks. Modal analysis is linked to the theory in the sense that the response of a structure is given by a "superposition"" of so-called modes of vibration. Each mode of vibration is associated with a shape and frequency. Modal analysis leads to an eigenvalue problem where the solutions are referred to as eigenfunctions accompanied by eigenvalues.

Modal analysis in theory follows the same line of thinking as in practice. The mathematical model leads to an eigenvalue problem and partial sums of modal solutions are supposed to converge to a solution of the model problem. This enables one to predict the response of a given structure (or body) to excitation. For the vibrating string model, this procedure is not only well-known but the theoretical foundation can be found in textbooks. To establish convergence of partial sums for other models (for example, the Timoshenko model) is not so simple. It depends on the completeness of the relevant sequence of eigenfunctions. In this presentation, we consider the spectrum of the axially loaded Timoshenko rod. Completeness of the sequence of eigenfunctions is proved and numerical results are presented.

The KdV-Burgers equation in the small dispersion/diffusion limits

JAC Weideman (Stellenbosch University)

The viscous Burgers equation models the interaction of a quadratic nonlinearity and diffusion. In the limit of small diffusion, the Burgers equation approaches the entropy solution of the inviscid Burgers equation. On the other hand, the KdV equation models the interaction of the same nonlinearity but with dispersion instead of diffusion. In the limit of small dispersion, the KdV equation does not approach the entropy solution. Instead, the limiting solution has a highly oscillatory profile. In this talk the focus is on the situation when both diffusive and dispersive terms are present. Because the entropy solution has a jump discontinuity (shock), computing nearby solutions is highly challenging. Preliminary results will be presented.