

## 28th Annual Conference

of the

## South African Society for Numerical and Applied Mathematics

hosted at the

## University of Stellenbosch

5 – 7 April 2004

## PROGRAM 2004

	Monday		Tuesd	lay	Wednesday		
08:00-08:50	REGISTRATION						
	E	202	E202		E202		
Chair:	$(L\epsilon)$	aurie)	(Weider	(Weideman)		(Goosen)	
08:30-08:50	08:50 Oper	ing Remarks	Ödm	Ödman		Krämer	
09:00-09:50	K	ress	Tokieda		Fornberg		
10:00-10:30	Т	ΈA	TEA	TEA		TEA	
	E	202	E20	2	E202		
Chair:	(D	river)	(van Ren	sburg)	g)		
10:30-11:20	Beardon		Schoor	nbie	SANUM AGM		
	E202	E205	E202	E205	E202	E205	
Chair:	(Abelman)	(Banasiak)	(Anguelov)	(Witten)	(Murray)	(van der Merwe)	
11:30-11:50	Davison	Davison Van Rensburg		Lett	Bora	Hendrickx 2	
12:00-12:20	Kara	Van der Merwe	Snyman	Mugwagwa	Abelman	Banasiak	
12:30-14:00	LUNCH		LUNC	CH	LUNCH		
	E202	E205	E202	E205	E202	E205	
Chair:	(Maritz)	(Rohwer)	(Herbst)	(Schoombie)	(Snyman)	(Kara)	
14:00-14:20	Van der Walt	Hendrickx 1	Muller 1	Erasmus	D Laurie	Manale	
14:30-14:50	Anguelov Hacking		Van Rooyen 1	Bouguerra	Rohwer	Santos	
15:00-15:20	Minani	Yeh	Muller 2	H Laurie	Sayomi	Hili	
15:30-16:00	TEA		TEA	Α			
	E202	E205	E202	E205			
Chair:	(Weideman)	(van Rensburg)	(Herbst)	(Schoombie)			
16:00-16:20	Driver	Driver Momoniat		Bashier			
16:30-16:50	Jordaan Muite		Van Rooyen 2	Witten			
17:00-17:20	Johnston	Tladi	Yadav	Tchuenche			

Special Session I: Patient Positioning for Proton Therapy, Tuesday 14:00–17:30, E202 Special Session II: Biomathematics, Tuesday 11:30–17:30, E205

#### Registration, tea breaks, lunches

Registration is on Monday, 5 April, from 8:00 to 8:50 in the tea room, next to room E202 (see map at the back of the program). Then proceed to E202 on the second floor, where all the plenary talks will be held. All tea breaks will also be held in the tea room, next to room E202. Lunch will be taken in The Workshop, Merriman Street (see map at the back).

#### **Conference Dinner**

The conference dinner is on Tuesday evening at Le Pommier Restaurant, on the Helshoogte road. Transport will be arranged and will leave at 18:30 from the parking area below the Applied Mathematics department, to arrive at the restaurant at 19:00.



#### Thursday outing

Depending on the number of people who are interested, there will be a tour to Franschhoek on Thursday, 8 April 2003 (the day after the conference). The main feature of this tour would be a wine tasting, after which we can amble into Franschhoek to find lunch at one of the many restaurants in this old French town.

The cost of the outing will be the cost of lunch and approximately R50 for transport, depending on how many people take advantage of this opportunity. Most wine estates also charge a small fee for tasting.

#### Email

Email and internet access is available in room A218, Building 39 (see map). Login and passwords will be provided by the organisers.

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## Numerical Solution of Emden's Equation

Shirley Abelman

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The numerical solution of Emden's equation  $y'' + (2/x)y' + y^n = 0$  is considered for different values of n.

#### Hausdorff continuous functions and the solution of PDEs through the order completion method

Roumen Anguelov

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The concept of Hausdorff continuity of interval valued functions was originally developed within the Approximation theory and is linked to approximations with respect to the Hausdorff distance between the graphs of the functions. It generalizes the concept of continuity of real valued functions using a minimallity condition with respect to inclusion of graphs. The Hausdorff continuous interval valued functions are not unlike the usual real valued functions. For instance, they assume real values on a dense subset of the domain and are completely determined by the values on this subset. It turns out that these functions can closely be linked to the study of nonsmooth functions in Real Analysis. Their application is based on the order completeness or Dedekind order completeness properties of the classes discussed in the talk. We can recall that the usual spaces of real valued functions considered in Analysis or Functional Analysis, e.g. spaces of continuous functions, Lebesgue spaces, Sobolev spaces, are with very few exceptions not Dedekind order complete. We discuss the Dedekind order completion of sets of continuous functions and the solution of nonlinear partial differential equations through the order completion method. Oberguggenberger and Rosinger showed in 1994 that arbitrary nonlinear PDEs defined by continuous, not necessarily smooth or analytic expressions, have solutions that can be assimilated with Lebesgue measurable functions. This powerful earlier existence results can now significantly be improved with respect to the regularity of solutions by showing that the solutions are in fact Hausdorff continuous.

## Shattering phase transition in applied sciences

#### Jacek Banasiak

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In many mathematical models of applied sciences there occurs a loss of the quantity described by the model that is not explained by the laws of nature used to build it. Typically it is attributed to formation of objects that are beyond the "resolution" of the model, e.g. creation of a dust of small isolated particles or, on the contrary, creation of a gel of very large paricles. We shall provide an analytic theory of this this phenomena by relating them to the characterization of the generator of the associated dynamical system and discuss a number of applications ranging from polymer degradation through solid drugs break down in organisms to population dynamics and phytoplankton evolution.

### Computation and Synchronization in Complex Dynamical Networks

Eihab Bashier

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In the past few years, the discovery of small-world and scale-free properties of many biological networks has stimulated increasing interest in further studying the underlying organizing principles of various complex networks. This has led to significant advances in better understanding the relationship between the topology and the dynamics of such networks. We have examined the recent dynamical models of regulatory networks and explored the computational abilities of these dynamical networks of different network topologies.

## Random iterations of contraction mappings

#### Alan Beardon

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The familiar Contraction Mapping Theorem is that if a map f decreases distances by at least a fixed factor k, where k < 1, then the orbits under the iterates  $f^n$  of f converge to a fixed point of f. A contraction is a map that decreases distances, but not necessarily by a fixed amount. What can we say about iterates of a contraction? What can we say about 'random iteration'  $f_1 \ldots f_n$ , where the  $f_i$  are chosen (perhaps randomly) from a given family of contractions? In this expository talk, I shall discuss some of the results of this type that occur in different branches of mathematics.

### What are the long-term outcomes of a disease modifying (non-sterilising vaccine?

Latifa Bouguerra

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Studies of HIV vaccines in animal models suggest that it is difficult to induce complete protection from infection (sterilising immunity). Several vaccines have been shown to have beneficial effect by reducing the viral load following infection and slowing or preventing disease progression. Shortcomings such as short duration of vaccine protection and the evolution and trasmission of escape mutant virus may reduce the efficacy of disease modifying HIV vaccines. We have developed on a mathematical model of a disease modifying HIV vaccine that incorporates both the intra-host and inter-host dynamics of infection in order to investigate the long-term outcome of a disease modifying vaccine. Our preliminary results suggest that the vaccine induced viral load reduction in infected individuals and any changes on sexual risk behaviour will have the strongest impact on epidemic outcome over the first 25 years after introduction of a vaccine. In addition, we can use the model to hypothesise on population structure effects on the long-term outcome of vaccination.

### Hydrodynamic Aspects of Diffraction of Water Waves by a Submerged Spherical Body

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Laplace's equation is of immense application in fluid dynamics in general and in water wave problems in particular. The forces exerted by the surface waves on a structure in water are very important for designing these structures. Accurate prediction of wave loads becomes indispensable in order to design safe structures. The researchers have been trying to evaluate the various loads and coefficients associated with the interaction of water waves with geometries like submerged sphere, circular cylinder, elliptic cylinder and caissons. We consider a submerged sphere of radius a in ocean water of finite depth d. The fluid flow is assumed incompressible, homogenous, inviscid and irrotational. Theflow being of potential kind, a boundary value problem, in which Laplace's equation is the governing equation, is set up. For small amplitude waves, solving of Laplace's equation subject to the bottom boundary condition, body boundary condition and the free surface conditions gives a good understanding of the problem. The use of linear diffraction theory is justified for most of the water wave problems. Knowing the solution for the velocity potential  $\phi$ is instrumental in the evaluation of hydrodynamic coefficients and loads on the sphere which has a lot of significance in designing the structure. Analytical expressions for the incident potential, diffraction potential and exciting forces due to the effects of diffraction arising out of the interaction of water waves are derived by solving the BVP. Theory of multipole expansions is used in obtaining the velocity potential in terms of an infinite series of associated Legendre polynomials with unknown coefficients. The orthogonality of associated Legendre polynomials renders the problem to a much simplified one. Two translational motions, namely surge and heave motions, are considered. Numerical results for the exciting forces are presented in tabular form for various depth to radius ratios.

## Symmetries Using Lie Derivatives and Forms

#### Alexander Davison

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An alternative method, originally devised by B. Kent Harrison and Frank B. Estabrook, for calculating Lie symmetries of DEs is presented, involving Lie derivatives of differential forms. The method is modified and extended so that other types of symmetry may be calculated as well, including potential symmetries, approximate symmetries and Noether symmetries.

## Linear combinations of orthogonal polynomials

#### Kathy Driver

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It is well known that if  $p_0, p_1, \ldots$  is a sequence of real monic polynomials, with  $p_n$  of degree n, that is orthogonal with respect to a Borel measure  $\mu$  supported on an interval  $I(\mu)$  in  $\mathbb{R}$ , then the n zeros of  $p_n$  are real, distinct and lie in  $I(\mu)$ . Further,  $p_n$  and  $p_{n+1}$  have no common zeros and between any two adjacent zeros of  $p_{n+1}$  there is a single zero of  $p_n$ , usually called the interlacing property.

We consider the interlacing properties of the zeros of some linear combinations of orthogonal polynomials. We prove interlacing properties for the zeros of  $\hat{p} = a_s p_s + \cdots + a_m p_m$ ,  $a_s a_m \neq 0$ ,  $s \leq m \leq n$  and s < n with those of  $p_n$  in the cases when  $\hat{p}$  and  $p_n$  have common zeros and when they do not. We also consider the interlacing of the zeros of the polynomials  $ap_n + bp_{n+1}$  and  $a_s p_s + \cdots + a_{n+1} p_{n+1}$  and we show that if  $ad - bc \neq 0$ , the polynomials  $ap_n + bp_{n+1}$  and  $cp_n + dp_{n+1}$  have no common zeros and their zeros are interlaced.

# Network models, spatial structure and population genetics

Louise Erasmus

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Network statistics together with individual-based modelling can be employed to demonstrate the genetic consequences of spatially structured and interacting populations. We employ such a model to demonstrate that low dispersal rates and long dispersal intervals reduce genetic diversity. Connected populations retain greater genetic diversity but were subject to a loss of their genetic integrity. In contrast, increased distances between populations promote genetic integrity, but result in decreased genetic diversity. The correlation between genetic composition and distance, caused by distance-dependent dispersal, only arose in poorly- and well-connected populations. In moderately connected populations, the number of potential dispersal routes obscures the effect of distance on genetic exchange. The combination of network statistics and individual based modelling used here provides a unique tool for exploring the adaptive and evolutionary implications of species and community spatial patterns.

# Radial Basis Functions: A new way to solve PDEs to spectral accuracy on irregular domains.

Bengt Fornberg

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Radial basis functions (RBFs) originated in the 1970s as a method for interpolating scattered data. More recently, both our knowledge about RBFs and their range of applications have grown tremendously. They easily generalize to multiple dimensions, handle irregular domains, and can be spectrally accurate. We will discuss the basics of RBF interpolation, different aspects on the flat basis function limit, and some computational algorithms for the resulting linear systems. We will conclude by comparing an RBF approach with some other methods for the numerical solution of Poisson's equation.

[Mon, 14h30, E205]

### The Syntax of Colour Codons

Ron Hacking

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In theory, at least, optical computers offer many advantages over their electronic counterparts, especially in the space environment. They are at least ten times faster, with a much greater information storage and carrying capacity; they require less energy to drive them; photon paths can intersect without interference, making them more suited to parallel processing; photonic devices deteriorate at a slower rate than electronic ones, especially in the solar wind; and, from a security viewpoint, it is almost impossible to monitor a light beam without corrupting it, whereas conventional communications are much more vulnerable to interception.

This paper focuses on another advantage. Employing Werner Gitt's 5-level information hierarchy – statistics, syntax, semantics, pragmatics and apobetics – and recognizing that Shannon et al has said all that needs to be said about bits at the statistical level, we propose the use of base-4 quits, or colour codons, to attack the second level, that involving syntax. It is only when we can successfully incorporate a syntactic template into the memory and processing elements that we can even begin to think about installing a level-3 semantic dimension into the machine.

We shall reveal the richness of information structures coded in quits, pointing out the generation of a wealth of super-Pascal strata by elementary finite-state Turing machines, which make use of the primary colours, R, G and B; plus X, for black. Whilst studiously avoiding the pitfalls of group theory, we shall nevertheless show how letters, words, sentences, paragraphs ... all the way up to semantically-devoid 'stories' can be easily generated mathematically, then interpreted linguistically in terms of grammatical rules and 'parts of speech'. If time permits, we will attempt to justify our claims by introducing Zipf plots and Shannon entropy orders. Unless we go mad first!

### Vector Fitting: a popular iterative least-squares rational approximation technique

Wouter Hendrickx\*, Tom Dhaene, Frans Arickx and Jan Broeckhove

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In System Theory, it is common practice to approximate the frequency domain response of a Linear Time-Invariant (LTI) system by a rational pole-zero function. Finding such an approximation is inherently a difficult problem due to the non-linearity of the approximant. To remove the non-linearity, the denominator is often fixed at some well-chosen polynomial or the system is linearized in some way. Of course, this can degrade the quality of the approximant, or can even make accurate approximation impossible. A few years ago, Gustavsen and Semlyen (see references) introduced an iterative technique, called Vector Fitting (VF), to construct rational approximations based on multiple frequency domain samples. VF is nowadays widely investigated and used in the Power Systems and Microwave Engineering communities. Numerical experiments show that VF has favorable convergence properties. However, so far, no theoretical proof for its convergence, or conditions to guarantee convergence, have been published. VF consists of an iterative pole relocation scheme. In each iteration step a linear least-squares problem is solved, to come up with more accurate approximations of numerator and denominator. New estimates of the poles are based on the approximations of the previous iteration. In this contribution we position the VF technique in a broader least-squares rational approximation framework. This way, we want to facilitate further exploration of the theoretical properties of the VF technique. Furthermore, we offer some insight into the initial choice of pole locations of the VF algorithm and into the conditioning of the systems that need to be inverted. References Gustavsen, Bjorn and Semlyen, Adam, Rational approximation of frequency domain responses by vector fitting, IEEE transactions on power delivery, Vol. 14, No. 3, July 1999 Semlyen, Adam and Gustavsen, Bjorn, Vector fitting by pole relocation for the state equation approximation of nonrational transfer matrices, Circuits Systems And Signal Processing, Vol. 19, No. 6, 549-566, 2000

[Wed, 11h30, E205]

### Modeling Complex LTI Systems using Multi-Precision Floating-Point Arithmetic

Wouter Hendrickx\*, Dirk Deschrijver, Tom Dhaene, Jan Broeckhove, Frans Arickx

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Complex Linear Time Invariant (LTI) systems can be well approximated in the frequency domain by rational pole-zero models. Most fitting techniques used to calculate the coefficients of rational models have some major drawbacks. The use of continued fractions as interpolants is a computationally efficient method, since it provides the ability to precisely specify the frequency response at particular frequencies. However it can cause widely varying responses at intermediate frequencies. Moreover, this technique is not guaranteed to work in all cases, and it even breaks down if the number of samples gets quite high. Recursive interpolation schemes such as Bulirsch-Stoer or algorithms of the Neville type are computationally less efficient, especially when a large number of data points are needed, since the model coefficients can no longer be calculated in a straightforward way. Solving the coefficients with a least-squares approach is, seen from a numerical point of view, even worse, since it requires the inversion of a Vandermonde-like matrix. However, this technique has the advantage that a desired physical behavior can easily be imposed, by only allowing poles and zeros to occur in complex conjugate pairs. In this paper, orthogonal fitting techniques are used to make the set of normal equations best conditioned, by expanding the numerator and denominator polynomial in a generalized Forsythe orthonormal basis. Numerical accuracy issues are resolved by fitting the models in a multi-precision floating-point arithmetic environment, which combines most advantages of the techniques mentioned above and avoids the use of splines.

# Hellinger distance estimation of nonlinear dynamical systems

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The present work deals with the minimum Hellinger distance (MHD) parameter estimation of nonlinear dynamical systems with highly correlated residuals. In order to estimate the parameter of interest, we fit the high correlated residuals by an EXPAR (EXponential AutoRegressive) time series model. Under some assumptions which ensure the stationarity, the existence of the moments of the stationary distribution and the strong mixing property of the fitted residuals, we establish the almost sure convergence and the asymptotic normality of the MHD estimates.

#### Quasi-orthogonality and zeros of some $_{3}F_{2}$ hypergeometric polynomials

Sarah Jane Johnston<sup>\*</sup> and Kathy Driver

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The properties and location of the zeros of hypergeometric polynomials are linked with those of the classical orthogonal polynomials in some cases, notably  ${}_{2}F_{1}$  and  ${}_{1}F_{1}$  hypergeometric polynomials which have been extensively studied. In the case of  ${}_{3}F_{2}$  polynomials, less is known about their properties, including the location of their zeros, because there is, in general, no direct link with orthogonal polynomials. We consider two classes of  ${}_{3}F_{2}$ hypergeometric polynomials, each of which has a representation as a sum involving  ${}_{2}F_{1}$ polynomials. A quasi-orthogonality result is proved for the class  ${}_{3}F_{2}(-n, a, b; a - 1, d; x)$ ,  $a, b, d \in \mathbb{R}, n \in \mathbb{N}$  and the location of its zeros is discussed. Based on numerical evidence generated by Mathematica, we make a conjecture on the orthogonality properties of this class of polynomials. We prove (quasi) orthogonality of the factors of the class  ${}_{3}F_{2}(-n, b, \frac{b-n}{2}; b-n, \frac{b-n-1}{2}; x), b \in \mathbb{R}, n \in \mathbb{N}$ , and deduce the location and movement of its zeros as the parameter b varies.

# Zero location and asymptotic zero distribution of several classes of $_{3}F_{2}$ hypergeometric function

Kerstin Jordaan

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We establish the location of the zeros of certain families of  ${}_{3}F_{2}$  hypergeometric polynomials that admit representations as various kinds of products involving  ${}_{2}F_{1}$  polynomials. In several cases, the zeros of the  ${}_{2}F_{1}$  polynomials in these products interlace. We also investigate the asymptotic zero distribution of families of  ${}_{3}F_{2}$  functions that are connected in a formulaic sense with Gauss hypergeometric polynomials. We analyse the weak asymptotics of the polynomial  ${}_{3}F_{2}(-n, n+1, 1/2; b+n+1, 1-b-n; z)$ .

## Symmetries, conservation laws and reduction of pdes

#### A H Kara

#### School of Mathematics and Centre for Differential Equations, Continuum Mechanics and Applications, University of the Witwatersrand kara@maths.wits.ac.za

The method of 'invariants' to analyse differential equations (d.e.s) is now commonly used; in most cases the invariants referred to are a consequence of conserved forms of the d.e. These provide a way of reducing the d.e. which in the case of partial d.e.s (p.d.e.s) may mean a decrease in the number of independent variables or, as in the case of ordinary d.e.s (o.d.e.s), refers to the order of the d.e. Evolution type equations do not admit Lagrangians which is somewhat unfortunate as the existence and knowledge of a Lagrangian makes the task of finding invariants easier via Noether's Theorem. Nevertehless, we show that some of these and other equations (like Burger's equation which models shock wave phenomena and the KdV equation which models shallow water behaviour) may still be analysed using the Lagrangian methods.

#### Exception Free Interval Computations and Closed Interval Systems

Walter Krämer

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Reliable numerical computations are most frequently based on interval computations. So called enclosure methods are able to compute automatically bounds for true solution sets. E. g. linear and nonlinear systems of equations can be solved numerically with mathematical rigour. The coefficients of the systems are allowed to vary within prescribed intervals (due to errors in measurements, rounding errors, approximation errors and so on the coefficients are in general no point data). In the talk we will discuss extended interval systems which can be used to perform interval computations even in cases where interval operations or interval functions are to be computed over arguments containing points outside the domain of the underlying point operations or point functions. Division by an interval containing zero or the computation of the square root of the interval [-1,2] are possible in such systems without throwing an exception. In contrast to the ordinary interval system the extended interval systems are closed and exception free. An extended interval system based on containment computations is implemented in the filib++ class library. Some applications will demonstrate the power of such an approach. Links to original work on containment sets, value sets, and so on as well as links to available software will be given.

### Recent developments in the numerical solution of inverse obstacle scattering problems

Rainer Kress

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For the approximate solution of the inverse obstacle scattering problem to reconstruct the boundary of an impenetrable obstacle from the knowledge of the far field pattern for the scattering of time-harmonic waves within the last decade a number of new reconstruction algorithms has been developed, analyzed and implemented. Roughly speaking one can distinguish three groups of methods. **Iteration methods** interpret the inverse obstacle scattering problem as a nonlinear ill-posed operator equation and apply iterative schemes such as regularized Newton type or Landweber methods for its solution. These methods yield the best reconstructions. However, they require the solution of the corresponding forward problem in each iteration step and a priori information on the geometry of the obstacle. **Decomposition methods**, in principle, separate the inverse problem in an ill-posed linear problem to reconstruct the scattered wave from its far field pattern and the subsequent determination of the boundary of the scatterer from the boundary condition. These methods do not require the solution of the forward problem and some of them perform well without a priori information on the geometry of the obstacle. Typical representatives of this approach are the potential method of Kirsch and Kress and the point source and multipol methods due to Potthast. Finally, the third group consists of the more recently developed sampling and probe methods such as the sampling method of Colton and Kirsch, the factorization method of Kirsch and the probe methods of Ikehata and of Potthast and Sylvester. These method perform well without any a priori information on the geometry. However, in general, they require more data than methods of the two other groups. The talk will give a survey by describing in detail one representative of each group including a discussion on the various advantages and disadvantages.

## Calculation of Kronrod-Radau and Kronrod-Lobatto quadrature formulas

Dirk Laurie

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A method for computing Kronrod extensions of *n*-point Radau and Lobatto rules in  $O(n^2)$  operations is presented. The method is applicable to any weight function for which enough three-term recursion coefficients are known.

### Critical discussion of mathematical models for spatial scaling laws in ecology

Henri Laurie

Department of Mathematics and Applied Mathematics, University of Cape Town Tony Rebelo and Walter Smit National Botanical Institute

The so-called species-area curve for number of species N in an area A

$$N = cA^z$$

is a typical example of a spatial scaling law for a density. Such laws purport to hold for spatially coherent subsets of a given domain, and occur in physics (phase transitions, for example) as well as ecology. I show some recent results based on Protea Atlas data (hence the collaboration with Tony Rebelo and Walter Smit of the National Botanical institute), which demonstrate spatial variability and surprising spatial signal in spatially referenced estimates of the parameters. This contradicts the assumptions under which the parameters were estimated. I also present a derivation of (SA) based on a similarity assumption, and interrogate that assumption. These results suggest that scaling laws might require assumptions stronger than can be met by spatial data from ecological studies.

### Continuous Cycling of Grouped vs. Solitary Strategy Frequencies in a Predator-Prey model

Christophe Lett\*, Pierre Auger and Jean-Michel Gaillard

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We present a model of predator and prey grouping strategies using game theory. As predators respond strategically to prey behavior and vice versa, the model is based on a co-evolution approach. Focusing on the "many eyes - many mouths" trade-off, this model considers the benefits and costs of being in a group for hunting predators and foraging prey: predators in a group have more hunting success than solitary predators but they have to share the prey captured; prey in a group face a lower risk of predation but greater competition for resources than lone prey. The analysis of the model shows that the intersections of four curves define distinct areas in the parameter space, corresponding to different strategies used by predators and prey at equilibrium. The model predictions are in accordance with empirical evidence that an open habitat encourages group living, and that low risks of predation favor lone prey. Under some conditions, continuous cycling of the relative frequencies of the different strategies may occur. In this situation, the proportions of grouped vs. solitary predators and prey oscillate over time.

## Formal Software Development Using SPIN

**Deneys** Maartens

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Formal methods of software development have been used in the development of critical systems for some time, but it remains unpopular. This is mostly due to the extensive mathematical skills required for the construction of the verification model and the large amount of overhead that is associated with the implementation of formal verification methods. Thanks to advances in theorem proving technology and the availability of substantial processing power, automated validation is now a practical possibility. We present an introduction to implementing such a model based approach for the development of the new robot-based patient positioning system at iThemba LABS.

## On integration of linear and nonlinear differential equations

J.M. Manale

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An alternative approach to the standard group method procedure for differential equations is proposed and tested on one of the simplest, yet difficult to solve analytically: the Airy equation. The resulting solutions are compared with those obtained through the series approximation expansion method and the Airy functions that arise through WKB approximation method. The foundation on which this alternative procedure is hinged, is that, instead of resolving the determining equation into individual monomials called defining equations for infinitesimals, rather transform the determining equation into a well-posed Cauchy problem, possible through the integrating factor criterion, then solve for the integrating factor or symmetries using Hadamard's theory for Cauchy problems, which also broadly extends to Dirichlet, Neumann and Tricomi problems.

# Hausdorff continuous solutions of Hamilton-Jacobi equations

Froduald Minani

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We consider the Hamilton-Jacobi equation with discontinuous initial data. Following the general viscousity approach we define a weak solution in the set of Hausdorff continuous functions.

## Similarity Solutions for Non-Unifrom Surface Tension Driven Spreading of a Thin Film

E. Momoniat

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In this paper we investigate similarity solutions admitted by a second-order nonlinear partial differential equation modelling the non-uniform surface tension driven spreading of a thin film. The effects of gravity are included in our model. We consider the cases where surface tension dominates; gravity dominates; and where gravity and surface tension effects balance. The resulting system of ordinary differential equations are solved numerically subject to the required boundary conditions.

## Modelling the dynamics of CTL epitope dominance in HIV infections

Tendai Mugwagwa

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Gareth Witten Department of Mathematics and Applied Mathematics, University of Cape Town. gareth@maths.uct.ac.za

Cytotoxic T lymphocytes (CTLs) play a key role in immune control of HIV early in infection. It has recently been shown that the location, timing and order of immune responses to different regions of the HIV proteome are correlated with clinical indicators of disease progression. During the course of infection the epitopes to which an infected individual mounts the strongest immune responses shift with time. We have modelled the host and viral factors that determine the immune response as well as the rate at which dominant epitopes are replaced in order to gain a better understanding of the implications of these processes for disease progression. We anticipate that this work will help to inform vaccine design efforts that are directed at stimulating strong and effective CTL responses early in infection.

### The Secondary flow in a short aspect ratio cylindrical lid driven cavity at small but finite Reynolds number

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In the cylindrical lid driven cavity, a viscous incompressible Newtonian fluid is driven by the rotating motion of an end wall. At low Reynolds number, the end wall drives a primarily azimuthal flow with a weak axial and radial vortical motion. As the Reynolds number is increased, the secondary flow vortex breaks down and a new flow pattern is created. An improved understanding of this flow is not only of scientific interest, but has possible applications to breakdown of airplane wingtip trailing vortices and in mixing. In this study, an analytical regular perturbation solution for the Stokes flow and for the first order inertial correction to the Stokes flow is calculated. The results of the calculation show that the vortex breakdown mechanism, a stagnation of the secondary flow by an adverse pressure gradient, is present at small Reynolds number. They also show that the boundary discontinuity between the rotating lid and the stationary sidewalls has little effect on the flow field. The results suggest a qualitative explanation for the shape of the experimentally determined vortex breakdown diagram as the Reynolds number and aspect ratio change.

### An Overview of the Patient Positiong system for Proton Therapy being developed at iThemba Labs

Neil Muller

#### Department of Applied Mathematics, University of Stellenbosch neil@dip.sun.ac.za

Proton therapy is a useful method for treatmennt of various types of tumor and lesion. Due to the high cost of convential gantry systems, itHemab Labs is developing a proton therapy facility based on a fluxed beamline and poisitioning the patient using an industiral robot manipulator. computer Vision techniques are sued to ensure that the patient is accurately positioned. In this presentation, we provide an overview of the project and desfribe how some of the components fit together.

## Calibrating a Stereo Rig and CT Scanner with a Single Calibration Object

#### Neil Muller

#### *iThemba Labs* neil@dip.sun.ac.za

We attach a stereo rig to a CT scanner to track patient motion during a scan. We wish to be able to relate the scan to the observed stereo motion. This requires that we calibrate the stereo rig to the CT scanner. The cameras in the stereo rig must also be calibrated with respect to one another to obtain accurate results. We propose a single calibration object that accomplishes this.

## GPS: Where does Optimization place you?

#### Dana Murray

#### University of the Free State murraydm.sci@mail.uovs.ac.za

Type your abstract here. GPS position fixing is based on pseudorange measurements from GPS satellites to a GPS receiver on or close to the earth's surface. These pseudoranges are combined with an accurate satellite orbit model to obtain model equations in terms of the receiver's coordinates. These model equations are slightly inconsistent, hence they are typically solved with an optimization procedure. The various steps in actually obtaining a GPS position fix are discussed.

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## **Bayesian Statistics and Phenomenology in Cosmology**

Carolina Ödman

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Cosmology is a fast-moving field of research that has entered a new era sometimes referred to as precision or quantitative cosmology. New observations and experimental data are released at regular intervals. Bayesian methods are used on the data to constrain the vast parameter space of cosmological models. In this presentation, I shall describe how this is done and give an overview of recent results. Markov Chain Monte Carlo (MCMC) methods have become the main statistical tool for such analyses. In particular, I shall present a project in which we make use of a phenomenological approach to assess the importance of experimental errors in the cosmological data and inferred physical quantities. A phenomenological parametrisation of the data turns out to uncover systematic errors in the observational data that would otherwise be overlooked.

### Computation of Noise Parameters from the Highest Resolution Level.

#### Carl Rohwer

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It is generally assumed that the highest resolution levels of a Multiresolution Decomposition contains most of the noise present in a measurement. Assuming symmetry for the distribution of the identically ,independently distributed additive noise, we can estimate other parameters like the standard deviation from the first resolution level only. This useful property of a particular Pulse Transform is demonstrated and computational procedures are derived.

[Wed, 14h30, E205]

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### Simulation of the Damage Evolution Problem in Elastoviscoplastic bars Employing the Generic Interface

Santos, F. G. Barbosa, J. M. A. Brito Jr., E. R.

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Computer simulation systems for complex problems using the finite element method usually needs a large number of auxiliary methods such as mesh generation methods, time integration schemes, solution schemes for elliptic, parabolic and hyperbolic equations, solvers for linear and nonlinear algebraic systems, error estimation and adaptation schemes and so on.. Therefore, any numerical investigation of such a problem needs the performance analysis of various combinations of methods, in the search for more efficient and accurate solution procedures. The conventional programming methods consider only lower levels of abstraction, instead of extending the abstraction consideration as far as the level of the simulator programming and configuration – considering the simulator as a methods articulator or as a reconfigurable and reprogrammable workflow. One of the consequences of this practice for complex problems is that the exchange of methods invariably implies in a radical and extensive reprogramming. In this work an algorithm computer representation in the form of directed acyclic graphs – following the pattern GIG – is applied to a reasonably complex problem and its quality is analyzed in what concerns performance and reusability.

### Computation of the Kernel Function; a comparison of Numerical methods and traditional methods

Olawumi Sayomi

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The resistivity Kernel Function, a key function in the direct interpretation of 1-dimensional resistivity sounding curves, is traditionally determined by the convolution of linear digital filters with the apparent resistivity, these are widely used but limited by the bandwidth of the filters which invariably leads to interpolation of the field curve t. Here the Kernel function is determined by numerical integration, a process not entirely new but seldom used due to computation time/accuracy, two schemes are examined here which permits the determination of the Kernel function at any spacing value. Several classes of curves computed using both the traditional method and the numerical procedures are compared for accuracy and computation time, the results interestingly shows that the numerical procedures are a fast and viable alternative to computation by filters.

## Adaptive dynamics: Playing the evolution game.

Schalk Schoombie

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When modelling the evolution of a species in a certain environment, it can be done either on the genetic or the phenotype level. In the latter case, a particular phenotype can be often be represented by a parameter in an appropriate population model. This parameter can be referred to as a strategy. Evolution is then seen as a two-player game between a resident population of which all members use the same established strategy, and a small number of mutants with a different strategy. The mutants win the game when they are able to invade the population. A function with two variables (mutant and resident strategies respectively) serves as an invasion criterion, and its sign indicates whether the mutants can invade or not. This mathematical study of a population with one or more phenotypes (strategies), which change over a period of time, is often referred to as adaptive dynamics. In this talk we discuss some recent developments. We look at the possibility of optimal strategies and its attainability during evolution, which leads to the concept of a singular point, which may be an attractor or a repulsor. The behaviour of the evolution model in the region of a singular point can be entirely characterised by the second derivative of the invasion criterion in terms of the mutant strategy, and by the second derivative of the same function with respect to the resident strategy. Finally we discuss computational techniques, which may be used to calculate some of these singular points.

### A conjugate gradient methodology for the solution of constrained optimization problems with severe noise

Jan Snyman

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A new implementation of the conjugate gradient method is presented that economically overcomes the problem of severe numerical noise superimposed on an otherwise smooth underlying objective function of a constrained optimization problem. This is done by the use of a novel gradient-only line search technique, which requires only two gradient vector evaluations per search direction. This line search method, in which the gradients may be computed by central finite differences with relatively large perturbations, allows for the effective smoothing out of any numerical noise present in the objective function. The proposed new implementation has been tested using a large number of well known test problems. For the initial tests no noise was introduced, and for high accuracy requirements it was found that the proposed new conjugate gradient implementation performed as robust and reliable as traditional penalty function implementations. With the introduction of severe relative random noise in the objective function the results are surprisingly good, with accuracies obtained that are more than sufficient compared to that required for engineering design optimization problems with similar noise. Keywords: conjugate gradient method, numerical noise, line search

# Convergence of a partially discretize scheme of an age-physiology dependent population dynamics.

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We partially discretize the equation of population dynamics of an age-physiological factor dependent model and give conditions under which a second order accuracy arises. A special case, the Kermack McKendrick-Von Foerster type equation is also studied.

#### Adiabatic chaos and transport in quasi-geostrophic baroclinic flows: the role of homoclinic and heteroclinic bifurcations

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Deep understanding of mesoscale and synoptic coherent structures including dynamics of oceanic eddies, atmospheric vortex blocking, rings of the Agulhas Current, and the Gulf Stream ring systems has been based on modons which are nonlinear solitary wave solutions of the quasi-geostrophic potential vorticity equations. Consider prototypical asymptotic model governing dissipative quasi-geostrophic baroclinic flows for eddying current

$$\begin{aligned} \frac{\partial q}{\partial t} + J(\psi, q) &= \frac{Ek}{Ro} \bigtriangleup q + \gamma \rho, \\ \frac{\partial \rho}{\partial t} + J(\psi, \rho) &= \frac{1}{Ed} \bigtriangleup \rho + \lambda q, \\ q &= \bigtriangleup \psi - \frac{\partial^2 \psi}{\partial z^2} + \beta y, \end{aligned}$$

$$u = -\frac{\partial \psi}{\partial y}; \quad v = \frac{\partial \psi}{\partial x}; \quad \rho = -\frac{\partial \psi}{\partial z};$$

where the nondimensional fields  $\underline{u}$ , q,  $\rho$  and  $\psi$  are fluid velocity, potential vorticity, density and streamfunction, respectively. The geophysically relevant parameter Ro is the Rossby number, Ek, the Ekman number,  $\beta$ , the reference reciprocal Coriolis parameter, Ed, the

 $<sup>^1\</sup>mathrm{Research}$  partially supported by Dr Dije DJ Mashao, Department of Electrical Engineering, University of Cape Town

Tladi

eddy diffusivity,  $\gamma$  and  $\lambda$ , small parameters. One of the advantages of the technique for the proof customized for the system of partial differential equations is that it emphasizes an advection form for accounting for the characteristic propagation directions, and it leads in a natural way to the following kinematic problem

$$\begin{aligned} \frac{dX}{dt} &= -\frac{\partial}{\partial Y}\psi(X,Y,z,t),\\ \frac{dY}{dt} &= \frac{\partial}{\partial X}\psi(X,Y,z,t),\\ \frac{X(\alpha,t)_{t=0}}{dt} &= \alpha. \end{aligned}$$

This formulation is desirable from the framework of the theory of adiabatic dynamical systems developed by Tasso Kaper *et al* and Lagrangian transport in baroclinic flows. Since the established baroclinic dissipative quasi-geostrophic system is a hybrid elliptichyperbolic equation the assertion of the proof follows from employing Alberto Valli *et al* convergent iteration scheme to obtain velocity from the elliptic system and then solve the coupled dissipative hyperbolic equations. Given that the operator  $\frac{\partial}{\partial t} + J(\psi, .)$  denotes advection along the fluid particle trajectories, the superposition principle and the technique of characteristics yield well-posedness of solution through the initial value problem. Additionally, the invariant manifolds of the dynamical system,  $\underline{X}(\alpha, t)$ , may be interpreted as the location at time t of a processing tool initially launched from  $\alpha$ . By the virtue of hyperbolic saddle criterion and the courtesy of Melnikov integral, we give existence proof for homoclinic and heteroclinic bifurcations. Illustrating that a simple zero exists for the Melnikov integral provides a proof for existence and persistence of homoclinic and heteroclinic orbits. The Melnikov function can be interpreted as representing the distance between the stable and unstable manifolds for the perturbed system. When these manifolds meet in a single dimension, the first-order terms in the perturbation parameter are adequate, along with the contraction mapping principle of Banach-Cacciopoli.

[Tue, 09h00, E202]

## Soft and hard rolling

Tadashi Tokieda

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Rolling is a very common type of motion we see every day, yet its physics is still illunderstood and its mathematics can be extraordinarily difficult to analyse. I shall show many toy demonstrations of soft and hard bodies rolling and explain some surprises that have been analysed only recently and point out other surprises that nobody understands yet.

#### Convergence structure of the set of Hausdorff continuous functions

Jan Harm van der Walt

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Order convergence can be defined on the set of Hausdorff continuous functions in a very convenient way by using that this set is order complete. More precisely, a net in the set of Hausdorff continuous functions is convergent if the limit inferior and the limit superior (which are guaranteed to exist) are equal. Their common value is called the limit of the net. The order convergence is potentially a very powerful tool for the construction of Hausdorff continuous solutions to PDEs. Hence our interest in it. We will show that the order convergence does not satisfy the Moore-Smith axioms and consequently it can not result from a topology. Furthermore, it also does not satisfy the Kuratowki's axioms for a limit space. However, we will show that the order convergence is closely linked to the Hausdorff metric on the set of Hausdorff continuous functions and also introduces in this set the structure of a filter space.

#### A quadratic eigenvalue problem for a system of differential equations

A J van der Merwe

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We consider a quadratic eigenvalue problem for a system of differential equations

$$\mu^2 u = -u'' + \phi',$$
  

$$\mu^2 \phi = -\frac{1}{\gamma} \phi'' - \alpha u' + \alpha \phi,$$

with boundary conditions

$$u(0) = \phi(0) = 0,$$
  

$$\mu^2 m u(1) + u'(1) - \phi(1) + \mu k_0 u(1) = 0,$$
  

$$\mu^2 I \phi(1) + \phi'(1) + \mu k_1 \phi(1) = 0.$$

It is important to note that the eigenvalue  $\mu$  appears in the boundary conditions.

This eigenvalue problem arises when analyzing the effect of boundary damping on a cantilevered Timoshenko beam with a tip body at the free end. The properties of the spectrum are of significance for the stabilization and control of such a system. In a recent paper it is proved that for a Timoshenko beam (no tip body) the spectrum is discrete and countable and split into two disjoint sets. This two-banded structure of the spectrum has also been observed in empirical studies.

Using the finite element method we show empirically that the two-banded structure of the spectrum is retained for the present system. We also propose an explanation of this two-banded structure by referring to the properties of the eigenfunctions of the associated undamped system.

# Eigenvalue problems for a system of differential equations

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We consider eigenvalue problems for the system of differential equations

 $-u'' + \phi' = \lambda u,$  $-\frac{1}{\gamma}\phi'' - \alpha u' + \alpha \phi = \lambda \phi,$ 

with boundary conditions. There are clearly different possibilities for the boundary conditions. One example that will be discussed is

$$u(0) = \phi'(0) = 0,$$
  
 $u(1) = \phi'(1) = 0.$ 

We show how detailed information of the distribution and multiplicity of eigenvalues can be obtained, using elementary mathematics. It should be noted that some properties of the eigenvalues and eigenfunctions can be obtained by using results from functional analysis. (It is possible to write the eigenvalue problem in the form  $x = \lambda Ax$ , with A a compact linear operator on a Hilbert space.) But there is a limit to the information that can be obtained from the abstract problem.

For at least one of the eigenvalue problems we derive exact solutions and for others we derive transcendental equations for the eigenvalues. The oscillatory properties of eigenfunctions related to consecutive eigenvalues may come as a surprise to many.

The eigenvalue problems under consideration arise from a modal analysis of the Timoshenko model for a beam. Since 1921 this model interested engineers and mathematicians alike and it resulted in the publication of numerous papers right up to 2004. References to old and recent papers will be provided. The results for the Timoshenko model will also be compared to results for other models.

## Using CCD Cameras in High Radiation Areas

#### Ruby van Rooyen

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In radiation therapy an intelligent vision system takes over the role of eyes inside the therapy room when human presence is not allowed. The accuracy and consistency of these vision systems are closely tied to the quality of the images captured. High quality images can easily be obtained with CCD cameras. This is however a catch-22 situation as the CCD chips themselves are highly sensitive to radiation damage. Typical indications of radiation damage is the occurrence of hot pixels. Although no CCD is immune to hot pixels, the occurrence of these pixels increase in direct relation to the radiation damage sustained by the CCD.

## Constructing a Non-Linear Distortion Correction Model

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One of the biggest irritations with the use of commercial zoom lenses is the presence of high degrees of image distortion. These distortions can vary from barrel distortion at the short focal length end to pincushion distortion at longer focal lengths. In the design of an automated distortion correction technique for photogrammetric implementation using commercial zoom lenses, a major objective is ensuring accuracy, efficiency and reliability. The distortion correction are calculated using a look-up table from the corrected pixel values to the input distortion pixel. This look-up table is calculated from an inverse mapping of the distortion correction model, and gives coordinates corresponding to pixel values in the distorted image.

### Balancing HIV-1 production with clearance implies rapid viral clearance in lymphoid tissue

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At the viral set-point, viral production by productively infected cells has to balance viral clearance. Viral clearance rates have been estimated by a variety different experimental techniques and somewhat different estimates have been obtained. In addition, virus is known to be associated with follicular dendritic cells in the lymphoid tissue. We propose a multi-compartmental model in order to estimate the clearance from lymphoid tissue by simulating experimental data and to estimate the clearance from lymphoid tissue during primary infection given a bolus amount of virus.

#### Using PRM Path Planning Methods in an Industrial Robot that is Carrying a Patient for Radiotherapy Treatment

Nikhil Yadav

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A number of methods have deen developed during the 1960s to the 1980s for planning safe paths for a rigid or articulated robot in a field of obstacles. These methods generally become intractable as soon as the complexity of the problem advances beyond using small numbers of robots, or stationary obstacles. The advent of probabilistic roadmap methods, called PRM methods, in the 1990s profoundly changed this situation by allowing the general situation of multiple robots, articulated robots with large numbers of links, and of movable obstacles to become tractable even on modest computing facilities. In this presentation we explain PRM path planning methods. The adaptation of these mehods for use in the specific situation of an articulated industrial robot carrying a patient for radiotherapy treatment is highlighted.

Yeh

#### An Algorithm for the Multi-Stage Supply Chain Network Reliability Problem

Wei-Chang Yeh

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In recent years, many developments in logistics were connected to the need for information in an efficient supply chain flow. The supply chain is often represented as a network (called a supply chain network, SCN) that is comprised of nodes that represent facilities (suppliers, plants, distribution centers and customers). Arcs connect these nodes along with the production flow. A multi-stage SCN (MSCN) is a sequence of multiple SCN stages. The flow can be only transferred only between two consecutive stages. The proposed MSCN problem is to find the reliability of the MSCN after the choice of facilities (plants and distribution centers) to be opened and the distribution network design which satisfied the demand with minimum cost.

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Registration and lectures: between Building 35 & 36 (indicated by a cross)

Lunches: The Workshop on Merriman (indicated by a cross)

Email and internet: 2<sup>nd</sup> floor (eastern wing) of Building 39

**Transport:** to conference dinner and Thursday outing leaves from parking in front of Building 39

**Student center:** with postal and banking services in Building 5