Program & Abstracts



of 30th Annual Conference of



(South African Society for Numerical and Applied Mathematics)

Department of Mathematical Sciences, University of Stellenbosch, South Africa

3, 4, 5 April 2006

Sponsors: The Center of Experimental Mathematics of the University of Stellenbosch.

	Monday 3 April			Tuesday 4 April					Wednesday 5 April				
08:00-08:30	Registration												
08:30-08:50	ʻbst)	8:50 Opening Remarks Borggaard Folkmar Bornemann John Norbury Technical University of Munich Vertical University, United Kingdom Fast Image Inpainting Vertical University, United Kingdom Ideal coordinates for pattern forming reaction diffusion equations of Ginzbe Landau type		bst)	Borggaard				eman)	Cliff			
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10:00-10:30						TEA							
10:30-11:20	Veideman)	Ronald Cools University of Leuven, Belgium Embedded cubature formulas			303A (Norbury)	Kevin Burrage University of Quee Modelling and sim biology	<i>nslar</i> ulati	nd, A on is	Australia ssues in cell	303A (Weideman)	Fernando P University oj Multiwavele Green's Fund fast algorithm	erez f <i>Colo</i> ts, Ga ctions ms for	<i>rado, USA</i> ussians and : A new kind of · PDEs
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12:00-12:20		solving non-linear design optimization problems in engineering			303A (Abelman	303B (F	Maths ir	Kimpolo	303A	Patidar	303B (Nel
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15:30-16:00	TEA												
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17:00-17:20	Dir	Oladele		Sjoberg									

Registration, lectures and tea breaks

Registration is on Monday 3 April 2006 from 8:00 to 8:50 in the Foyer of General Engineering Building (on the corner of Banghoek and Joubert roads). Then proceed to Room 303A where all the plenary talks will be held. All tea breaks will also be held in the Foyer of General Engineering Building.

Lunches

Lunch will be at Die Bloukamer, in the Neelsie student center (marked with a cross on the map) and are included for all regular and student conference participants.

Conference Dinner

The conference dinner is scheduled for Tuesday, 04 April 2006 and will be held at the Devon Valley Hotel. We will leave from the parking in front of the General Engineering Building at 18h00 to arrive at the restaurant at 18h30. All those who needs a lift and those who can offer a lift must please meet in front of the General Engineering Building in time to leave at 18h00.

The dinner is included in the regular registration fee, but not in the student registration fee. However, a few extra dinner tickets are available (for accompanying persons or student registration) at an extra cost of R170 per head.



Email

Email and internet access is available in room A218, 2nd floor (eastern wing) of the General Engineering building (see map). Login and passwords will be provided by the organisers.



Welcome

Welcome to SANUM 2006!

Monday 3 April

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Opening

FAST IMAGE INPAINTING

Folkmar Bornemann (Technical University of Munich, Germany)

Inpainting turns the art of image restoration, retouching, and disocclusion into a computer-automated trade. Mathematically, it may be viewed as an interpolation problem in some fancy function spaces thought suitable for digital images. It has recently drawn the attention of the numerical PDE community, which has led to impressive results. However, stability restrictions of the suggested schemes yield computing times so far that are next to prohibitive in the realm of interactive digital image processing. We address this issue by constructing an appropriate transport equation that combines a fast solver with high perceptible quality of the resulting inpainted images.

The talk will survey the background of the inpainting problem and prominent pde-based methods before entering the discussion of the suggested new methods. Many images will be shown along the way, in parts with online demonstrations.

EMBEDDED CUBATURE FORMULAS

Ronald Cools (University of Leuven, Belgium)

In many situations one is not happy with just one quadrature formula to approximate an integral. One uses pairs, or longer sequences, to obtain error estimates. It is convenient if the different formulas have some points in common; it saves function evaluations. The Gauss-Kronrod-Patterson sequence is the well known illustration of this.

Gauss quadrature formulas are well study and the theory of orthogonal polynomials plays an important role in that. The multivariate setting is more complicate. Generalizing the theory of orthogonal polynomials to cubature formulas didn't work out that well. Work on embedded cubature formulas is very limited. In cubature, many quality criteria are around nowadays and especially quasi-Monte Carlo methods, including lattice rules, received a lot of attention recently.

In the first part of this talk I will survey what is known about constructing embedded cubature formulas that are exact for algebraic polynomials. Not much happened in that area recently. Recently however, a fast algorithm was developed to construct lattice rules for high dimensions and with a huge number of points. The algorithm can easily be adapted to construct embedded lattice rules. The second part is devoted to this

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> recent work. One of the beauties of mathematics is that it can uncover connections between seemingly disparate applications. One of the most fertile grounds for unearthing connections is computational algorithms where one often discovers that an algorithm developed for one application is equally useful in several others. One such algorithm is centroidal Voronoi tessellations (CVTs) which are special Voronoi diagrams for which the generators of the diagrams are also the centers of mass (with respect to a given density function) of the Voronoi

> cells. CVTs have many uses and applications, several of which we discuss. These include data compression, image segmentation, clustering, cell biology, territorial behavior of animals, resource allocation, grid generation in volumes and on surfaces, meshless computing, hypercube sampling, and reduced-order modeling. We also discuss deterministic and probabilistic methods for determining CVTs.

NOVEL GRADIENT-BASED ALGORITHMS FOR SOLVING NON-LINEAR DESIGN OPTIMIZATION PROBLEMS IN ENGINEERING

Jan Snyman (University of Pretoria, South Africa)

Classical gradient-based algorithms are not generally suitable for engineering design optimization problems, because they may involve a very large number of variables, and/or require the evaluation of computationally very expensive objective and constraint functions. The presence of numerical noise and discontinuities in the functions, further complicate the application of standard methods. In addition multiple local optima may exist. This paper reviews a suite of novel gradient-based trajectory and approximation algorithms developed

by the author [1], which allows for overcoming some of the above difficulties. A number of examples of the application of these numerical methods to practical engineering design problems is presented. These include optimal structural design problems, the optimization of multi-body dynamical systems, and the solution of optimization problems in computational fluid dynamics (CFD).

1. Jan A. Snyman: Practical Mathematical Optimization basic optimization theory and classical and new gradient-based algorithms, Springer, Cambridge, Massachusetts, 2005, (ISBN 0-387-24348-8), 257 pages.

PROPORTIONAL SEPARATORS

Carl Rohwer (Department of Mathematical Sciences, University of Stellenbosch) chr@sun.ac.za

A separator S is an axis- and scale independent operator that is idempotent and co-idempotent, or SS=S and (I-S)(I-S)=I-S. This is as near as one can hope to get to a projection if S is nonlinear. Such an operator has only two eigenvalues, 0 and 1. Many important cases are not homogeneous. The simple properties demanded yield some trivial consistencies as well as some curious inconsistencies. Some separators for image processing yield a stronger consistency that is inherited by compositions, and proves that a wide class of Discrete Pulse Transforms (DPTs) acts linearly on the cone generated by the pulse components of a particular sequence. This is useful for a "Mathematics of Vision" as envisaged by Marr.

THE DISCRETE PULSE TRANSFORM IN IMAGE PROCESSING

J.P. du Toit (University of Stellenbosch)

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The LULU-operators and related discrete pulse transform has proven useful in the analysis of one-dimensional signals. Now, the same methods are applied on a real-world image processing problem. In the second harmonic (SH) imaging of PB_x Cd_{1-x} Te ternary alloys, two crystal types are observed: Pb-rich crystals and Cd-rich crystals. Two SH images with differing azimuthal angles are obtained of the same area. The Cd-rich phase comes in two growth directions, which manifests as different strength responses in the two SH images. Combining the information from the two images makes it is possible to identify the crystal types present. The discrete pulse transform was helpful with noise removal, edge detection and determining the crystal types.

ALGORITHMIC IMPROVEMENTS OF THE DISCRETE PULSE TRANSFORM

J.P. du Toit (University of Stellenbosch)

D. Laurie (University of Stellenbosch)

C.H. Rohwer (University of Stellenbosch)

M3

3 The discrete pulse transform has proved a useful tool in the multi-resolution analysis of signals. A naive implementation of the operators involved results in $O(n^2)$ running time. Recently $O(n \log n)$ and O(n) algorithms have been developed. This allows the discrete pulse transform to be useful in many cases where the long running time of the naive algorithm is a hindrance.

Parallel I

ON A-STABLE SYMMETRIC ONE-STEP METHODS OF ORDER FOUR

S.K. Shindin (School of Comp. & Appl. Math., Wits University) G.Yu. Kulikov (School of Comp. & Appl. Math., Wits University)

In the paper we present a new family of one-step methods which are sufficiently accurate. These methods are of the Runge-Kutta type. However, they have only explicit internal stages that leads to a cheap practical implementation. On the other hand, the new methods are of classical order 4 and stage order 2 or 3. They are A-stable and symmetric. All of these mean that they are applicable to solve both nonstiff and stiff ordinary differential equations (including reversible problems) and possess all the necessary practical features making them quite attractive.

VARIABLE STEP SIZE NORDSIECK METHODS: STABILITY AND ORDER REDUCTION PHENOMENON

G.Yu. Kulikov (University of the Witwatersrand, Johannesburg)

S.K. Shindin (University of the Witwatersrand, Johannesburg)

In this paper we study an order reduction phenomenon arising in Nordsieck methods when they are applied to ordinary differential equations on nonuniform grids. It causes some difficulties of using stepsize selection strategies in practical computations. We prove that the problem mentioned above is just a consequence of the fact that the concepts of consistency and quasi-consistency are not equivalent for such methods. Therefore we show how to improve Nordsieck methods with this property in order to avoid order reduction. We consider both explicit and implicit ways of doing this and study zero-stability of modified Nordsieck Adams-Moulton methods. The paper is also supplied with numerical examples which clearly confirm the presented theory.

PRACTICAL TESTS WITH GLOBAL ERROR EVALUATION TECHNIQUES IN MULTISTEP METHODS

A. Ramanantoanina (School of Comp. & Appl. Math., Wits University)

G. Yu. Kulikov (School of Comp. & Appl. Math., Wits University)

In this talk, we present numerical tests for different global error evaluation techniques in multistep methods.
 M4 The algorithms include Richardson extrapolation, using two different methods, solving for the correction,
 Zadunaisky's technique and solving the linearized discrete variational equation. The tests are performed on different grids, covering both uniform and non-uniform ones, and using different test problems with known solutions to be able to compare the global error with its estimate. We discuss their practical performance for weakly and strongly stable multistep methods and make conclusions.

Dirk Laurie's Birthday II

SPURIOUS BEHAVIOUR OF LEAPFROG SCHEMES FOR SOME PER-TURBED HARMONIC OSCILLATORS

Schalk Schoombie (University of the Free State)

Eben Mare (University of Pretoria)

We consider a special leapfrog scheme for the perturbed harmonic oscillator $\ddot{x} + \omega^2 x = \epsilon f(x, \dot{x})$. For the van M5der Pol equation it leads to an interesting but spurious amplitude modulation. This can be explained using 1 a discrete multiple scales techique, combined with a phase plane analysis. For other equations of this type there is no spurious behaviour. It would be interesting to find out what are the conditions on the function fwhich would cause spurious behaviour. In this talk we report the results of some preliminary investigations.

COMPUTING INFINITE RANGE INTEGRALS OF PRODUCTS OF BESSEL **FUNCTIONS**

Joris van Deun (Katholieke Universiteit Leuven) Ronald Cools (Katholieke Universiteit Leuven)

We present an algorithm to compute infinite range integrals of an arbitrary product of Bessel functions of the first kind and real order. The main ingredients in this algorithm are the well-known asymptotic expansion 2 and the observation that the infinite part of the integral can be approximated using the incomplete gamma function. Accurate error estimates are included in the algorithm, which is implemented as a Matlab program.

A NEW FIVE STAGE CONTINUOUS RUNGE-KUTTA METHOD FOR INI-TIAL VALUE PROBLEMS

Fatokun Johnson Oladele (Nasarawa State University, Keffi, Nigeria / AIMS) johnson@aims.ac.za

A five-step implicit Runge-Kutta method is presented in this paper. The continuous scheme for this is formulated using the power series approach. To make the mxn matrix consistent, a perturbation term was introduced for economisation thus leading to a (k+2) by (k+2) system of equations. The method is self-starting, of order five and it is convergent. The error constant is approximately-1/73 which guarantees better accuracy than the conventional Adams methods of the same order. Keywords: Implicit Runge-Kutta methods, Legendre polynomial and functions, power series, perturbation term, economisation, Convergence.

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3

 ${\rm Parallel}~{\rm II}$

SUPPORT TO ROCK EXCAVATIONS PROVIDED BY THIN ADHESIVE LINERS

David P Mason (School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg)

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Shotcrete has been used for support in mines for about fifty years. Recently, the use of thin sprey on liners has increased. The support mechanism for thin sprey on liners is investigated. A cylindrical rock excavation is considered. The fractured rock near the surface of the excavation is modelled as a thin layer with different Young's modulus from that of the surrounding rock. When the liner is applied it penetrates the fractures and returns the rock to approximately its original strength. The excavation is perturbed by shear field and by a tensile field. Using plane strain theory in linear elasticity the stress concentration factor at the surface of the excavation without the liner and with the liner are calculated and compared. It is found that the application of the liner reduces the stress concentration factor.

GEOTECHNICAL STRESS MODELLING USING THE MATERIAL POINT METHOD

Francois Malan (Modelling Unit, ISS International Ltd. Stellenbosch, South Africa) francoism@issi.co.za

The modelling of deformation and stress is of major importance in many fields of enigeering, including research in mining and rock mechanics. Two major approaches for discretizing and modelling a continuum are the Finite Element Method and the Finite Difference Method. We will focus on the Material Point Method (also called the Particle in Cell method), which is a Finite Difference code. The working of the MPM method will be explained, and some examples will be shown of applications in which it is useful. A recently developed constitutive model called IDRM (Integrated Damage Rheology Model), which we will be implementing in MPM, will be briefly described, and some applications discussed.

MODELLING THE PISTON EFFECT OF ROCK FALLS

Dr A Sjoberg (University of Johannesburg (APK campus)) Members of MISGSA2006 studygroup (University of Johannesburg (APK campus))

M6 This talk presents an overview of the results obtained by a studygroup at MISGSA2005. In mining, large caverns often form. When the roof of such a cavern collapse, it causes air blasts in connecting tunnels which

3 caverns often form. When the foor of such a cavern compse, it causes all blasts in connecting tunnels which can cause loss of lives and destroy equipment. We present a few models to describe the air pressure in the cavern during collapse.

Tuesday 4 April

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OPTIMIZATION-BASED METHODS FOR REDUCED-ORDER MODELING

Jeff Borggaard (Department of Mathematics, Interdisciplinary Center for Applied Mathematics, Virginia Tech)

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Reduced-order modeling is a technique for developing low-dimensional models for large- or infinitedimensional systems. For nonlinear problems, the method of choice is projection with the low-dimensional basis obtained using proper orthogonal decomposition of a given set of simulations. Reduced-order modeling has found its way into a number of useful applications including weather forecasting, control theory and understanding fundamental mechanisms in complex flows. There are a number of limitations with the technique, however. It is inherently sensitive to the basis that is used (which also depends on which simulations are chosen). In this talk, we give an overview of reduced-order modeling and the proper orthogonal decomposition (essentially the singular value decomposition). Following this overview, we describe a number of improvements to the traditional basis selection method based on the underlying optimization problem used for traditional proper orthogonal decomposition. We consider a straight-forward optimization problem and a technique known as the priciple interval decomposition.

IDEAL COORDINATES FOR PATTERN FORMING REACTION DIFFU-SION EQUATIONS OF GINZBERG-LANDAU TYPE

John Norbury (Oxford University, United Kingdom)

We look for good meshes on which to solve numerically for patterns with steep moving internal interfaces. The T1meshes are in space, and may evolve in time. Analytical solutions on the slow manifold for the system are also found, as these are attractors for the dynamics, and can be used to test numerical solutions for accuracy.

Robustness of the model is discussed, especially as a key small diffusion parameter tends to zero.

2

Plenary

MODELLING AND SIMULATION ISSUES IN CELL BIOLOGY

Kevin Burrage (University of Queensland, Australia)

The Stochastic Simulation Algorithm (SSA) is a crucial discrete modeling technique for simulating the interactions of small numbers of molecules in cellular environments and in particular genetic regulation. In this talk we discuss modifications to the SSA that will allow us to model external noise with respect to genetic regulation (through errors in transcription), delays in transcription and translation and address 1 multiscale issues associated with spatial heterogeneity (due to spatial ultrastructure within a cell). We will also attempt to give a brief overview of how noise arises and is modelled in a Cell Biology setting.

T2

Tuesday 4 April

A LOG LIKELIHOOD GRADIENT EVALUATION BY USING THE EX-TENDED SQUARE-ROOT INFORMATION FILTER

M. V. Kulikova (University of the Witwatersrand, Johannesburg) *mkulikova@cam.wits.ac.za*

T3 A newly developed algorithm for evaluating the Log Likelihood Gradient (score) of linear discrete-time dynamic systems is presented, based on the extended Square-Root Information Filter (eSRIF). The new result can be used for efficient calculations in gradient-search algorithms for maximum likelihood estimation of the unknown system parameters. The theoretical results are given with the examples showing the superior performance of this computational approach over the conventional one.

APPLICATION OF STATISTICAL CONTRASTS TO MEAN REFRACTIVE STATE

Dr Shirley Abelman (University of the Witwatersrand, Johannesburg) abelman@cam.wits.ac.za

Т3 2 Testing for differences between specific groups or combinations of groups is referred to as comparison or contrast testing. Statistical significance of comparisons can be assessed by first forming contrasts and then testing for their significance. A contrast essentially tests whether or not two means are significantly different, where each mean could be a weighted average of two or more means. The aim of this talk is to introduce mutually orthogonal linear contrasts of sample data to optometric research. By constructing particular contrasts, mean refractive states are compared before instillation, before and after instillation, and after instillation of a cycloplegic into the right eye of a subject.

 ${\rm Parallel}~{\rm I}$

SWARMING MODELED BY PARTIAL DIFFERENTIAL EQUATIONS

Paul Milewski (Department of Mathematics, University of Wisconsin)

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We introduce a new continuum model for swarming behavior in organisms. The model is written as a conservation law for the density of organisms that includes a nonlocal, anisotropic aggregation term (modelling the motion of the organisms due to a directionally biased sample of the nearby population) and a local nonlinear diffusion term (modelling the repulsion of organisms in regions of very high density). Numerical and asymptotic analysis shows a wealth of dynamics including compactly supported travelling waves (swarms). The speed of the swarm is bounded above and decreases as the total mass of the swarm increases. These swarms are singular at both their leading and trailing edge, and this has to be treated with care in numerical computations. Although the work considers a one-dimensional one-way model, we point out how to extend it to more physical situations.

DETERMINISTIC DYNAMICS IN QUESTIONNAIRES IN SOCIAL SCI-ENCES

Professor David Sherwell (School of Computational and Applied Mathematics, University of the Witwatersrand)

Charles Lebon Mberi Kimpolo (School of Computational and Applied Mathematics, University of the Witwatersrand)

We present models to translate a questionnaire into a sequence of "Yes/No" replies, or "zero/one" digits. Such a sequence is a point in a mathematical sequence space. A longitudinal questionnaire is then a point that evolves in that space. Distances can be defined and the point jumps in this space. We have developed a mathematical apparatus to analyse this abstract motion and to map the abstract space to the real line and other planes. If the motion is random then one can apply a diffusion equation. We will apply these ideas in demographic studies. To illustrate, migration is not well-modelled by a diffusion process because individuals do not random walk, geographically. Yet their social conditions as revealed by a questionnaire may random walk. Then the deterministic equation of applied mathematics might apply. As formulated above, we will extend the formulation and apply it to real demographic data.

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MESH INDEPENDENCE OF KLEINMAN-NEWTON ITERATIONS FOR **RICCATI EQUATIONS**

Lizette Zietsman (Department of Mathematics, Interdisciplinary Center for Applied Mathematics, Virginia Tech) lzietsma@vt.edu

In this talk we consider the convergence of the infinite dimensional version of the Kleinman-Newton al-T5gorithm for solving the algebraic Riccati operator equation associated with the linear quadratic regulator 1 (LQR) problem. In particular, we establish mesh independence for this algorithm. The importance of dual convergence and preservation of exponential stability (POES) with regard to strong convergence of the functional gains and mesh independence of the algorithm are discussed. These results are applied to systems governed by delay equations and numerical results are presented.

WEAK SOLUTIONS OF SECOND ORDER HYPERBOLIC PARTIAL DIF-FERENTIAL EQUATIONS

N F J van Rensburg (University of Pretoria)

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T5

2

Consider an initial boundary value problem for a second order hyperbolic partial differential equation. The problem may have a classical solution, a weak solution or a 'very" weak (mild) solution. Mild solutions are not merely of academic interest. Mathematical models for the motion of elastic bodies can be extremely complex. These complex model problems rarely have classical solutions and mild solutions are likely to occur. It is important to be aware of this fact since the rate of convergence of numerical approximations depends on the 'smoothness" of solutions. The wave equation is used as an example. Here it is easy to construct solutions with different 'degrees of smoothness". For a general linear vibration problem, a mild solution is not a solution of the usual variational problem. We present a weaker variational form and consider implementation of the finite element method.

A NUMERICAL STUDY OF A PLATE-BEAM SYSTEM

A J van der Merwe (Cape Peninsula University of Technology) vandermerwea@cput.ac.za

The classical models for plates and beams which are derived from linear elasticity, are not suitable for applications where the higher order natural frequencies are of importance. In such cases the complexities of three dimensional models are sometimes avoided by using lower dimensional models containing shear effects, i.e. the Timoshenko beam model and Reissner-Mindlin plate model. The eigenvalue problem used for calculating the natural frequencies for the Reissner-Mindlin plate model, is a system of three planar differential equations. For the Timoshenko beam model the associated eigenvalue problem is a system of two one dimensional ordinary differential equations. In a numerical study of a plate-beam system some interesting properties of the natural frequencies are investigated.

THE ROLE OF NOISE IN BIOLOGY

Dr Pamela Burrage (University of Queensland) pmb@maths.uq.edu.au

In this talk I discuss three approaches to mathematical modelling of biological applications, covering the cases where there are small numbers, intermediate numbers and large numbers of molecules. The resulting mathematical models require numerical simulation, and in particular, I will discuss weak and strong numerical solutions of stochastic differential equations, including the implementation of a splitting technique. The numerical solution of stochastic differential equations requires a set of random numbers, and when there are two or more Wiener processes, it is critically important to sample the random numbers carefully to ensure the required statistical properties are satisfied. I will discuss this in relationship to both strong and weak order solutions. I will conclude the talk by presenting numerical simulations for two biological applications.

GENERALIZED SENSITIVITY FUNCTIONS AND PARAMETER ESTIMA-TION

Franz Kappel (Institute for Mathematics and Scientific Computation) franz.kappel@uni-graz.at

Realistic models for the cardiovascular-respiratory system describing reactions of the system to external influences, as for instance ergometric work loads, orthostatic stress or hemorrhage, have to be rather complex. Validation of such models typically involves estimation of a large number of parameters on the basis of a limited number of measurements. Generalized sensitivity functions as introduced in [Th-C] characterize the dependence of the estimated parameters on the measurements. This is in contrast to classical sensitivities which characterize the dependence of the model output on the parameters. Generalized sensitivities also allow to determine time intervals where the measurements carry the most information for a specific parameter. In the presentation we demonstrate these possibilities for the example of a basic model for the reaction of the cardiovascular system to an ergometric work load (for the model see [K-P]). References 1. Kappel, F., and Peer, R. O., A mathematical model for the fundamental regulation processes in the cardiovascular system, J. Math. Biology 31(1993), 611 - 631. 2. Thomaseth, K., and Cobelli, C., Generalized sensitivity functions in physiological system identification, Annals of Biomedical Engineering 27(1999), 607 - 616.

Τ6

1

Parallel II

REFINABLE FUNCTIONS REGULARITY BY MEANS OF FOURIER ANALYSIS

Akwum Onwunta (University of Stellenbosch)

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In this talk, we present the regularity (or smoothness) of a given continuous compactly supported refinable function by using two methods which are based on the decay rate of its Fourier transform. We shall achieve this by first studying the embedding of certain Sobolev spaces into a Hölder regularity space. Our first method is based on maximizing the mask symbol (Laurent polynomial) on the unit circle in the complex plane, whereas our second method uses the spectral radius of a certain transfer matrix obtained from the mask symbol. We also compare our Fourier-based results with those obtained from a subdivision-based result. Illustrative numerical examples are provided.

HERMITE INTERPOLATION AND A METHOD FOR EVALUATING CAUCHY PRINCIPAL VALUE INTEGRALS OF OSCILLATORY KI [IN-COMPLETE]

George Okecha (University of Fort Hare) GOkecha@ufh.ac.za

T7 An alternative method to the method proposed in [10] for the numerical evaluation of integrals of the form 1 $\int_{-1}^{1} exp(i\phi t) f(t) dt$, where f(t) has a simple pole in [-1,1] and $\phi \in R$ may be large, has been developed. The method is based on a special case of Hermite interpolation polynomial and it is comparatively simpler and entails fewer function evaluations and thus faster, but the two methods are comparable in accuracy. The validity of the method is demonstrated in the provision of two numerical experiments and their results.

ASYMPTOTIC ZERO DISTRIBUTION OF SOME HYPERGEOMETRIC POLYNOMIALS

Sarah Jane Johnston (School of Mathematics, University of the Witwatersrand) sjohnston@maths.wits.ac.za

Т7 2 In recent papers Martínez-Finkelshtein and Kuijlaars and their co-authors have used Riemann-Hilbert methods to derive the asymptotic zero distribution of Jacobi polynomials $P_n^{(\alpha_n,\beta_n)}$ when the limits $A = \lim_{n\to\infty} \frac{\alpha_n}{n}$ and $B = \lim_{n\to\infty} \frac{\beta_n}{n}$ exist and lie in the interior of certain specified regions in the AB-plane. We prove that the zeros of ${}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};z\right)$ asymptotically approach the section of the lemniscate $\left\{z: |z(1-z)^2| = \frac{4}{27}; \operatorname{Re}(z) > \frac{1}{3}\right\}$ as $n \to \infty$. Our result corresponds to one of the transitional or boundary cases for Jacobi polynomials in the Kuijlaars Martínez-Finkelshtein classification.

Parallel III

THE ROLE OF ATAGES OF PROGRESSION IN MODELS FOR HIV PREVA-LENCE

Angelina Mageni Lutambi (Physical and Mathematical Analysis, University of Stellenbosch) Fritz Hahne (Physical and Mathematical Analysis, University of Stellenbosch)

We present two models that describe the transmission and spread of the HIV epidemic in the human population. In the first model all infected people are assumed to cause the same infection rate, while in the second model the rate depends on the stage of infection. The stability analysis and the numerical results are presented for the general case of a varying population.

ASPECTS OF USING A PORTAL X-RAY TO CONFIRM PATIENT POSITION

Neil Muller (University of Stellenbosch)

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Jantine Frahn (iThemba Labs)

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Proton therapy is well suited to the treatment of several lesions, but requires that the patient be accurately positioned. Current practice at iThemba Labs relies on the manual verification of the position using a portal X-Ray. This is both slow and potentially error-prone. In this paper, we examine some of the problems that arise when looking to replace this with automated image registration.

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Opening

COOLING OF THERMOPLASTIC MATERIAL IN AN INFLATABLE SPACECRAFT

Eugene M. Cliff (Virginia Tech)

J. T. Borggaard (Virginia Tech)

On-orbit assembly of very large spacecraft, such as the International Space Station, is costly in time, money and risk. Advanced thermoplastics offer the possibility to efficiently compress/package the structure for launch and then to deploy it on-orbit. In this talk we describe an analysis of post-deployment cooling in a space environment. We consider a cylindrical structural component (a truss longeron) composed of several thin shells. The resulting formal model includes a pair of unsteady, two-dimensional heat equations, with source terms that account for conductive, convective and radiative interactions. Several coupled, ordinary differential equations account for thermal behaviour of the metallic end-caps and of the enclosed gas. Numerical discretization uses finite-elements in space and backwards (implicit) Euler in time. Experimental results from a vacuum chamber test are used to tune parameters in the numerical model and comparisons are shown. Thermal response near the junctions with the end-caps is of particular interest.

WAVES IN SHALLOW WATER

Harvey Segur (University of Colorado, Boulder, USA)

The Korteweg-de Vries (KdV) equation has been studied extensively over the last 40 years, because it is the prototype of an "integrable" partial differential equation. As such, it has many miraculous properties, including exact N-soliton solutions, infinitely many conservation laws, etc. The Kadomtsev-Petviashvili equation is a higher-dimensional generalization of the KdV equation, and it shares this list of miraculous properties. But both equations were originally derived as models of long waves in shallow water. The purpose of this talk is to explore what these integrable models tell us about ocean waves in shallow water. Applications include the 2004 tsunami, and rip currents.

MULTIWAVELETS, GAUSSIANS AND GREEN'S FUNCTIONS: A NEW KIND OF FAST ALGORITHMS FOR PDES

Fernando Perez (University of Colorado, Boulder, USA)

Wavelet bases provide sparse representations of a wide class of integral and pseudo-differential operators (for reasons similar to those which lead to their use in image compression). Thanks to this, they can be used to construct fast algorithms for the application of such operators in one spatial dimension.

When combined with a technique for constructing Gaussian-based approximations of functions, it becomes feasible to extend the above ideas to multidimensional problems. I will discuss how the union of these two ideas permits the development of very flexible fast algorithms for the application of Green's functions and other operators in multiple dimensions, with controlled accuracy and no conditioning problems.

These new techniques have been already tested in problems ranging from Poisson's equation to Schrödinger's equation. We continue to develop both the basic algorithmic ideas and their implementation, looking to develop a very flexible set of methods for the fast and accurate solution of PDEs of interest in physical applications.

W2

1

SECOND-ORDER NUMERICAL SCHEME FOR SINGULARLY PER-TURBED REACTION–DIFFUSION ROBIN PROBLEMS

Srinivasan Natesan (Department of Mathematics, Indian Institute of Technology, Guwahati, India)

Rajesh K. Bawa (Department of Computer Science and Engineering, Punjabi University, Patiala, India)

Singular perturbation problems (SPPs) arise in several branches of engineering and applied mathematics which include fluid dynamics, quantum mechanics, elasticity, chemical reactor theory, gas porous electrodes theory, etc. To solve these types of problems various methods are proposed in the literature. In this work, we develop a hybrid numerical scheme for singularly perturbed reaction-diffusion boundary-value problem with smooth data. In general, the solution of these problems may exhibit two boundary layers of exponential type at both end points. Here, we are interested in an hybrid scheme which is a combination of the cubic spline scheme with the classical central difference scheme on a piecewise Shishkin mesh. The newly developed cubic spline scheme satisfies discrete maximum principle only in the boundary layer regions and fails in the regular region. This is because the mesh intervals are coarse in the regular region and to satisfy the discrete maximum principle one has to restrict the mesh size in relation with the perturbation parameter. To overcome this difficulty, we use the cubic spline scheme only in the boundary layer regions and the classical central difference scheme in the regular regions. Truncation errors are derived, and the present method provides second-order uniform convergent result throughout the domain of interest. Numerical examples have been carried out to show the efficiency of the method.

NUMERICAL METHODS FOR SINGULARLY PERTURBED DIFFEREN-TIAL DIFFERENCE EQUATIONS

Kailash C. Patidar (Department of Mathematics and Applied Mathematics, University of the Western Cape)

Mohan K. Kadalbajoo (Indian Institute of Technology Kanpur)

Kapil K. Sharma (MAB Université Bordeaux 1, France)

We consider some problems arising from singularly perturbed differential difference equations. First we construct (in a new way) and analyze a 'fitted operator finite difference method (FOFDM)" which is first order UNKNOWN LATEX SYMBOL-uniformly convergent. With the aim of having just one function evaluation at each step, attempts have been made to derive a higher order method via Shishkin mesh to which we refer as the 'fitted mesh finite difference method (FMFDM)". This FMFDM is a direct method and UNKNOWN LATEX SYMBOL-uniformly convergent with the nodal error as $\mathcal{O}(n^{-2} \ln^2 n)$ which is an improvement over the existing direct methods (i.e., those which do not use any acceleration of convergence techniques, e.g., Richardson's extrapolation or defect correction, etc.) for such problems on a mesh of Shishkin type that lead the error as $\mathcal{O}(n^{-1} \ln n)$ where n denotes the total number of sub-intervals of [0, 1].

W3

2

27

ON A PRIME NUMBER GENERATOR

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W4

Studies show that the most promising approaches to finally establishing a prime number generator may not come from pure mathematics, but from physicists. This is after researchers discovered a deep connection between Riemann zeros and energy levels of a quantum mechanical system. To test this hypothesis, we seek here a function f(x) such that the roots of the equation $\phi'' = f(x)\phi$ satisfy prime numerical conditions. The astouding results we got after giving this equation a Lie group theoretical methods treatment is that f(x)assumes values of the form 1/x, thus reducing this equation into a Schrödinger type equation.

TO BE ANNOUNCED

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 $\frac{W4}{2}$

Parallel III

SOLUTION OF WATER WAVE SCATTERING PROBLEM WITH VARYING OCEAN DEPTH BY FINITE FOURIER COSINE TRANSFORM

Swaroop Nandan Bora (Indian Institute of Technology, Guwahati) Subahs Chandra Martha (Indian Institute of Technology, Guwahati)

In this paper, the problem of oblique water wave diffraction by a small deformation of the bottom of a laterally unbounded ocean is considered using linear water wave theory. It is assumed that the fluid is incompressible and inviscid, and the flow irrotational. A perturbation analysis is employed to obtain the velocity potential, reflection and transmission coefficients up to the first order in terms of integrals involving the shape of functions representing the bottom deformation by using the Green's integral theorem. Two particular forms of the shape function are considered and the integrals for the reflection and transmission coefficients are evaluated for these two different functions. Among those cases, for the particular case of a patch of sinusoidal ripples at the bottom, the reflection coefficient up to the first order is found to be an oscillatory function in the quotient of twice the wave number along x-axis and the ripple wave number. When this quotient becomes one, the theory predicts a resonant interaction between the bed and free surface, and the reflection coefficient becomes a multiple of the number of ripples and high reflection of the incident wave energy occurs if this number is large. Known results for the normal incidence are recovered as special cases. The numerical solutions for the reflection and transmission coefficients are also evaluated against wave numbers and angles of incidence.

TURBULENCE SIMULATION USING RAPID DISTORTION THEORY

Mosa Chaisi (Maths & Computer Science Dept., National University of Lesotho, Roma 180, Lesotho)

Derek D Stretch (Department of Civil Engineering, University of KwaZulu-Natal, King George V Avenue, Durban 4041)

A fairly simple method of simulating a turbulent domain of a fluid body is demonstrated; which is also fairly inexpensive in terms of computing resources needed to carry it out. The underlying theory employed in dealing with the Navier-Stokes (N-S) equations is what is referred to as Rapid Distortion Theory - linearised N-S equations. Amongst the many physical scenarios where this can be useful, momentum exchange across a liquid-liquid interface, turbulent mixing and surface renewal are explored with the help flow visualization in the form of vector plots, flow animation and particle tracking techniques.

STEADY FLOW IN TUBES OF VARYING CROSS-SECTION: EFFECT OF VARIABLE VISCOSITY

Kgomotshwana Thosago (Academic Development Unit, University of Limpopo – Turfloop Campus)

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W5 In this paper, I examine steady flow of a variable viscosity incompressible fluid in a tube of small aspect ratio. On the basis of certain simplifying assumptions, the governing equations of continuity and momentum are obtained. Analysis has been carried out for low Reynolds number flow and the expressions for various flow characteristics are obtained using perturbation technique. It has been observed that an elevation in fluid viscosity will increase the magnitude of wall shear stress and pressure drop. The effect of initial forces on these flow characteristics varies with the tube geometry.

POLYNOMIAL CONTAINMENT AND QUASI-INTERPOLATION IN RE-FINEMENT SPACES

Désirée Moubandjo (Department of Mathematics, University of Stellenbosch) Johan de Villiers (Department of Mathematics, University of Stellenbosch)

In wavelet construction methods based on multi-resolutional analysis, it is important that the refinement space, defined as the span of the integer shifts of the underlying refinable function, locally contains the polynomials up to a certain degree. Using a purely time-domain approach, we show here that it suffices to demand merely that the corresponding Laurent polynomial mask symbol possesses a zero of order N at -1, to guarantee polynomial containment up to degree (N-1) for the associated refinement spaces. In the process, we derive a generalised Marsden identity for refinable functions, in which all the coefficients can be computed recursively. Moreover, we provide an explicit formulation of the optimally local quasi-interpolation operator mapping the space of real-valued functions on \mathbb{R} into the refinement space. Illustrative examples will be given.

ON THE CONVERGENCE OF THE CASCADE ALGORITHM

Guy Blaise Dongmo (Stellenbosch University)

Johan de Villiers (Stellenbosch University)

We present, separately for the scalar and vector cases, sufficients conditions on the Laurent polynomial mask symbol and the initial iterate for cascade algorithm convergence, and therefore also for refinable function existence and subdivision convergence. Moreover, we obtain explicitly the geometric constant appearing in the estimate for the geometric convergence of the cascade iterates to the corresponding refinable function. For the vector case, we consider an application of our result to the issue of Hermite subdivision convergence. For the scalar case, we apply our result to a one-parameter family of mask symbols to obtain a refinable function existence and subdivision convergence parameter interval, yielding in particular also masks with at least one negative coefficient.

INTEGRABILITY PROPERTIES AND EXACT SOLUTIONS OF THE KHOKHLOV-ZABOLOTSKAYA EQUATION

J C Ndogmo (University of the Western Cape) jndogmo@uwc.ac.za

W6

We investigate the integrability properties of the Khokhlov-Zabolotskaya equation and show that it satisfies certain necessary conditions for the Painlev $\tilde{A}(c)$ property. We also perform some classical similarity 3 reductions and obtain exact solutions. Although the symmetry algebra of this equation depends on three distinct arbitrary functions, we do not impose any restrictions on them in the similarity analysis, and this lead to more general results which are compared with those obtained using the direct method of Clarkson and Kruskal.

Parallel V

OBJECT ORIENTED EXTENSION OF LEGACY NUMERICAL SOFTWARE WITH PYTHON

Neilen Marais (CEMAGG group, E&E Engineering, Stellenbosch) David B. Davidson (CEMAGG group, E&E Engineering, Stellenbosch)

Software Engineering aspects in Computational Electromagnetics (CEM) are becoming more important as the complexity of CEM codes continue to increase. Object Orientated Programming (OOP) methods promise to alleviate the challenges posed by more complex software systems, but offer little help for legacy codes. Python, an object-oriented very high level language (VHLL) is both easy to learn and a productive environment for experienced programmers. It has extensive standard and third party libraries and very liberal Free Open Source (FOSS) licensing. Python itself is an interpreted language, but has easy to use facilities for incorporating external compiled code into a Python program by means of language wrappers. Thanks to Python's powerful namespace features, it is relatively easy to use a program structure in Python that differs completely from the structure of the original external code. This can be used to retroactively provide object oriented structure to a legacy code written in a programming language that does not support object orientation. The application of this method is described in the context of eMAGUS, a microwave Finite Element Method code written in Fortran 90.

DECOMPOSITION OF THE FIRST-ORDER RAVIART-THOMAS SPACE ON A TETRAHEDRAL MESH, WITH AN APPLICATION IN [INCOM-PLETE]

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The standard, Raviart-Thomas, face-associated basis functions of mixed first-order are commonly employed to model divergence-conforming fields on tetrahedral meshes. The function space being modeled contains both solenoidal- and non-solenoidal field components. In this presentation, a scheme is described whereby an equivalent, alternative set of basis functions may be constructed, such that these two constituent sub-spaces are each represented explicitly (similar to the tree-cotree decomposition in the curl-conforming setting). As an application, an approach to stabilizing the finite element method (FEM) for solving the linearized acoustic vector wave equation, is presented. Using the standard basis, this formulation's conditioning deteriorates as the frequency is lowered. Using the new basis, with its property of explicitly modeling the divergence operator's null-space, one may render the FEM system matrix positive definite in the low-frequency limit via simple diagonal normalization. Some numerical results will be shown.

33

W7

1

CALCULATION OF THE INVARIANTS OF PSEUDO-ORTHOGONAL GROUPS

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1

Although I.M. Gel'fan has given a formula for an explicit determination of the invariants of orthogonal and W8 pseudo-orthogonal groups, only a very little number of these invariants have actually been computed in the literature, probably due to the level of complexity of this formula, and the computed invariants are usually erroneous and only for low dimensional groups. We propose an algorithm that returns a fundamental set of invariants for any given pair (p,q) of indices of the pseudo-orthogonal group. We also present another algorithm that can be useful for checking the invariants of any system of operators.

CORRECTED ROOT AND BELL'S DISK ITERATION METHODS FOR THE SIMULTANEOUS DETERMINATION OF THE ZEROS OF A POLYNOMIAL

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W8 This paper considers the Root and Bell's disk iteration methods enhanced by incorporating a correction 2term and a choice of a disk inversion formula in the methods for the simultaneous computation of the zeros of a polynomial. The error propagation is proved to be the same in both methods. When the refinement process of correction is efficient, it is this mode of correction we have desired to propose.

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