The 40th Annual South African Symposium of Numerical & Applied Mathematics (SANUM)

22nd to 24th March 2016

Department of Mathematical Sciences
Stellenbosch University, South Africa

UNIVERSITEIT
STELLENBOSCH
UNIVERSITY

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The organising committee acknowledges sponsorship from the Centre of Excellence - Mathematical and Statistical Sciences

Welcome to SANUM! The SANUM meeting came into existence in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York), happened to visit South Africa at the same time. A number of local researchers used the opportunity to organise a meeting at the (then) University of Natal, at which a total of ten papers were presented. From here the meeting grew in strength, with numerous famous numerical and applied mathematicians passing through. Aside from missing a single year (1996) and on one occasion (1997) being a mere special session of a larger conference, the meeting has been held annually, which makes 2016 the 40th occurrence of this major event in the alendar of South African numerical and applied mathematics.

Conference Venue: The 2016 conference is held on the ground and first floors of the Mathematical Sciences building (building number 67 on the campus map – next page). Registration takes place on 22nd March from 08:00 to 08:45 in the foyer outside the ground floor lecture hall.

Wifi details: SSID: Applied_Maths, Key/password: Confer3nc320!6

Tea & Coffee: Tea & Coffee will be in room 1006, just outside the ground floor lecture hall. Early morning coffee will available 8:00–8:45 on Tuesday and 8:30–9:00 on Wednesday and Thursday.

Lunch Venue: Lunch will be at the Katjiepiering Restaurant, the Stellenbosch Botanical Gardens, approximately a 10 minute walk from the conference venue. Transport will be made available for those who require it. On the first day, please assemble in the foyer so that we can walk to the gardens en masse. **Please ensure your name badge is worn and visible.**

Conference Reception: A drinks reception with light snacks will be held at 17:30 on the 22nd March in the Katjiepiering Restaurant. **Please ensure your name badge is worn and visible.**

Conference Dinner: The conference dinner will take place at Tokara restaurant on the Helshoogte Pass (approximately a 10 minute drive from the conference venue) at 19:00 on the 23rd March. Transportation will be organised during the conference. Dress code is smart / casual. **(No shorts or sandals, please!)**

Campus security: Emergency number: +27 (0)21 808 2333. Enquiries: +27 (0)21 808 4666.

Luggage: On the final day (only) luggage may be stored in room 1006A. (Not room 1006!)

40 SANUM meetings:

- Durban, 10–11 April 1975
- Durban, 8–9 April 1976
- Durban, 6–7 April 1977
- Durban, 18–20 July 1978
- Durban, 18–20 July 1979
- Durban, 21–23 July 1980
- Durban, 20–22 July 1981
- Durban, 19–21 July 1982
- Durban, 18–20 July 1983
- Ballito, 2–4 July 1984
- Umhlanga Rocks, 8–10 July 1985
- Umhlanga Rocks, 14–16 July 1986
- Umhlanga Rocks, 13–15 July 1987
- Umhlanga Rocks, 11–13 July 1988
- Umhlanga Rocks, 17–19 July 1989
- San Lameer, 9–11 July 1990
- Umhlanga Rocks, 15–17 July 1991
- Durban, 13–15 July 1992
- San Lameer, 12–14 July 1993
- Umhlanga Rocks, 4–6 July 1994
- Scottburgh, 10–12 July 1995
- Cape Town, 15–17 April 1998
- Stellenbosch, 29–31 March 1999
- Stellenbosch, 3–5 April 2000
- Stellenbosch, 9–11 April 2001
- Stellenbosch, 3–5 April 2002
- Stellenbosch, 31 March–2 April 2003
- Stellenbosch, 5–7 April 2004
- Stellenbosch, 30 March–1 April 2005
- Stellenbosch, 3–5 April 2006
- Stellenbosch, 2–4 April 2007
- Stellenbosch, 2–4 April 2008
- Stellenbosch, 6–8 April 2009
- Stellenbosch, 15–17 April 2010
- Stellenbosch, 23–25 April 2011
- WITS, 2–4 April 2012
- Stellenbosch, 3–5 April 20013
- WITS, 14–16 April 2014
- Pretoria, 30 March–1 April 2015
- Stellenbosch, 22–24 March 2016

CAMPUS MAP

SUPPORT SERVICES

1 Administration, Block A
2 Administration, Block B
3 Administration, Block C
4 JS Gericke Library
5 Neelsie Student Centre
6 Campus Security
7 Centre for Prospective Students
8 Centre for Student Counselling and Development: Reception
9 Development Office
10 Careers Office
11 Academic Counselling and Career Development
12 Therapy and Personality Development
13 Academic Support
14 Centre for Teaching and Learning
15 Campus Health Services
16 Maties Sport
17 DF Malan Memorial Centre
18 Old Conservatoire
18 Bureau for Economic Research
19 SU Art Galery
20 Eben Dönges Centre (Sasol Art Museum)
21 Telematic Services
22 Language Centre (Crozier Street Houses)
23 WAT, 115 Banghoek Road
24 Matie Community Services, Lückhoff School
25 SU Vehicle Fleet
26 Legal Aid Clinic
27 IT Student Help Centre
28 TAS-IT
29 Information Technology
30 Facilities Management
31 Buying and Supplying Services

SPORTS FACILITIES

33 Danie Craven Stadium
34 Coetzenburg Athletics Stadium
35 Coetzenburg Tennis Courts
36 PSO Club House
37 Hockey Club House and Fields
38 Swimming Pool
39 SU Gymnasium
40 Tennis Courts (Residences)
41 Old Mutual Sports Centre
42 Tennis Courts
43 Heidehof Rugby Fields

ACADEMIC BUILDINGS

44 Africa Centre for HIV and AIDS Management
45 Food Science
46 PO Sauer
47 Civil Engineering
48 Electrical / Electronic Engineering
49 Process Engineering
50 Mechanical and Industrial Engineering
51 Engineering, General
52 Knowledge Centre
53 JC Smuts – Biological Sciences
54 De Beers – Chemistry
55 Mike de Vries
56 Chemistry – first-years
57 Inorganic Chemistry
58 CGW Schumann
59 Van der Sterr
60 Accounting and Statistics
61 JS Marais
62 Polymer Science
63 Visual Arts
64 JH Neethling
65 AI Perold
66 Merensky
67 Mathematical Sciences and Industrial Psychology
68 Nursery
69 Natural Sciences
70 Chamber of Mines
71 RW Wilcocks
72 Old Main Building
73 HB Thom Theater
74 CL Marais Library
75 Journalism
76 GG Cillié
77 Arts and Social Sciences
78 Lombardi
79 Konservatorium (University Choir)
80 Theological Seminary
81 Paul van der Bijl Laboratories
82 Welgevallen Experimental Farm
83 Agronomy
84 Sport Science

WOMEN'S RESIDENCES

85 Huis Neethling
86 Monica
87 Harmonie
88 Heemstede
89 Huis ten Bosch
90 Lydia
91 Minerva
92 Nerina
93 Erica
94 Nemesia
95 Serruria
96 Tinie Louw Dining Hall
96 amaMaties hub
97 Sonop (Huis van Niekerk)
98 Irene

MEN'S RESIDENCES

99 Eendrag
100 Helshoogte
101 Simonsberg
102 Huis Visser
103 Huis Marais
104 Dagbreek
105 Majuba
106 Wilgenhof
107 Helderberg

MEN'S AND WOMEN'S RESIDENCES

108 New generation residence
109 Metanoia
110 Academia
111 Concordia
112 Goldfields
113 Huis McDonald

UNIVERSITY FLATS AND HOUSES

114 Lobelia
115 Crozierhof
116 Huis de Villiers
117 Waldenhof

LISTEN, LIVE AND LEARN

118 LLL town
■ LLL houses

FACULTIES (IN COLOUR) AND DEPARTMENTS

Accounting	59
African Languages	77
Afrikaans en Nederlands	77
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AGRISCIENCES	45, 46, 53, 55, 61, 64, 78, 81–83
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Institute for Plantbiotechnology	69
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Microbiology	53
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Soil Science	78
Sport Science	84
Statistics and Actuarial Science	60
THEOLOGY	80
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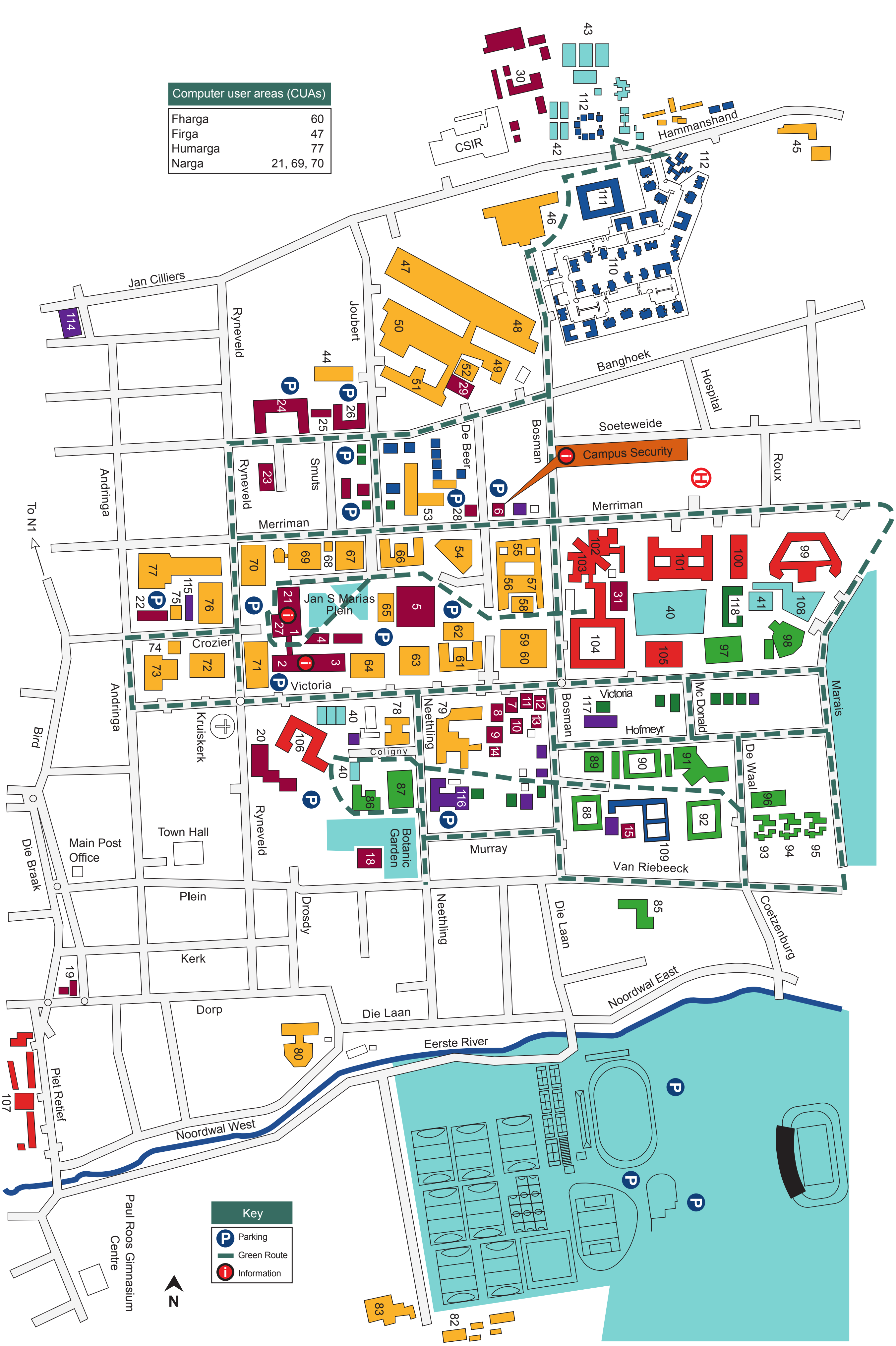
Viticulture and Oenology	64

Ⓟ Parking


Computer user areas (CUAs)
Fharga 60
Firga 47
Humarga 77
Narga 21, 69, 70
Key
Parking
Green Route
Information
N
Campus Security
CSIR
Jan Cilliers
Hammanshand
Banghoek
Hospital
Soeteweide
Roux
Merriman
Ryneveld
Joubert
Smuts
De Beer
Bosman
Andringa
To N1
Bird
Die Braak
Main Post Office
Town Hall
Jan S Marais Plein
Crozier
Victoria
Kruiskerk
Coligny
Neethling
Hofmeyr
McDonald
Marais
De Waal
Botanic Garden
Murray
Van Riebeeck
Plein
Kerk
Drosdy
Die Laan
Coetzenburg
Noordwal East
Noordwal West
Dorp
Eerste River
Piet Retief
Paul Roos Gimnasium Centre

Programme:

		TUESDAY		WEDNESDAY		THURSDAY	
From	**To**	**Room 1005**	**Room 2002**	**Room 1005**	**Room 2002**	**Room 1005**	**Room 2002**
		Tea, coffee, and registration: 8:00 - 8:45		Tea and coffee: 8:30 - 9:00		Tea and coffee: 8:30 - 9:00	
8:45	9:00	Welcome: Weideman					
9:00	9:50	Plenary: Trefethen		Plenary: Bornemann		Plenary: Higham	
9:50	10:40	Plenary: Van Assche		Plenary: Pelloni		Plenary: Banda	
10:40	11:10	Tea and coffee		Tea and coffee		Tea and coffee	
		Orthogonal polynomials	Finite elements	Misc.	Ecological dynamics	Modeling	Misc.
11:10	12:30	Clarkson	van Rensburg	Ali	Landi	Botha	Jacobs
11:30	11:50	Fasondini	van der Merwe	Angstmann	Latombe	du Toit	Elago
11:50	12:10	Driver	Stapelberg	Kurganov	Ramanantoanina	Gibson	Mergia
12:10	12:30	ter Horst	Labuschagne	Akinola	van Schalkwyk	Mahasa	Eneyew
12:30	14:00	Lunch @ Katjiepiering restaurant		Lunch @ Katjiepiering restaurant		Lunch @ Katjiepiering restaurant	
14:00	14:50	Plenary: Reddy		Plenary: Mansfield		Wine tasting?	TBC
		Orthogonal polynomials	Finance	Fluids	Machine learning		
14:50	15:10	Trefethen	Wilcox	Mason	Herbst / du Preez		
15:10	15:30	Jooste	Chipoyera	Hutchison	du Preez		
15:30	16:00	Tea and coffee		Tea and coffee			
16:00	16:20	Samuel	Harvey	Kgatle	Paquet		
16:20	16:40	Hale	Nuugulu	Fareo	Fabris-Rotelli		
16:40	17:00	Kelil	van Zyl	Tambue	Bassett		
17:00	17:20			Magagula	Theron		
17:30	20:00	Reception: 17:30 @ Katjiepiering					
19:00	23:00			Conference dinner: 19:00 @ Tokara			

Key	**Mins**
Plenary talk	50
Contributed talk	20
Special session	20
Tea/coffee	30
Lunch time	90

For changes to the timetable, see http://sanum.github.io/programme.html

Presentation Details:

- Please load your talks on the available machines **before** the session in which you are speaking.
- If you are using your own laptop for the presentation, please check the connections in advance.
- A laser pointer and a clicker will be available in each room.
- Contributed and special session talks will be kept **strictly** to 20 (15+5) mins.
- We ask please that the first speaker in each contributed session (counting the 14:50–17:20 block as one session) will chair, with the second speaker acting as chair for the first talk.
- Chairs for the special sessions will be arranged by the organiser of the session.

Plenary abstracts

Optimal control and inverse problems for evolution models

Mapundi Banda (University of Pretoria)

Optimal control and inverse problems governed by partial differential equations will be considered. The first part of the presentation will derive continuous and discrete schemes for optimal control problems subject to nonlinear, hyperbolic conservation laws. The initial condition will be treated as the control and a tracking-type problem also with non-smooth desired states will be approximated. The second part will consider a simultaneous identification of a coefficient and an initial condition of a partial differential equation from boundary measurement problem. An identification algorithm will be discussed. Some numerical simulations to show the effectiveness of the proposed algorithm will also be demonstrated.

Numerical Problems Inspired by Discrete Complex Analysis

Folkmar Bornemann (TU Munich)

Discrete Complex Analysis, that is, the quest for discretizing the whole theory and not just single manifestations of it has led to a wealth of interesting mathematical concepts and a rich nonlinear theory related to integrable systems. A major question is the construction of discrete analogues of the set of "standard" holomorphic maps and a good model problem is the power map z^{α}. The stable numerical evaluation of that map touches on many mathematical topics ranging from boundary value problems of discrete Painlevé equations to infinite-dimensional linear algebra and discrete optimization.

Challenges in Multivalued Matrix Functions

Nicholas J Higham (University of Manchester)

Multivalued matrix functions arise in solving various kinds of matrix equations. The matrix logarithm is the prototypical example. Another example is the Lambert W function of a matrix, which is much less well known but has been attracting recent interest. A theme of the talk is the importance of choosing appropriate principal values and making sure that the correct choices of signs and branches are used, both in theory and in computation. We will give examples where incorrect results have previously been obtained. We focus on matrix inverse trigonometric and inverse hyperbolic functions, beginning by investigating existence and characterization. Turning to the principal values, various functional identities are derived, some of which are new even in the scalar case, including a "round trip" formula that relates acos(cos A) to A and similar formulas for the other inverse functions. Key tools used in the derivations are the matrix unwinding function and the matrix sign function.

A new inverse scaling and squaring type algorithm employing a Schur decomposition and variable-degree Padé approximation is derived for computing acos, and it is shown how it can also be used to compute asin, acosh, and asinh. In numerical experiments the algorithm is found to behave in a forward stable fashion and to be superior to computing these functions via logarithmic formulas.

Lattice based Multispace and Applications

Elizabeth Mansfield (University of Kent)

In this talk I will describe the construction of a manifold called multispace which uses the Lagrange interpolation data of functions as coordinates and which has the jet space for functions embedded as a sub manifold. The purpose for the construction is the simultaneous calculation of smooth and discrete invariants of Lie group actions on the base space, via a moving frame, together with their differential (smooth) and recurrence (discrete) relations, with all appropriate continuum limits guaranteed by the construction. Two main applications have been described to date, to discrete integrable systems and to discrete variational systems. I will discuss the application to variational shallow water systems, showing how a simultaneous smooth and discrete Noether's theorem leads to the conservation of energy and momenta in the discrete scheme, with guaranteed continuum limits to the smooth energy and momenta. This is joint work with Gloria Mari Beffa (University of Wisconsin–Madison).

Beyond integrability: the far-reaching consequences of thinking about boundary conditions

Beatrice Pelloni (University of Reading)

In this talk, I will outline results obtained in the last fifteen years, all stemming from the original aim to include boundary conditions into the celebrated Inverse Scattering Transform, which is in essence a nonlinear Fourier transform.

The talk will revisit the Fourier transform on $\mathbb{R}$, embedding it in a general way of thinking about integral transform that relies on a formulation in the complex domain (called a Riemann–Hilbert formulation) and start from this idea to describe a generalised approach, now known as the unified transform, or Fokas transform. This circle of ideas has produced unexpected and very general results for the solution of linear boundary value problems, including interface and moving boundary problems, new ideas for numerical schemes, a way to study of nonselfadjoint differential operators, as well as going some way to achieving the original aim of solving nonlinear integrable problems in domains with boundaries.

I will describe the key ideas and some of the most unexpected results.

Discontinuous Galerkin Methods: An Overview and Some Applications

Daya Reddy (University of Cape Town)

Discontinuous Galerkin (DG) methods, as the name suggests, are a class of Galerkin finite element methods in which the condition of continuity of functions across element boundaries is relaxed. DG methods have become a powerful tool in the approximate solution of hyperbolic problems, the context in which they were first developed. DG methods have also been applied with much success to elliptic PDEs and systems, and it is this class of applications that will be the focus of this presentation. A simple model problem will serve as the vehicle for motivating the DG approach and comparing it to the conventional finite element method. A further application will be in the context of problems arising in solid and fluid mechanics, and in which dependence on a parameter may lead to singular behaviour of the approximate solution. It will be shown how uniformly convergent DG approaches can be constructed in such situations.

Ten things you should know about quadrature

Nick Trefethen (University of Oxford)

Quadrature is the term for the numerical evaluation of integrals. It's a beautiful subject because it's so accessible, yet full of conceptual surprises and challenges. This talk will review ten of these, with plenty of history and numerical demonstrations. Some are old if not well known, some are new, and two are subjects of my current research.

Simultaneous Gauss quadrature

Walter Van Assche (KU Leuven)

The talk will be about simultaneous Gaussian quadrature (introduced by Borges in 1994) for two integrals of the same function f but on two disjoint intervals. The quadrature nodes are zeros of a type II multiple orthogonal polynomial for an Angelesco system. We recall some known results for the quadrature nodes and the quadrature weights and prove some new results about the convergence of the quadrature formulas. Furthermore we give some estimates of the quadrature weights. Our results are based on a vector equilibrium problem in potential theory and weighted polynomial approximation. This is joint work with Doron Lubinsky (Georgia Institute of Technology, Atlanta GA, USA).

Contributed abstracts

Computing the complex eigenpair of a large sparse matrix in complex arithmetic

Richard Akinola (University of Jos)

It is well known that if the largest or smallest eigenvalue of a matrix has been computed by some numerical algorithms and one is interested in computing the corresponding eigenvector, one method that is known to give such good approximations to the eigenvector is inverse iteration with a shift. For complex eigenpairs, instead of using Ruhes normalization, it has been shown that the natural two norm normalization for the matrix pencil, yields an under-determined system of equation and by adding an extra equation, the augmented system becomes square which can be solved by LU factorization at a cheaper rate and quadratic convergence is guaranteed. In this work, we show that the square linear system in real arithmetic can actually be converted to complex system and solved at a much cheaper rate in complex arithmetic. An algorithm is given with an example which shows that an application of Newtons method in complex arithmetic converge quadratically.

Transformation Based Pre-processing for Mixed Integer Quadratic Programs

Montaz Ali and Eric Newby (University of the Witwatersrand)

This talk presents two pre-processing techniques for mixed-integer quadratic programs with non-convex objective functions, where the continuous part of the Hessian is invertible. The techniques aim at reducing the number of bilinear terms in the objective. Results show that one of the techniques decreases the solution times once the reduction in bilinear terms crosses a threshold.

From discrete time random walks to numerical methods for fractional order differential equations

Christopher Angstmann (University of New South Wales)

We present a method for finding an numerical scheme to solve a class of fractional order differential equations. The scheme is derived from the master equations that govern the evolution of the probability density of a discrete time random walk. Under the appropriate limit the master equation approaches the fractional order differential equation of interest. The limiting procedure guarantees the consistency of the numerical scheme, provided that the underlying random walk exists. Using this approach numerical schemes have been found for fractional Fokker–Planck equations, reaction subdiffusion equations, and fractional compartment models.

Machine Learning in Astronomy

Bruce Bassett (AIMS/SAAO/UCT)

Machine learning is rapidly becoming unavoidable in astronomy with the next generation of large surveys culminating in the exabyte SKA surveys. We discuss recent progress in using machine learning in astronomy and the challenges that remain.

Modelling Environmental Phenomena

Joseph F Botha (University of the Free State)

It is tempting to summarise the current debate on global warming in the printed and electronic media as implying that humans can understand and describe the future evolution of natural phenomena accurately and with confidence. Is such an objective really feasible in the natural world?

Calibration of the Variance gamma and Normal inverse Gaussian processes on an emerging market

Charlene Chipoyera (University of the Witwatersrand)

This paper focuses on two exponential Lévy processes namely the variance gamma (VG) and normal inverse Gaussian (NIG) because these two processes can be represented using time subordination of Brownian motion. The VG and NIG models are robust, simple and have a closed form for the probability density function. Despite the existence of numerical empirical studies to show that the NIG and VG models fit daily returns quite well, little research has been done to calibrate the NIG and VG models on an emerging market using intraday data. Recognizing this gap in literature, an analysis of plots of point estimates at different sampling frequencies will be made in this paper to assess the suitability of the NIG and VG models to fit the statistical features of asset returns. Simulated data for the NIG and VG processes at different sampling frequency will be used to test the performance of maximum likelihood estimator (MLE) and Method of Moment Estimators (MME).

Semi-classical orthogonal polynomials and the Painlevé equations

Peter Clarkson (University of Kent)

In this talk I shall discuss the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for some semi-classical weights, the coefficients in the recurrence relation can be expressed in terms of Wronskians that arise in the description of special function solutions of a Painlevé equation. The orthogonal polynomials discussed will include a semi-classical Hermite weight, a generalization of the Freud weight and an Airy weight.

Orthogonal polynomials and interlacing of zeros

Kathy Driver (University of Cape Town)

The interlacing of zeros of two polynomials of consecutive degree in an orthogonal sequence is a classical result that has important applications to Gauss quadrature. Stieltjes extended the concept of interlacing to zeros of two orthogonal polynomials of non-consecutive degree. In this more general context, common zeros of the two polynomials involved (if there are any) play a critical role when interlacing of zeros is under consideration.

This talk will focus on sequences of Laguerre polynomials $\{L_n^{(\alpha)}\}_{n=0}^{\infty}$, α fixed, $\alpha > -1$. Recent developments on the interlacing of zeros of orthogonal polynomials from different sequences within this classical family will be discussed and the connection between the interlacing of zeros and the existence of common zeros will be highlighted. The mixed three term recurrence relations satisfied by Laguerre polynomials corresponding to different values of the parameter α are used to derive bounds for the largest and smallest zeros of Laguerre polynomials. We discuss the interlacing of zeros, and the co-primality, of the quasi-orthogonal Laguerre sequences $\{L_n^{(\alpha)}\}_{n=0}^{\infty}$ for α fixed, $-2 < \alpha < -1$.

Colouring a map into four colours by means of Probabilistic Graphical Models

JA du Preez (Stellenbosch University)

The four-colour map theorem has an interesting history. The conjecture arose in the mid eighteen hundreds, for nearly 125 years resisted proof before succumbing in 1976 to become the first major computer-proven theorem. While we were working on a Probabilistic Graphical Model based superpixel system (an accompanying talk), we required an algorithm to colour maps using only four colours. Although much material is available on the proof of this famous theorem, an efficient algorithm for doing the actual colouring was not so readily forthcoming. Possibly the long delay of a formal proof should have alerted us to this possibility.

The standard approach seems to be a directed recursive search with backtracking. This, however, is often very slow, so much so that it would destroy all the computational benefit we hoped to gain from our particular superpixel approach. In order to address this, we compiled a PGM-based system to fairly efficiently colour such maps. Although we suspect that the system is able to generically colour all such maps, we have no proof of this. It was, however, able to colour all the maps we encountered in our image processing work.

Determining superpixels via Probabilistic Graphical Models

JM Pinks and JA du Preez (Stellenbosch University)

Superpixels often form a fundamental building block of image segmentation / interpretation tasks. A variety of approaches are available, differing markedly in terms of the quality of adherence to edges in the underlying image vs computational requirements. Of these the SLIC algorithm is fast [1] while still yielding high quality superpixels. There is also a history of graph-based approaches to this problem, ranging from graph-cut based techniques through Markov Random Field approaches.

In this talk we discuss an exploratory effort towards applying probabilistic reasoning to segment an image into superpixels. We use a "soft" low-level edge detector (inspired by, but different from the well known Canny algorithm) to scrutinize an initial SLIC segmentation for possibly badly segmented superpixels. These suspect superpixels are then re-interpreted using a Probabilistic Graphical Model that blends these low-level edges, the desire for closed-contour superpixel edges, the four-colour map theorem and the SLIC result. The quality of the segmentations we found in a small set of images (not a full or formal evaluation by any means), are promising. However, the computational expense remains fairly severe.

[1] Radhakrishna Achanta, Appu Shaji, Kevin Smith, Aurelien Lucchi, Pascal Fua, and Sabine Susstrunk, SLIC Superpixels Compared to State-of-the-Art Superpixel Methods, IEEE transactions on pattern analysis and machine intelligence, pp. 2274–2281, (2012).

Earthquake induced oscillations of high-rise buildings and other vertical structures

Sonja du Toit (University of Pretoria)

Engineers and researchers are interested in the effect of wind and earthquake induced oscillations on high-rise structures such as buildings, masts and chimneys. Obviously, the relevant structure should be able to withstand these oscillations. In a recent article the authors draw attention to other not so obvious considerations for example a large investment in non-structural components which may be damaged. In this talk the focus is on high-rise buildings subjected to earthquake induced oscillations. Reliable mathematical models are needed to determine the effect of such oscillations on buildings. Tall buildings are often modeled as vertical beams. Some authors use the Euler–Bernoulli beam model for a high-rise building. We consider the Timoshenko model and other simplified beam models. The beam models are adapted to make provision for the effect of gravity and longitudinal vibration. Modal analysis is used to compare these models.

A numerical technique to solve fractional order differential equations in epidemiology

D Elago, F Gideon, and KC Patidar (University of Western Cape)

There have been significant efforts towards studying the mathematical models in biology using ordinary and/or partial differential equations of integer orders. However, due to fact that many biological phenomena have memory or aftereffects, it makes more sense to model them by using fractional differential equations. In this talk, we will consider such a model and will present a robust solution approach to solve it. We will also discuss analysis of this method and then present some numerical simulation results.

Eulerian-Lagrangian modelling of the motion of a mixture of particles and a Newtonian fluid

Eyaya Birara Eneyew (University of Stellenbosch)

The time-volume averaged Navier-stokes equation is used to model the fluid phase, and the solid particles are tracked in a Lagrangian frame using Newtons equation of motion. OpenFOAM is used for the simulation of the flow of a mixture of particles and fluid. A dynamic adaptive mesh refinement is used to refine regions only near to the solid particles in order to decrease the computa- tional cost. In addition, a comparison of the linear system solvers in OpenFoam is done to choose a suitable solver for the two phase system. Some results of the validation and verification of the simulation will be presented..

A point process model for DPT pulses from water images

Inger Fabris-Rotelli (University of Pretoria)

The Discrete Pulse Transform (DPT) provides a multiscale decomposition of an image. We look into how the resulting pulses at different scales can be used, via point process modelling, to develop a texture model for water surface images.

Fluid-driven hydraulic fracture in a permeable medium

Adewunmi Fareo (University of the Witwatersrand)

A two dimensional fracture propagating in a permeable elastic medium by the injection of a viscous incompressible Newtonian fluid into it is investigated. The flow of the fracturing fluid is considered laminar. The leak off of fluid into the porous rock matrix is governed by Darcys law. Using lubrication theory a system of partial differential equations relating the width of the fracture, fluid pressure and the leak off velocity is obtained. The Lie group theoretic approach is employed to reduce the system of partial differential equations to a system of ordinary differential equations. Numerical solution for the evolution of the width of the hydraulic fracture is obtained.

A computational study of a class of multivalued tronquée solutions of the third Painlevé equation

Marco Fasondini (University of the Free State)

The third Painlevé transcendent, P_{III}, is the generally multivalued solution of the third member of the celebrated sextet of second order nonlinear ODEs known as the Painlevé equations. These equations have arisen in numerous applications since their discovery more than a century ago. In particular, P_{III} has featured in studies of statistical mechanics, general relativity and two-dimensional polymers among others. The solutions of the Painlevé equations usually have poles all over the complex plane but *tronquée* solutions are characterized by pole-free sectors. Tronquée solutions of the first or second Painlevé transcendent have been the subject of many studies but comparatively little has appeared in the literature concerning the recently discovered tronquée solutions of P_{III}. We present a method for computing P_{III} on any number of sheets of its Riemann surface and display known and novel tronquée P_{III} solutions.

A differential equation model for multi-class, multi-server queue networks with time dependent parameters.

Emma Gibson and Stephan Visagie (Stellenbosch University)

As part of an ongoing project to reduce patient waiting times at Zithulele Hospital, a mathematical model of the Out Patients/Casualty Department (OPD) has been created. The OPD model consists of an open network of multi-class, multi-server queues with non-stationary arrival and service rates. The size and complexity of this system makes it unsuitable for traditional queueing theory analysis, but promising results have been achieved through a combination of agent-based and discrete event simulation. In this presentation, an alternative model will be developed to obtain a continuous approximation of the flow of patients through nodes in the queue network. Time dependent parameters are expressed as piecewise functions and discrete variables are introduced to modify the behaviour of each queue based on the traffic intensity. The resulting system of discontinuous differential equations is solved numerically and compared with the simulation results.

An ultraspherical spectral method for fractional integral and differential equations of half-integer order on bounded domains

Nick Hale (Stellenbosch University) and Sheehan Olver (University of Sydney)

The ultraspherical spectral (US) method recently introduced by Olver & Townsend is based on the observations that (a) differentiation is a banded operator between expansions in two different ultraspherical polynomials bases (for example, Chebyshev first- and second-kinds), and (b) the transformation back to the original basis is the inverse of a banded operator. The result is a fast and well-conditioned spectral method for ordinary and partial differential equations.

In this talk we shall see that these two properties hold for fractional integration and differentiation of half-integer order when one instead considers a suitably weighted polynomial basis, allowing a fast algorithm for fractional integral and fractional differential equations of Riemann–Liouville type.

Reconciling order book resiliency and price impact

Michael Harvey and Dieter Hendricks (University of the Witwatersrand)

Understanding and quantifying the impact and persistence of trade events on limit order book dynamics is of critical importance for trading decisions. Specifically the trading trajectory chosen to cause least impact with a reasonable guarantee of execution. In this paper, empirical point processes are extracted from intraday tick data which represent key liquidity demand and resiliency events. Using these point processes, trades and quotes are modelled as a mutually-exciting four-variate Hawkes point process with a sum-of-exponentials kernel. The calibrated model allows us to quantify order book resiliency in terms of expected time frame and magnitude of quote replenishment in response to a trade event. By examining a period of increasing trade velocity on the Johannesburg Stock Exchange, we find that the empirically observed increase in low-volume price impact can be explained by a lack of commensurate quote replenishment following low-volume, price-moving trades. We conjecture that certain anomalous shape characteristics of empirical price impact curves can be explained by measuring quote replenishment following trades which move the mid-quote price.

A unified theory for turbulent wake flows described by eddy viscosity

Ashleigh Hutchinson (University of the Witwatersrand)

In this talk, we consider the conservation laws for the far downstream wake equations described by eddy viscosity. A basis of conserved vectors is constructed. The well-known conserved quantities for the turbulent classical wake and the turbulent wake of a self-propelled body are obtained by integrating the corresponding conservation law across the wake and imposing the boundary conditions. For the wake of a self-propelled body the additional condition that the drag on the body is zero is required to obtain the conserved quantity. A third conservation law, which possibly belongs to another type of wake, is discovered. The Lie point symmetry associated with the conserved vector is used to obtain the invariant solution and a typical velocity profile for this wake is provided. This wake appears to have common properties with the other two well-known wakes. We then analyse the invariant solutions to all three wake problems and prove that a simple mathematical relationship exists between them thus unifying the theory for turbulent wake flows.

Unsupervised Change Detection using a System of Nonlinear Partial Differential Equations

Turgay Celik and Byron Jacobs (University of the Witwatersrand)

We propose a space-time partial differential equation based unsupervised change detection method. The method is derived from a system of two-dimensional diffusion equations coupled with a nonlinear Fithugh–Nagumo type cubic nonlinear source term which exhibits a thresholding effect. This source term invokes two steady states that correspond to black and white regions, or regions of interest and otherwise. The proposed method is simulated using an explicit finite difference method which is provably stable under reasonable parameter choices. Although the proposed numerical scheme is rudimentary it is parallelizable which relieves the high computational when working with large data sets. The primary novelty in this work is in governing equation of the threshold parameter. This equation ensures that the threshold map is smooth throughout the computation and enables an unsupervised convergence criterion.

On the zeros of Meixner and Meixner-Pollaczek polynomials

Alta Jooste (University of Pretoria)

Josef Meixner was a German physicist and the Meixner polynomials were introduced by him in 1934. Meixner-Pollaczek polynomials can be considered as Meixner polynomials with an imaginary argument and both these systems of polynomials lie on the ${}_2F_1$ plane of the Askey scheme of hypergeometric orthogonal polynomials. We discuss the properties of the zeros of these polynomials as well as the quasi-orthogonality of these systems of polynomials, using a characterisation of quasi-orthogonality due to Riesz.

Characterizing properties of generalized Freud polynomials

PA Clarkson (Kent, UK), Abey Kelil, and KH Jordaan (University of Pretoria)

Generalized Freud polynomials can be generated via quadratic transformation from semi-classical Laguerre polynomials. In this talk, we discuss the relation of generalized Freud polynomials to the classical solutions of the fourth Painlevé equation and we investigate some of the properties of the polynomials. We show that the coefficients in the three-term recurrence relations satisfied by these polynomials can be expressed in terms of Wronkians of parabolic cylinder functions that arise in the description of special function solutions of the fourth Painlevé equation. We also derive a differential-difference equation and second-order linear ordinary differential equation satisfied by these polynomials.

Hyperbolic hydraulic fracture with tortuosity

Rahab Kgatle and David Mason (University of the Witwatersrand)

A tortuous hydraulic fracture with contact regions modelled by the hyperbolic crack law is investigated. We replace the tortuous hydraulic fracture by a two-dimensional symmetric model fracture with a modified Reynolds flow law to account for the presence of asperities at the fluid-rock interface and a modified crack law (the hyperbolic crack law) to account for the presence of contact regions in the fracture. A Lie point symmetry analysis is used to find the group invariant solution of the governing nonlinear diffusion equation. The hyperbolic fracture admits only one solution which describes fluid injected at a constant pressure at the fracture entry. Although modelling contact regions with the linear crack law yields many solutions for fluid injection and extraction, the hyperbolic crack law is generally considered to be a more realistic model for contact regions. The invariant solution depends on an unknown function which is determined numerically. Analysis of the width averaged fluid velocity leads to the derivation of an approximate analytical solution which agrees well with the numerical result.

An Operator Splitting Based Stochastic Galerkin Method for the One-Dimensional Compressible Euler Equations with uncertainty

Alina Chertock (North Carolina State University), Shi Jin (University of Wisconsin-Madison), and Alexander Kurganov (Tulane University)

I will present an operator splitting based stochastic Galerkin method for the one-dimensional compressible Euler equations with random inputs. The method uses a generalized polynomial chaos approximation in the stochastic Galerkin framework (referred to as the gPC-SG method). It is well-known that such approximations for nonlinear system of hyperbolic conservation laws do not necessarily yield globally hyperbolic systems: the Jacobian may contain complex eigenvalues and thus trigger instabilities and ill-posedness. We propose to split the underlying system into a linear hyperbolic system and two effectively scalar linear or nonlinear hyperbolic equations with variable coefficients and source terms. The gPC-SG method, when applied to each of these subsystems, results in globally hyperbolic systems. The performance of the new gPC-SG method will be illustrated with a number of numerical examples with uncertainties from the initial data or equation of state.

Mixed Finite Element Method for the transient response of a Timoshenko beam

Madelein Labuschagne (University of Pretoria)

We consider the transient response of the Timoshenko beam due to some excitation (e.g. wind or earthquake). It is well known that for the Finite Element Method, shear locking is a problem when a certain parameter is too large. One way to get past this problem is to use basis functions consisting of polynomials of higher degree (e.g., Hermite cubics) instead of piecewise linear basis functions. Convergence can still be very slow though. Our experiments show that the Mixed Finite Element Method with piecewise linear basis functions is incredibly effective, regardless of the value of the parameter.

Branching scenarios in eco-evolutionary prey-predator models

Fabio Dercole (Politecnico di Milano), Pietro Landi (Stellenbosch University), and Sergio Rinaldi (Politecnico di Milano)

We show in this paper how simulations of ODEs and continuations of systems of algebraic equations can be combined to study the evolution of biodiversity in multispecies systems where phenotypic traits are genetically transmitted. We follow the adaptive dynamics (AD) approach, which provides a deterministic approximation of the evolutionary dynamics of stationary coexisting populations in terms of an ODE system, the so-called AD canonical equation. AD also provides explicit conditions to test whether a stable evolutionary equilibrium of the canonical equation is a branching pointresident and mutant morphs coexist and further differentiate, thus increasing biodiversityor not. We analyze a standard parameterized family of prey-predator communities, described by the most standard ecological model, and propose an iterative procedure to obtain the branching portrait, explaining the dependence of branching scenarios on two (demographic, environmental, or control) parameters. Among a number of interesting results, in line with field studies and known ecological principles, we find that prey branching is induced by the predation pressure, and is favored when prey intraspecific competition is highly sensitive to the resident-mutant phenotypic mismatch, while predator branching is not possible when prey and predators are present in an equal number of morphs. This results in alternate prey-predator branching sequences with possible phases of prey unilateral branching. The guidelines for deriving a general method for analyzing the evolution of biodiversity are also discussed. The indications that can be obtained typically have a qualitative nature, but can be of help for the long-term conservation and management of biodiversity.

The Community Assembly Phase Space: a process-based, multi-dimensional representation of ecological community assembly

Cang Hui (Stellenbosch University), Guillaume Latombe,
and Melodie A McGeoch (Monash University)

Ecological communities are generally considered to be assembled by the interaction between neutral processes, considering only dispersal limitations and no niche difference between species, and niche processes, considering that species niche differ but assuming unlimited dispersal. As a result, the neutral-niche continuum proposes that natural communities lie along a continuum based on the relative importance of these processes, with pure neutral- and pure niche-driven communities at either extreme. In particular, the continuum does not accommodate interactions via feedback between processes and the environment, which can generate non-linear community patterns. By contrast, we introduce the Community Assembly Phase Space (CAPS), a multi- dimensional space that uses community processes (such as dispersal and niche selection) to define the limiting neutral and niche conditions and to test the continuum hypothesis. To do so, we compare the outputs of communities simulated in an individual-based model with a heterogeneous landscape, assembled by pure neutral, pure niche and composite processes. Individual-based models define the behaviour of separate individuals (here plants and plant propagules), and let communities emerge from the interaction between individuals and with the environment, enabling the natural formation of feedbacks during the colonisation process.

Differences in patterns under different combinations of processes in CAPS reveal hidden complexity in neutral-niche community dynamics. The neutral-niche continuum only holds for strong dispersal limitation and niche separation. For weaker dispersal limitation and niche separation, neutral and niche processes amplify each other via feedback with the environment. This generates patterns that lie well beyond those predicted by a continuum. Inferences drawn from patterns about community assembly processes can therefore be misguided when based on the continuum perspective. CAPS also demonstrates the complementary information value of different patterns for inferring community processes and captures the complexity of community assembly. It provides a general tool for studying the processes structuring communities and can be applied to address a range of questions in community and metacommunity ecology.

On the multi-domain bivariate spectral local linearisation method for solving system of non-similar boundary layer equations

Vusi Magagula, Sandile Motsa, and Precious Sibanda (University of KwaZulu-Natal)

In this work, a new approach for solving system of non-similar boundary layer equations over a large time interval is presented. The method is termed multi-interval Legendre-Gauss-Lobbatto bivariate spectral local linearisation method (MD-LGL-BSLLM). The method uses the concept of multi-interval, Legendre-Gauss-Lobbatto grid points, local linearisation technique, and combines it with the spectral collocation method on approximate functions defined as bivariate Lagrange interpolation.

For the first time, we demonstrate the application of the method by solving system of nonlinear partial differential equations that belong to a class of non-similar boundary layer equations. The model equations considered describe an unsteady free convective flow of a viscous incompressible fluid along a permeable vertical flat plate in the presence of soluble species. Convergence analysis and grid independence tests are conducted to establish the accuracy, convergence and validity of the proposed method. The solution for the limiting case when ξ is very large is used to validate the solution obtained using the MD-LGL-BSLLM. The proposed method is better than existing methods for solving a class of non-similar boundary layer equations over large time intervals since it converges faster and uses few grid points to achieve accurate results. The proposed method uses minimal computation time and its accuracy does not deteriorate with an increase in ξ.

Mathematical Model of Tumor-Immune Surveillance

Khaphetsi Joseph Mahasa (SACEMA), R Ouifkir (SACEMA),
A Eladdadi (The College of Saint Rose), and L de Pillis (Harvey Mudd College)

We present a novel mathematical model involving various immune cell populations and tumor cell populations. The model describes how tumor cells evolve and survive the brief encounter with the immune system mediated by natural killer (NK) cells and the activated CD8 + cytotoxic T lymphocytes (CTLs). The model is composed of ordinary differential equations describing the interactions between these important immune lymphocytes and various tumor cell populations. Based on up-to-date knowledge of immune evasion and rational considerations, the model is designed to illustrate how tumors evade both arms of host immunity (i.e. innate and adaptive immunity). The model predicts that: (a) An influx of external source of NK cells might play a crucial role in enhancing NK-cell immune surveillance; (b) Immune system alone is not fully effective against progression of tumor cells; (c) the development of immunoresistance by tumor cells is inevitable in tumor immune surveillance. Our model also supports the importance of infiltrating NK cells in tumor immune surveillance, which can be enhanced by NK cell-based immunotherapeutic approaches.

Turbulent axisymmetric free jet described by Prandtl's mixing length model with non-zero kinematic velocity

Des Hill, Miccal Matthews (University of Western Australia),
and David Mason (University of the Witwatersrand)

A turbulent axisymmetric free jet in an incompressible fluid is investigated. The effect of the turbulence on the mean flow is described by Prandtl's mixing length model for eddy viscosity. The kinematic viscosity of the fluid is included in the model even although it is much smaller than the eddy viscosity. The conserved quantity for the mean flow is derived from the elementary conservation law. The invariant solution generated by the Lie point symmetry associated with the elementary conserved vector is derived. The kinematic viscosity, although small, is non-zero and it determines the mixing length and the form of the invariant solution. After the form of the invariant solution has been obtained the approximation that the kinematic viscosity is zero can be made. A numerical solution is investigated.

Robust simulation of a singularly perturbed multi-scale model arising in ecology

WD Mergia and KC Patidar (University of Western Cape)

In ecology, quantities vary according to widely differing time scales due to tropical time diversification. Such quantities can be modelled by equations based on different time scales. In this talk, we aim to discuss a dynamical system describing slow-fast dynamics. In particular, we consider a predator-prey system that is highly susceptible to local time variations. The solution of this problem exhibits a stable relaxation oscillation for some parametric values in the positive octant. To understand the global structure of the solution theoretically, we use geometric singular perturbation theory. After having discussed these qualitative properties of the solution, we will discuss a special class of numerical methods. Finally, we will present several comparative numerical results.

Front fixing method for solving arbitrary order PDEs for pricing American stock options under some Lévy dynamics

F Gideon, SM Nuugulu, and KC Patidar (University of Western Cape)

American options are the most commonly traded options in the market. They are used to mitigate risk and to speculate about the future. In this talk, we will present a numerical method for solving a arbitrary order partial differential equation arising in pricing American options (on stocks) under some Lévy dynamics. The basic numerical method is based on the front-fixing finite difference algorithm. One of the attractive features of this method is that, the early exercise boundaries and the option prices can be computed simultaneously with high accuracy. We will also discuss the stability and convergence analysis of the method along with some reliable numerical results.

Indexable Matrix Factorization for Maximum Inner Product Search

Ulrich Paquet (Apple)

This talk will be about designing Machine Learning models whilst keeping the final system architecture on which they will be employed in mind. Well look at Machine Learning for Recommender Systems, in particular how models can be adapted to give sublinear runtime re- trieval performance: The Maximum Inner Product Search (MIPS) problem, prevalent in matrix factorization-based recommender systems, scales linearly with the number of objects to score. Recent work has shown that clever post-processing steps can turn the MIPS problem into a nearest neighbour one, allowing sublinear retrieval time either through Locality Sensitive Hashing or various tree structures that partition the Euclidian space. In this talk I'll show that instead of employing post-processing steps, substantially faster retrieval times can be achieved for the same accuracy when inference is not decoupled from the indexing process, and a probabilisitic model is instead set up to be natively geometrically indexable.

This was work that was completed during my time at Microsoft Research.

Formulating spread of species with habitat dependent growth and dispersal in heterogeneous landscapes

Andriamihaja Ramanantoanina (AIMS / Stellenbosch) and Cang Hui (Stellenbosch)

Habitat heterogeneity can have profound effects on the spreading dynamics of invasive species. In this talk we present an integro- difference model for the spread of a population in a one-dimensional heterogeneous landscape which consists of alternating favourable and unfavourable habitat patches. We assume that population growth and dispersal (including emigration probability and dispersal distance) are dependent on habitat quality, and discuss the role of the favourable patches and dispersal strategies on the spread of the population.

On Multi-Domain Polynomial Interpolation Error Bounds

Samuel Mutua and Sandile Motsa (University of KwaZulu-Natal)

In this paper, we present new theorems and proofs for the bivariate polynomial interpolation error bounds where the domain of approxi- mation has been decomposed into smaller non-overlapping intervals.

The error bound theorems presented here include those obtained using equispaced, Gauss-Lobatto and Chebyshev grid points. The aim is to identify the set of grid points that minimises the approximation errors in order to form a good bases of developing powerful numerical schemes when approximating the solutions to non-linear differential equations using interpolating polynomials.

The Gauss-Lobbato nodes have been widely used for solving differential equations using an interpolating polynomial as the approximating function. Therefore, we seek to determine if there is a better alternative that uses different interpolation nodes.

The theorems and proofs are built from the univariate error bound theorems using equispaced and Chebyshev that have been presented in literature. A new nodal polynomial whose roots are the Gauss-Lobbato nodes is constructed in an attempt to derive an analytic expression for the error bounds when Gauss-Lobatto nodes are used in interpolation. The idea is extended to obtain error bounds in bivariate polynomial interpolation in single and multi-domains.

The generalized error bound theorems for multi-variate interpolating polynomial are also presented. The Chebyshev nodes emerges to be a better choice of interpolation nodes compared to Gauss-Lobatto nodes. Thus, the Chebyshev nodes can be adopted as an alternative to Gauss-Lobatto nodes by numerical analysts when descritising a differential equation. The comparison of the three set of grid points has been presented in figures. A numerical example has been solved to demonstrate the validity of the proved theorems and the results have been presented on tables.

Error estimates for semi-discrete and fully discrete Galerkin finite element approximations of the general linear second order hyperbolic equation

Belinda Stapelberg (University of Pretoria)

In this presentation we derive error estimates for the semi-discrete and fully discrete Galerkin approximations of a general linear second order hyperbolic partial differential equation with general damping (which includes boundary damping). The results can be applied to a variety of cases e.g., vibrating systems of linked elastic bodies. The results generalize the work of Dupont (1973) and Basson and Van Rensburg (2013).

A Rosenbrock-Type Method for simulation of liquid-vapor flows with phase change in geothermal systems

Inga Berre (University of Bergen), Jan M Nordbotten (University of Bergen & Princeton University), and Antoine Tambue (AIMS & University of Cape Town)

Numerical analysis of multidimensional flow and heat transfer in porous media is intrinsically complicated and often prohibitive in terms of computational cost, particularly when phase-change occurs. One of the main reasons is the strongly nonlinear and coupled nature of the governing equations.

Another fundamental difficulty lies in the presence of moving and irregular interfaces between the single and two-phase subregions in a domain of interest, which degenerate the mathematical model. This degeneration leads to strict time step restrictions; consequently, efficient and stable time integrators are needed.

In this talk, a Rosenbrock-Type Method for time integration combined with the finite volume method (two-point approximation) space discretizations are used for numerical simulations of liquid-vapor flows with phase change in geothermal systems. As all Rosenbrock-Type methods, this scheme use the rational functions of the Jacobian and only two linear systems are normally solved at each time step, thus no need to solve nonlinear algebraic equations as with standard implicit methods. Furthermore, this scheme is L-stable and the order of convergence is preserved for any relatively good approximation of the Jacobian. We investigate the performance of the method for realistic geothermal model problems and the convergence of the method through numerical examples.

Robust control in multidimensional systems

Sanne ter Horst (North West University)

Several (robust) control problems for discrete-time 1-D linear systems can be translated to metric constrained interpolation problems, and can then be solved using existing solution methods for such interpolation problems. In the case of discrete-time multidimensional (n-D) linear systems the connection is less transparent, and even fails in some aspects. In this talk we will focus on a few of the complications and the connections between linear multidimensional systems and interpolation that survive. The talk is based on joint work with JA Ball.

From Hopping Hoops to Bouncing Bicycle Wheels

WFD Theron (Stellenbosch University)

The development of the mathematical model for loaded wheels (or hopping hoops) is reviewed briefly, starting with the massless hoops of Littlewood and Tokieda and culminating in the most recent damped elastic model of Theron and Maritz in an article which is to appear shortly.

The main feature of this presentation is a video clip of a loaded bicycle wheel rolling down a steep hill and bouncing a number of times, and the simulation of this motion as obtained from the mathematical model.

Numerical solution of ODES

Nick Trefethen (University of Oxford)

Without having planned it, the Chebfun team has found itself in recent years developing what appears to be the most convenient general-purpose tool available for solving ordinary differential equations, both initial- and boundary-value problems. I will say a word about the algorithms we use and demonstrate Chebfun in action. A new Chebfun-enabled book called "Exploring ODEs" is about 75% finished, coauthored with Asgeir Birkisson and Toby Driscoll.

Error estimates for the Galerkin finite element approximation for a linear second order hyperbolic equation with modal damping

Alna van der Merwe (Auckland University of Technology)

In this talk, I consider the Galerkin finite element approximation for a general linear vibration model with damping terms. It is known that solvability results for such a linear vibration model are influenced by the properties of the damping terms. The same holds for the convergence results of the Galerkin approximation.

It is complicated to model the damping that causes energy dissipation. Modal damping, also known as Rayleigh damping, is often a first step in the modeling process. In this case the damping terms consist of a linear combination of viscous type damping (air damping, external damping) and material damping (strain rate damping, Kelvin–Voigt damping, internal damping). An inertia norm error estimate for the semi-discrete approximation is derived, and subsequently, this error estimate is used to derive an inertia norm error estimate for the fully discrete approximation.

The results can be applied to the classical beam and plate models, as well as to models for systems of linked bodies, provided that the assumption of modal damping is satisfied. These error estimates can be applied to the generalised dual-phase-lag hyperbolic heat conduction model as well.

Well-posedness and FEM approximation for hyperbolic type model problems

Nic van Rensburg (University of Pretoria)

It does not come as a surprise that conditions for the existence of a solution of a model problem are virtually the same as conditions for convergence of the FEM approximation. Nevertheless, it does happen that researchers try to improve numerical results in a situation where a solution does not exist.

Only linear problems are considered. First we present general existence and convergence results for hyperbolic type problems – with examples. Next we consider FEM approximation for an ill-posed problem. Lastly, we consider cases where numerical results appear satisfactory but the solution does not have the required smoothness properties or the existence of a solution cannot be proved using known methods.

A mathematical modelling approach for determining cost-effective biological control release strategies against water hyacinth

Cang Hui, Linke Potgieter, and Heléne van Schalkwyk (Stellenbosch University)

The Amazonian water hyacinth, Eichhornia crassipes, is currently regarded as the world's, as well as South Africa's, worst aquatic weed. Man and animal suffer severely under its reign as it forms dense impenetrable layers across the surfaces of dams and rivers. The sustainable and cost-effective management of the water hyacinth weed remains a challenge in South Africa. One of the more successful methods of controlling this determined weed is biological control. A reaction-diffusion model, consisting of a system of delay partial differential equations, is being developed to mathematically describe the water hyacinth's population growth as well as its interaction with populations of the various life stages of the Neochetina eichhorniae weevil as biological control agent in a temporally variable and spatially heterogeneous environment.

The primary objectives are to establish a model which may be used to investigate the efficiency of different release strategies for various temperatures and water hyacinth distributions, providing guidance towards the optimal magnitude, frequency, timing and location of biological control agent releases, and to evaluate the cost-effectiveness of local mass rearing programs of biological control agents compared to the more classical approach of a once-off release after quarantine. This may determine whether the benefit of a faster decrease in water loss is worth the expenses pertaining to mass rearing programs.

Black-Scholes with the Yosida approximation

Gusti van Zyl (University of Pretoria)

Arbitrage constraints impose a natural semigroup structure on European option pricing problems [1, 3]. Usually this semigroup structure reflects as an abstract Cauchy problem. Example 7.4 of [2] can be seen, amongst others, as a demonstration of the advantages of a semigroup approach for numerical approximations, notably avoiding any need for time stepping and using the optimized matrix exponentiatial function of Matlab.

In this work we use the classical Yosida semigroup approximation to obtain numerical approximations. We consider the semigroup generated by a Black-Scholes equation with artificial boundary conditions on a bounded domain. The Yosida approximation, or rather the resolvent, of the infinitesimal generator is then discretized using a Legendre basis for $L_2[0, 1]$. Similarly to [2], the solution is then computed as a matrix exponential of the discretized operator.

In the talk we will report on results and note some advantages and disadvantages of this method.

[1] MB Garman, Towards a semigroup pricing theory, The Journal of Finance, vol. 40, pp. 847–861, (1985).
[2] TA Driscoll and N Hale, Rectangular spectral collocation, IMAJNA, vol. 36, pp. 108–132, (2015).
[3] M Einemann, Semigroup Methods in Finance. Diss. PhD thesis, Universität Ulm, (2008).

Price formation on electronic exchanges

T Gebbie, M Harvey, D Hendricks, and D Wilcox (University of the Witwatersrand)

We report on some stylised facts for the high frequency evolution of prices on an electronic trading platform. Investigation outcomes include the identification of an anomaly in the price impacts of small transaction volumes following a change in the billing model of the Johannesburg Stock Exchange (JSE). We first review evidence for the existence of a master curve for price impact. On attempting to re-estimate a master curve after trading fee reductions, it is found that the price impact corresponding to smaller volume trades is greater than expected relative to prior estimates for a sector of stocks.

Participants

- Richard Olatokunbo Akinola (University of Jos)
- Montaz Ali (University of the Witwatersrand)
- Christopher Angstmann (University of New South Wales)
- Mapundi Banda (University of Pretoria)
- Bruce Bassett (AIMS/SAAO/UCT)
- Folkmar Bornemann (TU Munich)
- Joseph F Botha (University of the Free State)
- Charlene Chipoyera (University of the Witwatersrand)
- Peter Clarkson (University of Kent, UK)
- Kathy Driver (University of Cape Town)
- JA du Preez (Stellenbosch University)
- Sonja du Toit (University of Pretoria)
- David Elago (University of the Western Cape & SAMS)
- Eyaya Birara Eneyew (Stellenbosch University)
- Inger Fabris-Rotelli (University of Pretoria)
- Adewunmi Fareo (University of the Witwatersrand)
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- Nick Hale (Stellenbosch University)
- Michael Harvey (University of the Witwatersrand)
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- Sarah Jane Johnston (University of South Africa)
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- Abey Kelil (University of Pretoria)
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- Alexander Kurganov (Tulane University)
- Madelein Labuschagne (University of Pretoria)
- Pietro Landi (Stellenbosch University)
- Guillaume Latombe (Monash University)
- Dirk Laurie (Stellenbosch University, AIMS, and LitNet)
- Vusi Magagula (University of KwaZulu-Natal)
- Khaphetsi Mahasa (SACEMA, Stellenbosch University)
- Elizabeth Mansfield (University of Kent)
- Milton Maritz (Stellenbosch University)
- David Mason (University of the Witwatersrand)
- Woinshet Defar Mergia (University of the Western Cape)
- Sandile Motsa (University of KwaZulu-Natal)
- Samuel Nuugulu (University of the Western Cape)
- Ulrich Paquet (Apple)
- Beatrice Pelloni (University of Reading)
- Andriamihaja Ramanantoanina (AIMS & Stellenbosch University)
- Daya Reddy (University of Cape Town)
- Carl Rohwer (Stellenbosch University)
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- Jan Snyman (University of Pretoria)
- Belinda Stapelberg (University of Pretoria)
- Antoine Tambue (AIMS & University of Cape Town)
- Wille Theron (Stellenbosch University)
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- Alna van der Merwe (Auckland University of Technology)
- Nic van Rensburg (University of Pretoria)
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- Nick Trefethen (University of Oxford)
- Gusti van Zyl (University of Pretoria)
- André Weideman (Stellenbosch University)
- Diane Wilcox (University of the Witwatersrand)
- Zhi-Xue Zhao (University of Pretoria)