The 46th Annual South African Symposium on Numerical & Applied Mathematics (SANUM)

02 to 04 April 2025

School of Computer Science and Applied Mathematics
University of the Witwatersrand, South Africa

Local organising committee:
Olumuyiwa Otegbeye, Erick Mubai, Adewunmi Fareo,
Matthews Sejeso, Mannoi Ramabolu & Reuben Khunou

The organising committee acknowledges generous sponsorship from the DSI-NRF Centre of Excellence - Mathematical and Statistical Sciences

1 Welcome note

For a program that was started by very few scholars in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York) visited South Africa, the theme and impact of the program has continued to be positively felt quite consistently since then. The South African Numerical and Applied Mathematics (SANUM) is an applied mathematics forum that brings together scholars and students from within South Africa and beyond to share and disseminate knowledge. The areas include numerical analysis, scientific computing, mathematical biology, symmetries, among several others. It is usually in the form of an annual symposium that includes both experienced researchers and students presenting their work through oral talks, posters, and networking.

The school of computer science and applied mathematics at the University of the Witwatersrand is a centre of excellence in South Africa and beyond, that hosts highly experienced researchers in various applied mathematics research areas, some of which can be found here.

The direct relationship between the theme of SANUM and the research themes of our school made it seamless for us in accepting to host the workshop and conference in the past and this year. As a school, we are grateful for the opportunity granted to us by the SANUM facilitators in hosting the prestigious annual event. We also appreciate the head of school of our school, Prof Raseelo Joel Moitsheki, for accepting the invitation and preparing a conducive environment for the smooth running of the program.

On behalf of the school, the scientific committee, the local organizing committee, and the administrators, I would like to welcome each and every one of the participants, invited speakers, workshop facilitator, and the university community to the SANUM 2025 workshop and conference. It is my strong belief that you all will have a productive and informative session with us and create strong networks. Better yet, the students will get critical and useful feedback to steer their research in the direction of academic excellence and strong research networks will be formed.

Special gratitude must be made to the financial sponsors of SANUM 2025; DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE MaSS), School of Computer Science and Applied Mathematics, Prof. Raseelo Moitsheki, Prof Jean Lubuma, and all the registered attendees for all coming together to ensure that we have a smoothly-run program.

Thank you all for being a part of this.

Yours sincerely,
Olumuyiwa Otegbeye,
Chair, SANUM 2025 local organizing committee.

2 About SANUM

South African Numerical and Applied Mathematics (SANUM) is a special interest group for South African and international academics in the areas of applied mathematics, numerical analysis, and scientific computing, and a yearly symposium with presentations and posters on these and related topics.

The symposium typically rotates amongst host institutions: Stellenbosch University, the University of Pretoria, the University of the Witwatersrand, and the University of Johannesburg. It serves as the annual symposium for the Numerical and Applied Mathematics research theme of the DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE MaSS).

The SANUM meeting came into existence in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York), happened to visit South Africa at the same time. A number of local researchers used the opportunity to organize a meeting at the (then) University of Natal, at which a total of ten papers were presented. From here the meeting grew in strength, with numerous famous numerical and applied mathematicians passing through. Aside from a few missing years, the meeting has been held annually, which makes SANUM a major event in the calendar of South African numerical and applied mathematics. With only a few exceptions, SANUM has taken place every year since, and 2025 marks the 46th anniversary of the meeting.

46 SANUM meetings:

1. Durban, 10–11 April 1975
2. Durban, 8–9 April 1976
3. Durban, 6–7 April 1977
4. Durban, 18–20 July 1978
5. Durban, 18–20 July 1979
6. Durban, 21–23 July 1980
7. Durban, 20–22 July 1981
8. Durban, 19–21 July 1982
9. Durban, 18–20 July 1983
10. Ballito, 2–4 July 1984
11. Umhlanga Rocks, 8–10 July 1985
12. Umhlanga Rocks, 14–16 July 1986
13. Umhlanga Rocks, 13–15 July 1987
14. Umhlanga Rocks, 11–13 July 1988
15. Umhlanga Rocks, 17–19 July 1989
16. San Lameer, 9–11 July 1990
17. Umhlanga Rocks, 15–17 July 1991
18. Durban, 13–15 July 1992
19. San Lameer, 12–14 July 1993
20. Umhlanga Rocks, 4–6 July 1994
21. Scottburgh, 10–12 July 1995
22. Cape Town, 15–17 April 1998
23. Stellenbosch, 29–31 March 1999
24. Stellenbosch, 3–5 April 2000
25. Stellenbosch, 9–11 April 2001
26. Stellenbosch, 3–5 April 2002
27. Stellenbosch, 31 March–2 April 2003
28. Stellenbosch, 5–7 April 2004
29. Stellenbosch, 30 March–1 April 2005
30. Stellenbosch, 3–5 April 2006
31. Stellenbosch, 2–4 April 2007
32. Stellenbosch, 2–4 April 2008
33. Stellenbosch, 6–8 April 2009
34. Stellenbosch, 15–17 April 2010
35. Stellenbosch, 23–25 April 2011
36. Wits, 2–4 April 2012
37. Stellenbosch, 3–5 April 2013
38. Wits, 14–16 April 2014
39. Pretoria, 30 March–1 April 2015
40. Stellenbosch, 22–24 March 2016
41. Wits, 28–30 March 2017
42. Stellenbosch, 04–06 April 2018
43. Pretoria, 27–29 March 2019
44. University of Johannesburg, 3–5 April 2023
45. Stellenbosch, 2–5 April 2024
46. **Wits, 2–4 April 2025**

3 Programme

Wednesday, 2 April 2025	
Time	**Activity**
08:15 – 09:00	**Registration**
09:00 – 09:30	**Opening address: Dr Robin Drennan,** Director of DVC: Research and Innovation, University of the Witwatersrand
09:30 – 10:30	**Plenary: Jan Nordstrom** **Title: Linear and Nonlinear Boundary Conditions: What's the difference?** **Chair:** *Olumuyiwa Otegbeye*
10:30 – 11:00	**Tea break** - *MSB UG floor*
Session 1 **Chair:** *Olumuyiwa Otegbeye*	
11:00 – 11:30	**Carel Olivier** *Comparing different numerical schemes for the 1D heat equation*
11:30 – 12:00	**Emma Nel** *Laguerre spectral collocation with improved stability*
12:00 – 12:30	**Yusuf Tijani** *Sensitivity and numerical simulation of unsteady reactive hydromagnetic Eyring-Powell flow in a non-horizontal channel*
12:30 – 13:00	**Mpho Mendy Nefale** *Overlapping Multi-Domain Paired Quasilinearization Method for the Solution of the MHDWilliamson-Nanofluid Flow Over an Exponentially Stretching Surface*
13:00 – 14:00	**Lunch -** *MSB UG floor*
Session 2 **Chair:** Matthews Sejeso	
14:00 – 14:30	**Arsene Tasse** *From COVID-19 to Post-COVID-19 Condition: A Modelling Approach with Mathematical Formulation of Follow-up Strategies*
14:30 – 15:00	**Nkululeko Qwabe** *Mathematical Perspective into the p53 Protein on the Expression of the Transcription Factor E2F in CycD/Cdk4(6) Subsystem*
15:00 – 15:30	**Mokiri Mathews Nkwana** *Solving higher order Korte De Vries equations using multi-domain spectral collocation method*
15:30 – 16:00	**Mathibele Nchabeleng** *A similarity solution for a pre-existing fluid driven fracture in a permeable rock*
16:15 – 16:30	**Photo session**
17:30 – 20:00	**Welcome cocktail -** *MSB UG floor*

Thursday, 3 April 2025	
Time	**Activity**
09:00 – 10:00	**Plenary: Abdul Kara** **Title: Symmetries & Conservation Laws of differential equations - alternate approaches and extensions** **Chair:** *Erick Mubai*
Session 3 **Chair:** *Erick Mubai*	
10:00 – 10:30	**Frank Smuts** *Complex Travelling Waves of the Ablowitz-Ladik Equations*
10:30 – 11:00	**Jessica James** *Planar Integrable Systems Associated with the sl(3,C) Algebra*
11:00 – 11:30	**Tea break** - *MSB UG floor*
11:30 – 12:00	**Mulweli Mudau** *Overlapping multi-domain simple iteration approach for solving ordinary differential equation*
12:00 – 12:30	**Nancy Mukwevho** *A pseudospectral method for time-fractional PDEs with shifted Chebyshev and Lagrange interpolating polynomials on overlapping decomposed domains*
12:30 – 13:00	**Yusuf Tijani** *Adaptive multidomain numerical solution for singularly perturbed fractional differential equation: Chebyshev pseudospectral method*
13:00 – 14:00	**Lunch** - *MSB UG floor*
14:00 – 15:00	**Plenary: Inger Fabris-Rotelli** **Title: Spatial Linear Networks in Applications** **Chair:** *Kendall Born*
Session 4 **Chair:** *Kendall Born*	
15:00 – 15:30	**Andre Weideman** *Computation and applications of Fourier-Pade Approximation*
15:30 – 16:00	**Nick Hale** *Optimal node selection for summation-by-parts formulations on general function spaces*
19:00 – 21:00	**Conference dinner** - *Hofmeyr House*

Friday, 4 April 2025	
Time	**Activity**
09:00 – 10:00	**Plenary: Ashleigh Hutchinson** **Title: Going with the flow: Integrating lab experiments into mathematical modelling** **Chair:** *Gideon Fareo*
Session 5 **Chair:** *Gideon Fareo*	
10:00 – 10:30	***Nic van Rensburg*** *Modeling planar motion of beams, cables, hoses and wires*
10:30 – 11:00	**Madelein Labuschagne** *Finite element analysis of rod vibration models*
11:00 – 11:30	**Tea break** - *MSB UG floor*
11:30 – 12:00	**Aletta Jooste** *Zeros of orthogonal polynomials and the polynomials associated to them*
12:00 – 12:30	***Khutsang Kanapi*** *Exploring the Propagation Properties of Lee Waves: From Phase Velocity to Critical Layer Interactions*
12:30 – 13:00	**Erick Mubai** *Two-dimensional turbulent thermal momentumless wake*
13:00 – 13:30	**David Mason** *The aerodynamics of wind turbine blades*
13:30 – 13:40	**Closing Remarks**
13:40 – 14:30	**Lunch -** *MSB UG floor*
Departure	

For changes in the schedule, see https://sanum.github.io/programme.html

Presentation details:

- Please ensure your presentation is loaded onto the available machine **before** the session in which you are speaking.
- If you intend to use your own laptop, please check the connections **in advance**.
- A laser pointer and clicker will be available.
- Contributed talks will be kept **strictly** to **30 minutes** (25 minutes for the talk + 5 minutes for questions).

4 General information

- **Registration:** Registration will take place outside WSS3.
- **Talks:** All talks will be held in WSS3.
- **Tea & Coffee:** Tea and coffee will be available at the times indicated in the schedule, on the UG floor of the MSB building.
- **Lunch Venue:** Lunch will be served on the UG floor of the MSB building.
- **Welcome function:** The welcome cocktail will take place on Wednesday at 17:30, on the UG floor of the MSB building.
- **Conference Dinner:** The conference dinner will be held at Hofmeyr House at 19:00 on Thursday. Hofmeyr House is located on East Campus.
- **Wifi details:** Connect to the **Wits-Guest** network using the following credentials: **Username: 971h4**, **Password: Vv2p651**. Eduroam is also available on campus.
- **Conference Photo:** Please gather outside WSS3 on Wednesday after the last talk for a group photo of all attendees.
- **Emergency contacts:** Wits Protection Services are available 24/7 at the following locations:
  - **East Campus:** Room 1, Robert Sobukwe Block — Tel: (011) 717 4444 / 6666
  - **West Campus:** Wits Science Stadium — Tel: (011) 717 1842

5 Workshop

- The workshop will take place in MSL 006 on Monday, 31 March and Tuesday, 1 April. Sessions will run from 09:00 to 16:00 on both days.
- Tea and coffee will be available during the following breaks, served on the UG floor of the MSB building:
  - Morning break: 10:30 – 11:00
  - Afternoon break: 16:00 – 16:30
- Lunch will be served from 13:00 to 14:00, also on the UG floor of the MSB building.

Workshop Schedule: Generative AI for Numerical Analysis

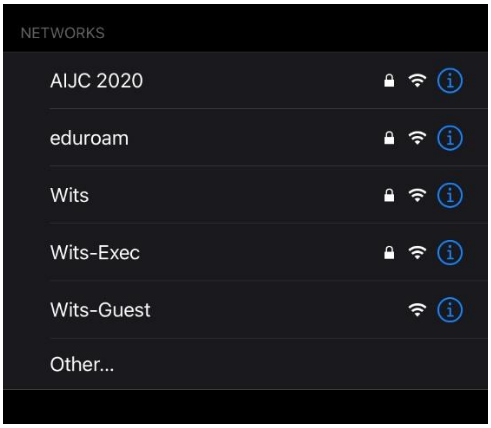
Monday , 31 March 2025	
Time	**Activity**
09:00 – 09:30	Welcome and Introductions
09:30 – 10:30	Exploring Best AI Tools For Mathematics
10:30 – 11:00	Tea Break – *MSB UG floor*
11:00 – 12:00	Prompt Engineering For Numerical Analysis
12:00 – 13:00	Hands on Practice
13:00 – 14:00	Lunch – *MSB UG floor*
14:00 – 15:00	AI for Error Analysis and Stability Investigation
15:00 – 16:00	Group Work
16:00 – 16:30	Tea Break – *MSB UG floor*
16:30 – 17:30	Group Work

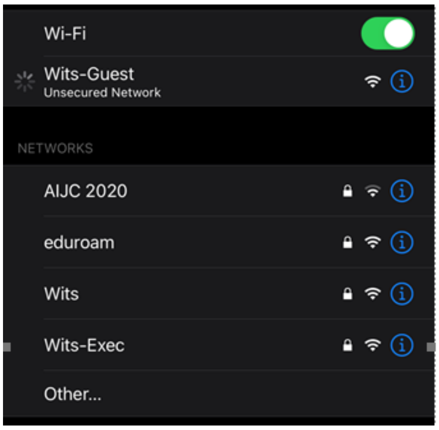
Tuesday , 1 April 2025	
Time	**Activity**
09:00 – 10:30	Deriving Numerical Schemes with AI Assistance
10:30 – 11:00	Tea Break – *MSB UG floor*
11:00 – 12:00	Using AI to Linearise and Programme Numerical Schemes
12:00 – 13:00	Hands on Practice
13:00 – 14:00	Lunch – *MSB UG floor*
14:00 – 15:00	Refining and Debugging AI-Generated Code for Numerical Solvers
15:00 – 16:00	Group Work
16:00 – 16:30	Tea Break – *MSB UG floor*
16:30 – 17:30	Group Work

Instructions on how to connect to Guest-Wi-Fi

Step 1:

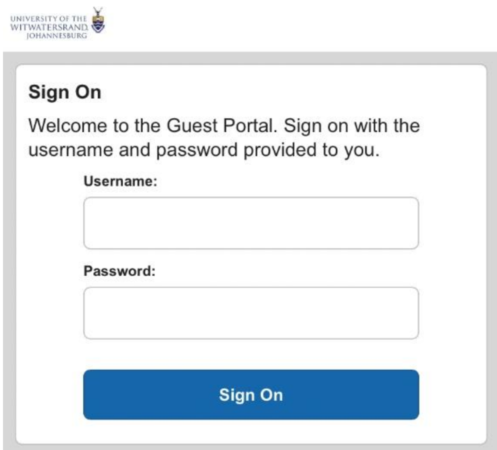
- Select the network SSID labelled ‘Wits Guest’.





Step 2:

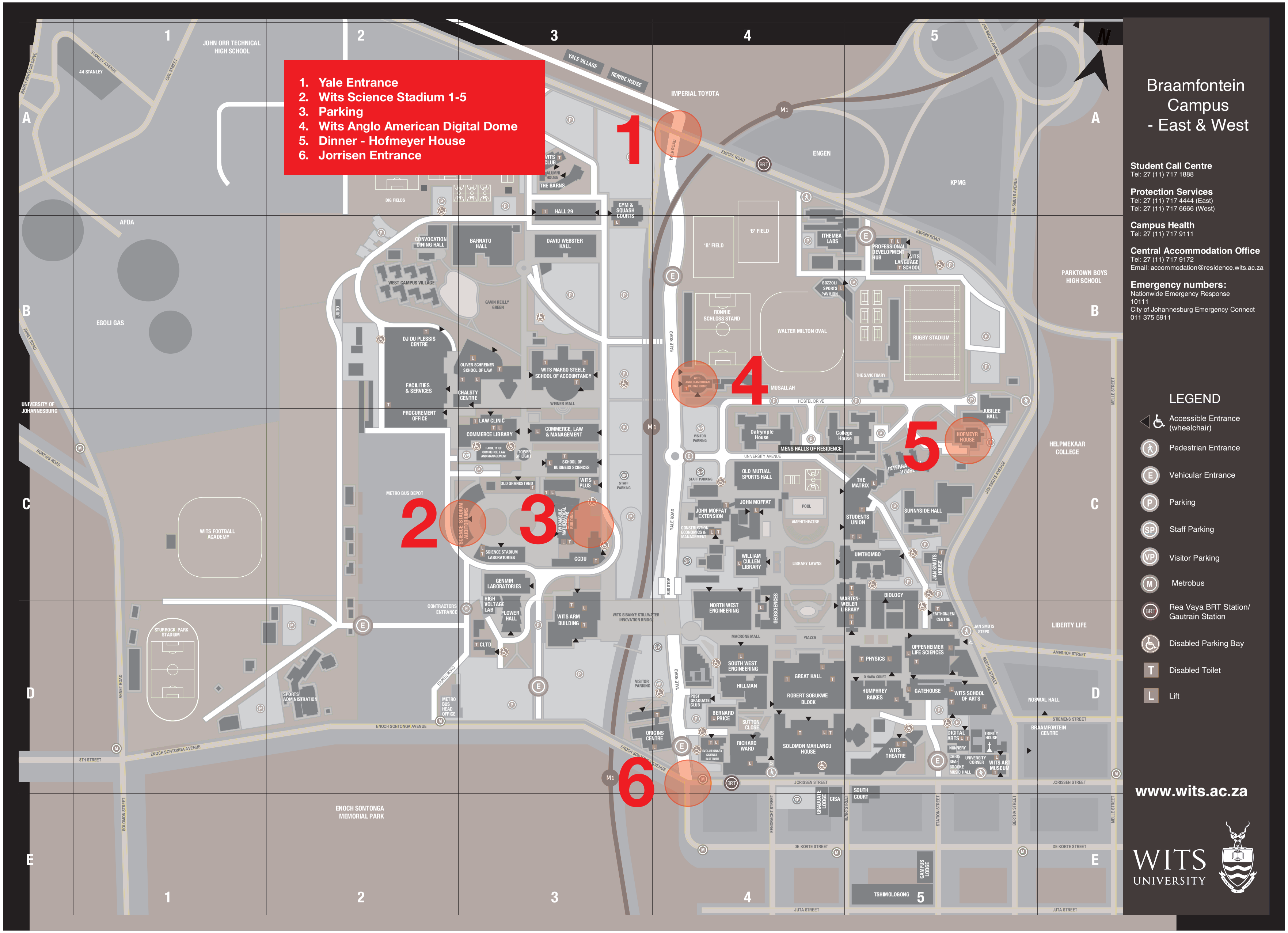
- After selecting the correct SSID, you will then be redirected to your browser, there where you will type the username and password you have received.



- In the event you are not automatically redirected to the browser page, you will have to open your browser and type in ‘cnn.com’. This should take you to the Guest login page.

Step 3:

- Once you have typed in the login details, the prompts thereafter are simple and easy to follow.



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Protection Services
Tel: 27 (11) 717 4444 (East)
Tel: 27 (11) 717 6666 (West)
Campus Health
Tel: 27 (11) 717 9111
Central Accommodation Office
Tel: 27 (11) 717 9172
Email: accommodation@residence.wits.ac.za
Emergency numbers:
Nationwide Emergency Response
10111
City of Johannesburg Emergency Connect
011 375 5911
LEGEND
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Pedestrian Entrance
Vehicular Entrance
Parking
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Disabled Toilet
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1. Yale Entrance
2. Wits Science Stadium 1-5
3. Parking
4. Wits Anglo American Digital Dome
5. Dinner - Hofmeyer House
6. Jorrisen Entrance
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WALTER MILTON OVAL
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RONNIE SCHLOSS STAND
YALE ROAD
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JAN SMUTS AVENUE
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JORISSEN STREET
DE KORTE STREET
JUTA STREET
MENS HALLS OF RESIDENCE
UNIVERSITY AVENUE
HOSTEL DRIVE
MUSALLAH
HALL 29
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SCHOOL OF BUSINESS SCIENCES
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NOSWAL HALL
BRAAMFONTEIN CENTRE
SOUTH COURT
GRADUATE LODGE
CISA
CAMPUS LODGE
TSHIMOLOGONG
STURROCK PARK STADIUM
SPORTS ADMINISTRATION

Workshop

On the use of LLMs for scientific code generation

Sandile Motsa (University of Eswatini)

The workshop will explore the use of large-language model (LLM) tools for developing scripts based on the application of well-defined prompts on the block hybrid method for solving initial-value problems.

Bio: Prof Motsa is currently serving as the Dean of the Faculty of Science and Engineering at the University of Eswatini. He holds Honorary Professorships at the UKZN and Wits University and is a Professor Extraordinaire at UNISA. He is a distinguished Applied Mathematician and a founding member of the Kingdom of Eswatini Academy of Science (KEAS). He specializes in the development of numerical methods for solving mathematical models in Science and Engineering. With over 200 publications and more than 5900 citations, Prof Motsa is an influential researcher and serves on the editorial boards of prestigious journals. He has contributed significantly to the Southern Africa Mathematical Sciences Association (SAMSA) as Vice President and President. He has supervised numerous Postdocs, Doctoral and Master's students. He is passionate about integrating AI and technology into education and research, regularly sharing his insights through his column "AI Corner" in the Eswatini Observer.

Plenary abstracts

Symmetries and Conservation Laws of differential equations - alternate approaches and extensions

Abdul Kara (University of the Witwatersrand)

The relationship between symmetries and conservation laws has been a subject of interest during the 1800s and culminated in Noether's Theorem of 1918 for variational DEs. In this presentation, we discuss this relationship with alternate approaches, e.g., 'partial Lagrangians' and Noether type operators. We also consider the extension of this relationship to DEs with 'small parameters via approximate symmetries', discrete equations and fractional DEs.

Bio: Abdul H Kara completed all his studies at Wits University and in all, but one year, has been at Wits from Junior Lecturer through to Professor; the one year being served as a school teacher. His PhD thesis involved a range of topics around the Symmetries of Differential Equations, Euler-Lagrange equations and their relationship with Conservation Laws. Abdul has published with collaborators from China, the US, Russia, Pakistan and with his students from SA and abroad. He continues to apply his work in mathematical physics, engineering and relativity and extend his ideas to Discrete Equations and Fractional Differential Equations.

Going with the flow: Integrating lab experiments into mathematical modelling

Ashleigh Hutchinson (University of Manchester)

It is often thought that mathematical models, where equations are derived using fundamental physical laws, lead to accurate representations of reality. After all, if we base our equations on well-established principles such as the conservation of mass, momentum, and energy, and we find solutions to these equations which are exact or numerical with an acceptably small margin of error, what could possibly go wrong? In this talk, we will examine several mathematical models that describe the behaviour of slow-moving viscous fluids in confined geometries. I will then reveal what happens when these models are tested against laboratory experiments. As we compare theory to experiment, we will discover that even when our models are built on solid physical foundations, they can neglect crucial factors that significantly impact real-world behaviour. As a result, even a small set of laboratory experiments can play a crucial role in understanding why mathematical models may fail to represent reality. I will show how experiments and theory can be used in tandem, where lab experiments can assist in identifying missing physical mechanisms, narrowing our focus, and ultimately resulting in mathematical models that are a better representation of reality.

Bio: Dr Ashleigh Jane Hutchinson is a Lecturer in Applied Mathematics at the University of Manchester, UK. In 2016 she earned her PhD from the University of the Witwatersrand, Johannesburg, South Africa. In 2021, she was awarded a prestigious Newton International Fellowship, allowing her to conduct research at Cambridge University, UK. Her research is primarily focused on fluid mechanics, particularly problems involving low-Reynolds number flows and non-Newtonian fluids, which have a variety of applications in both nature and industry. Dr Hutchinson's approach to solving complex problems is multidisciplinary, combining theoretical modelling, laboratory experiments and numerical simulations. In addition to fluid dynamics, she has a keen interest in other areas of applied mathematics, including population dynamics, energy conservation, and mining.

Spatial Linear Networks in Applications

Inger Fabris-Rotelli (University of Pretoria)

This talk will cover the use of linear networks in Spatial Statistics. A variety of methods will be discussed involving analysis in a linear network space. Applications in informal roads, criminology and disease mapping will be presented.

Bio: Prof Inger Fabris-Rotelli is currently an associate professor in the Department of Statistics, University of Pretoria. She has been at the Department of Statistics since 2004. She holds a PhD Mathematical Sciences, obtained in 2013, an MSc Applied Mathematics, a double BSc (Hons) in Mathematical Statistics and Applied Mathematics and a BSc Applied Mathematics. She has supervised 68 honours, 26 Masters and 7 PhD students to completion, and is currently supervising 10 honours, 6 Masters and 7 doctoral students, and has a particular passion for postgraduate supervision. She has published 33 peer-reviewed journal articles, 26 peer-reviewed conference proceedings papers and 1 book chapter.

She was on the executive of the South African Statistical Association (SASA) from 2012 – 2018, and from 2019-2024 sat as a director on the ICCSSA (Institute of Certificated and Chartered Statisticians in South Africa) board. She is the immediate past-president of SASA and CEO of ICCSSA. She is also a member of ISI and IMS internationally, and the Golden Key Society, SASA, SAMS, GISSA, SAMSA, GASA and RLadies Johannesburg co-chair locally. She is a SACNASP council member elected 2021 – 2025 as well as a SACNASP registered scientist. Her research interests are in spatial statistics and GIS, as well as remote sensing and general image processing, including spatial epidemiology and criminology.

Linear and Nonlinear Boundary Conditions: What's the difference?

Jan Nordstrom (Linköping and University of Johannesburg)

We present a straightforward energy stable weak implementation procedure of open boundary conditions for initial boundary value problems. The new boundary procedure generalize the well-known characteristic boundary procedure for linear problems to the nonlinear setting. We discuss the differences and exemplify with boundary conditions for the shallow water equations, the Euler and Navier-Stokes equations and equations for multi-phase flows.

Bio: Jan Nordström is Professor Emeritus in Computational Mathematics at the Department of Mathematics, Linköping University, Sweden, a Distinguished Visiting Professor at the Department of Mathematics and Applied Mathematics, University of Johannesburg, South Africa and an Honorary Professor in Computational Mathematics at University of Cape Town, South Africa. He is also a member of the Academy of Science of South Africa. His main interest is in Initial Boundary Value Problems (IBVPs), and in particular the fundamental effect of boundary and interface conditions on well-posedness and stability. He stresses the necessity to understand the IBVP during the development of numerical approximations.

Contributed abstracts

Zeros of orthogonal polynomials and the polynomials associated to them

Aletta Jooste (University of Pretoria)

Every sequence of real polynomials $\{p_n\}_{n=0}^{\infty}$, orthogonal with respect to a positive weight function $w(x)$ on the interval (a,b), satisfies a three-term recurrence equation. I discuss the role played by the polynomials *associated* to p_n, especially as coefficient polynomials in the three-term recurrence equation involving polynomials p_n, p_{n-1} and $p_{n-m}, m \in \{2,3,\ldots,n-1\}$.

I will also show how Christoffel's formula is used to obtain mixed three-term recurrence equations involving the polynomials p_n, p_{n-1} and $g_{n-m,k}, m \in \{2,3,\ldots,n-1\}$, where the sequence $\{g_{n,k}\}_{n=0}^{\infty}, k \in \mathbb{N}_0$, is orthogonal with respect to $c_k(x)w(x) > 0$ on (a,b) and c_k is a polynomial of degree k in x. Lastly, the conditions on k, necessary and sufficient for these equations to be in such a form, that they can be applied in the study of the location of the zeros of the appropriate polynomials, will be discussed.

Computation and applications of Fourier-Padé Approximation

Andre Weideman (Stellenbosch University)

The talk is on Fourier–Padé approximation, with specific focus on computational aspects and applications. Topics that will be discussed are the mitigation of the Gibbs phenomenon, post-processing of numerical solutions of differential equations, and singularity detection in the complex plane.

From COVID-19 to Post-COVID-19 Condition: A Modelling Approach with Mathematical Formulation of Follow-up Strategies

Arsene Tasse (University of the Witwatersrand) Musyoka Kinyili (University of the Witwatersrand) and Jean Lubuma (University of the Witwatersrand)

The COVID-19 pandemic left behind another health burden, the post-COVID-19/long-COVID condition. We construct an extended $SEIR$-type model for COVID-19 coupled with additional compartments that account for long-COVID conditions of infected, hospitalized, and recovered COVID-19 cases. Apart from exponentially distributed follow-up strategies, the key intervention to stop the progression to long-COVID condition is formulated through a general treatment function. We prove that the unique disease-free equilibrium (DFE) is locally asymptotically stable (LAS) whenever $\mathcal{R}_0$, the basic reproduction number is less than 1. When $\mathcal{R}_0 > 1$, at least one LAS endemic equilibrium (EE) exists. Using more explicit thresholds, we specify the number of positive equilibria and prove the global asymptotic stability (GAS) of the DFE. We propose a nonstandard finite difference scheme that is dynamically consistent. Numerical simulations that support the theory are provided. They show the GAS of the DFE and EE in the expected ranges of the thresholds.

Comparing different numerical schemes for the 1D heat equation

Carel Olivier (Akademia & North-West University)

A wide variety of numerical methods are available for the numerical solution of partial differential equations, including finite difference methods, spectral methods and finite element methods. When choosing a method, a compromise is reached between accuracy and efficiency, where the latter is typically sacrificed in favour of the former, and vice versa. Accuracy are typically measured in terms of convergence rates, while efficiency depends on stability and complexity. In this paper, a simple test is introduced in order to compare the performance of different numerical schemes that takes the above-mentioned factors into account. To illustrate the test, we consider a number of numerical schemes for solving the one dimensional heat equation subject to periodic boundary conditions.

The aerodynamics of wind turbine blades

David Mason (University of the Witwatersrand) and Nicholas Whittaker (University of the Witwatersrand)

A brief outline is given of the derivation of the singular integral aerofoil equation. The solution for a symmetric aerofoil using an infinite series of Chebyshev polynomials of the first kind is reviewed. The extension of the solution to an aerofoil with non-zero camber will be presented. Applications to the horizontal and vertical axis wind turbine will be discussed.

Laguerre spectral collocation with improved stability

Emma Nel (Stellenbosch University) and Nick Hale (Stellenbosch University)

In this work, we investigate the use of Laguerre spectral collocation methods for solving ordinary and partial differential equations on the semi-infinite line. Although spectral methods are generally used for their high accuracy, the unbounded domain and the exponential weighting associated with Laguerre polynomials introduce some challenges. Specifically, the direct computation of the entries in the Laguerre spectral differentiation matrix leads to underflow and overflow issues in IEEE-754 arithmetic for large degrees N, even though the entries themselves are well-behaved. We present strategies to overcome these numerical challenges, enabling computations for much larger values of N. Additionally, we explore truncated Laguerre spectral collocation, where only a subset of the roots of degree-N Laguerre polynomials is used in the discretisation. By employing these approaches, one can achieve significantly higher values of N in Laguerre spectral methods, ultimately leading to better numerical results.

Two-dimensional turbulent thermal momentumless wake

Erick Mubai (University of the Witwatersrand) and David Mason (University of the Witwatersrand)

In this talk, we present a Lie point symmetry analysis of the turbulent momentumless thermal wake flow and provide numerical results. We use the Prandlt mixing length hypothesis to model turbulence brought about by momentum and thermal mixing. The Lie point symmetry analysis includes obtaining conserved vectors of the governing partial differential equations and their associated Lie point symmetries. Using the associated Lie point symmetries to reduce the governing partial differential equations to ordinary differential equations (ODEs) guarantees that the ODEs can be integrated at least once by Sjöberg theorem. We employ a shooting method that uses conserved quantities as targets to solve the resulting ODEs.

Complex Travelling Waves of the Ablowitz-Ladik Equations

Frank Smuts and Igor Barashenkov (University of Cape Town)

The Ablowitz-Ladik equation is a fundamental differential-difference equation, which arises as an integrable discretization of the Nonlinear Schrodinger Equation. We construct exact periodic-wave and soliton solutions of the Ablowitz-Ladik, for both of its focusing and defocusing cases. Unlike cnoidal waves and solitons that were obtained by earlier authors, our solutions have a phase variable with nonlinear dependence on time and index. In particular, the solitons describe depressions propagating in the motionless condensate. We start with the construction of the standing waves, and then modify them to include a nonzero velocity. The existence of the traveling waves hinges on the availability of a two-point map governing the wave's modulus; this map gives rise to standing waves centered arbitrarily relative to the lattice sites. With a properly chosen phase variable, the position of the wave becomes a linear function of time.

Planar Integrable Systems Associated with the sl(3,C) Algebra

Jessica James (University of Cape Town)

The first and second complex sine-Gordon equations are integrable systems on the plane with explicit vortex solutions. The complex sine-Gordon-1,

$$\nabla^2\psi + \frac{(\nabla\psi)^2\psi^*}{1-|\psi|^2} + \psi(1-|\psi|^2) = 0 \tag{1}$$

is a relatively simple equation. The associated Lax representation is formulated on the $sl(2,\mathcal{C})$ algebra. The complex sine-Gordon-2,

$$\nabla^2\psi + \frac{(\nabla\psi)^2\psi^*}{2-|\psi|^2} + \frac{1}{2}\psi(1-|\psi|^2)(2-|\psi|^2) = 0 \tag{2}$$

is a deep reduction of a multifield system associated with $sl(3,\mathcal{C})$. In (1) and (2), ∇ is a two-dimensional gradient operator: $\nabla = \{\partial_x, \partial_y\}$.

Equation (1) admits a symmetric spinor representation that is employed to derive the Bäclund transformations, and, eventually, to construct wide classes of multi-vortex solutions. The spinor representation for (2) is not known.

In this presentation, I discuss soliton theory, integrability, the inverse scattering problem and Lax pairs as tools to analyse these two complex sine-Gordon equations. I discuss the spinor representation of the complex sine-Gordon 1, and how it can be used to construct multi-vortex solutions. I also discuss a spinor representation for the complex sine-Gordon 2 equation, which may potentially provide Bäcklund transformations and hence allow us to construct multi-vortex solutions.

Exploring the Propagation Properties of Lee Waves: From Phase Velocity to Critical Layer Interactions.

Khutsang Kanapi (University of the Witwatersrand)

Lee waves are fascinating atmospheric phenomena that occur when stable, stratified air flows over a mountain or ridge, generating oscillations in the atmosphere. There are two types of lee waves: trapped lee waves and vertically propagating wales. This presentation focuses on the latter. We consider a two-dimensional (2D) inviscid model for lee waves and simplify the analysis using a stream function, reducing the governing equations to the Taylor-Goldstein equation. We derive the dispersion relation, which in turn provides the phase velocity and group velocity of the waves. Since vertically propagating waves continue until they reach the inversion layer, where they break and cause clear air turbulence hazardous to aviation, we addressed two scenarios. The first case, before the critical layer, was solved using the WKBJ approximation to find the wave solutions. The second case, at the critical layer, where the mean fluid velocity equals the phase velocity, was tackled using the Frobenius method. The application of the Frobenius method extends the existing literature by considering higher-order terms, thereby offering improved accuracy.

Finite element analysis of rod vibration models

Madelein Labuschagne (University of Pretoria) and Kirstin Hohls (University of Pretoria)

A general one-dimensional continuum model is considered for the planar motion of rods, encompassing structures such as beams, cables, hoses, and wires. In this talk, the general model is compared to two simplified models for small oscillations, using newly developed finite element algorithms. These algorithms offer a simpler alternative to existing approaches, such as those by Simo and Vu-Quoc.

The comparison focuses on how different models predict wave propagation under varying axial forces (tensile and compressive) and the resulting differences in wavelength. Large oscillations of highly slender rods is also considered.

A similarity solution for a pre-existing fluid driven fracture in a permeable rock

Mathibele Nchabeleng (University of Pretoria) and Adewunmi Fareo (University of the Witwatersrand)

The problem of a two-dimensional, pre-existing, fluid-driven fracture propagating in a permeable rock is investigated. The fracturing fluid is a viscous, incompressible Newtonian fluid and the flow of fluid inside the fracture is laminar. The elasticity of the rock is modelled using the Cauchy principal value integral derived from linear elastic fracture mechanics. With the aid of lubrication theory, a nonlinear partial integro-differential equation relating the fracture half-width to the pressure and leak-off velocity is derived. Similarity solutions are derived for the fracture half-width, pressure and leak-off velocity and are used to reduce the partial integrodifferential equation to an ordinary integro-differential equation. In order to close the problem, a model in which the leak-off velocity is proportional to the fracture half-width and the gradient of the fluid–rock interface was used. Numerical results are obtained for the fracture length, fracture half-width and the net fluid pressure.

Solving higher order Korte De Vries equations using multi-domain spectral collocation method

Mokiri Mathews Nkwana (University of Limpopo) and Musawenkosi Mkhatshwa (University of Limpopo)

The aim of this study is to introduce multi-domain spectral collocation method for solving higher order Korteweg De Fries (KDV) equations defined on large time frames. In particular single equations of order seven are considered. The KDV equations are firstly altered to a linearized form of iterative scheme using quasi-linearization method (QLM).The time domain is decom- posed into non-overlapping sub-intervals, while on the contrary space domain is disintegrated into overlapping sub-intervals. The solutions at each time sub-interval are computed independently, where continuity condition is applied to produce initial conditions for the subsequent subintervals, whereas the solutions in the space interval are to be determined simultaneously across overlapping sub-intervals. The unknown functions and corresponding derivative functions are approximated by bi-variate lagrange interpolation polynomial at each selected Gauss–Lobatto grid points. The solutions are then correlated with the exact solutions to manifest accuracy. Furthermore, the efficacy, stability and accuracy of the method are demonstrated by presenting computational error analysis, condition numbers and the computational time for the solution of KDV equations.

Overlapping Multi-Domain Paired Quasilinearization Method for the Solution of the MHDWilliamson-Nanofluid Flow Over an Exponentially Stretching Surface

Mpho Mendy Nefale (University of the Witwatersrand), Olumuyiwa Otegbeye (University of the Witwatersrand), Shina Daniel Oloniiju (Rhodes University) and Mekonnen Shiferaw Ayano (University of Eswatini)

This study examines a non-Newtonian fluid model representing shear-thinning behaviour with applications in both biological and industrial fields. While prior research has investigated the fluid's dynamics in various contexts, the combined effects of the Williamson fluid model and additional parameters remain underexplored. This work focuses on the analysis of magnetohydrodynamic Williamson-nanofluid flow over an exponentially stretched surface. To solve the transformed model equations, an advanced numerical approach, the overlapping multi-domain paired quasilinearization method (OMD-PQLM), is employed. This method enhances the traditional paired quasilinearization method (PQLM) by subdividing the integration interval with overlapping techniques, enabling efficient and accurate solutions to the nonlinear ODEs. Key parameters studied include the Hall parameter, chemical reaction parameter, thermophoresis particle parameter, Brownian motion parameter, and magnetic parameter. Notably, the Williamson parameter significantly affects velocity profiles, temperature distribution, and concentration, with its shear-thinning properties altering flow characteristics and leading to higher temperature and concentration distributions. The OMD-PQLM demonstrated superior accuracy compared to the standard PQLM. This comprehensive analysis provides valuable insights into the OMD-PQLM's effectiveness and the behaviour of Williamson-nanofluid systems under various conditions, contributing to advancements in nanofluid applications, air conditioning, cooling processes, and thermal energy storage.

Overlapping multi-domain simple iteration approach for solving ordinary differential equation

Mulweli Mudau (University of the Witwatersrand), Olumuyiwa Otegbeye (University of the Witwatersrand) and Shina Daniel Oloniiju (Rhodes University)

Systems of nonlinear differential equations that are defined on large spatial domains tend to be tedious to solve using some numerical methods. Studies have shown that by introducing domain decomposition techniques, existing methods can overcome the computational difficulty. The simple iteration method has also been shown to be an effective method in solving systems of nonlinear differential equations with high accuracy of solutions. The present study introduces the overlapping multi-domain simple iteration method for solving systems of nonlinear ordinary differential equations (ODEs) over large domains. The OMD-SIM approach is applied to model two-dimensional, incompressible, steady, reactive Eyring-Powell nanofluid flow on a vertical Riga plate, represented by a system of nonlinear ODEs. The numerical solutions obtained using OMD-SIM are compared to those of the SIM method over the large computational domain. The stability, convergence speed, and accuracy of the methods are evaluated by examining the condition number of matrices, residual norms, and convergence errors. The results demonstrate that OMD-SIM schemes exhibit superior stability, faster convergence, and higher accuracy compared to SIM schemes. Further analysis reveals that OMD-SIM requires fewer overlapping points, whereas SIM relies on an optimal number of collocation points for efficient convergence. The study highlights that OMD-SIM is an efficient and stable approach for solving systems of nonlinear ODEs over large computational domains. This OMD-SIM method outperforms the standard SIM in terms of computational efficiency and accuracy, indicating that partitioning large domains into small overlapping multi-domains improves accuracy and computational efficiency.

A pseudospectral method for time-fractional PDEs with shifted Chebyshev and Lagrange interpolating polynomials on overlapping decomposed domains

Nancy Mukwevho (Rhodes University), Olumuyiwa Otegbeye (University of the Witwatersrand) and Shina Daniel Oloniiju (Rhodes University)

Time-fractional partial differential equations (TFPDEs) are powerful mathematical tools for modelling a wide range of physical and biological phenomena that exhibit memory effects, anomalous diffusion and non-local behaviour. These classes of equations are crucial in capturing dynamics where the influence of past states affects future evolution, making them essential in many areas of applied science, such as heat transfer, viscoelasticity and anomalous diffusion. This study proposes a pseudospectral method that combines the weighted sums of the Chebyshev and Lagrange polynomials to numerically approximate the solutions of TFPDEs. The spatial domain is partitioned into uniform, overlapping subdomains, where the solution in each subdomain is represented as a weighted sum of the Lagrange interpolating polynomials. On the other hand, the time domain is treated as a whole without decomposition, and the solution in the temporal dimension is expanded using the first-kind shifted Chebyshev polynomials. We validate the accuracy and performance of the method through a series of test cases, covering both linear and nonlinear TFPDEs in one and multiple spatial dimensions. These examples showcase the method's capability to handle the computational challenges associated with TFPDEs and underline its potential for broader applications in problems involving fractional dynamics. Specifically, the proposed technique is applied to resolve TFPDE, which models heat transfer on a disk, a problem relevant to modelling heat conduction in circular plates and semiconductor wafers. A time-dependent Gaussian heat source concentrated in a specific region of the disk is introduced to accurately simulate practical thermal diffusion dynamics. The gradual increase of the source term over time offers a more realistic representation of the evolving thermal diffusion process.

Modeling planar motion of beams, cables, hoses and wires

Nic van Rensburg (University of Pretoria) and Kirstin Hohls (University of Pretoria)

The process of modeling includes a continuum mechanics based derivation of the model, mathematical analysis, development of algorithms and analysis of numerical computations. We present a brief overview of this process for the local linear Timoshenko (LLT) rod which can be applied to to beams, cables, etc. The model consists of a system of nonlinear (hyperbolic type) partial differential equations.

The research grew out of research on linear models and started approximately ten years ago involving four doctoral students and another collaborator. The essential part of the work is contained in doctoral theses and articles from 2019 to 2025. Some detail will be provided on the following articles: LLT model 2021, algorithm for oscillations of the LLT model 2023, spectral theory for the associated linear model 2025.

Optimal node selection for summation-by-parts formulations on general function spaces

Nick Hale (Stellenbosch University), Charis Harley (University of Johannesburg), Prince Nchupang (Stellenbosch University) and Jan Nordström (University of Johannesburg and Linköping University)

Gauss–Lobatto quadrature nodes are known to be optimal in the case of closed summation-by-parts (SBP) formulations based on polynomial function spaces. Here we show that, for general function spaces, a corresponding generalized Gauss–Lobatto quadrature rule determines the optimal nodes for the SBP formulation. We present an algorithm for the computation of such quadrature rules and demonstrate their efficacy in a few applications.

Mathematical Perspective into the p53 Protein on the Expression of the Transcription Factor E2F in CycD/Cdk4(6) Subsystem

Nkululeko Qwabe (NorthWest university) and Kesh Govinder (NorthWest university)

Inhibitors play a significant role in the monitoring of cellular pathways. In this investigation, we evaluate the impact of p53 protein production in the CycD/Cdk4(6) subsystem. This protein assumes a crucial role in the regulation of cellular processes, including DNA repair, cell quiescence, and apoptosis. The occurrence of these processes takes place during the $\mathbf{G_1}$ phase, which represents the growth stage of a cell. Although the expression of the p53 protein is not fully understood, it is known to be pulsatile and produced in response to DNA damage. As a result, the severity of cell DNA damage directly affects the level of p53 protein expression within the cell system. To further comprehend the cancerous activity of a cell in this subsystem, we have developed a novel mathematical model for the behaviour of the CycD/CDK4(6) subsystem. Our observations reveal that the system exhibits a Hopf-bifurcation, which is consistent with laboratory findings. In addition to the mathematical analysis, we have also conducted simulations of the cell processes, which successfully mimic the results obtained in the laboratory. We hope that this model will contribute to a deeper understanding of the impact of the p53 protein on cellular processes in averting oncogenic activities.

Adaptive multidomain numerical solution for singularly perturbed fractional differential equation: Chebyshev pseudospectral method

Yusuf Tijani (Rhodes University), Olumuyiwa Otegbeye (University of the Witwatersrand) and Shina Oloniiju (Rhodes University)

Singularly perturbed differential equations pose significant challenges for many numerical and semi-analytical solution methods in classical calculus. The complexity of these equations increases when dealing with fractional differential equations, which exhibit unique properties, including the non-local nature of the arbitrary non-integer order differential operators. This study presents an adaptive multidomain Chebyshev pseudospectral method to effectively approximate the solutions of singularly perturbed fractional differential equations. In each subdomain, the fractional differential operator in the Caputo sense accounts for both local derivative effects and memory by dividing the discrete fractional operator into two distinct components, one representing local behaviour and the other capturing memory effects. The adaptivity of the method is controlled by ensuring that the maximum residual error in each domain remains below a predetermined tolerance value. If the maximum error exceeds the set tolerance, the size of the subinterval is reduced by a specified ratio, and the solution is recomputed within that interval. To assess the performance of the adaptive method, a comparative study with the multidomain pseudospectral method with uniform step length is used. The accuracy of the adaptive multidomain Chebyshev pseudospectral method is validated through a series of numerical tests on various fractional differential equations, thus demonstrating the effectiveness of the approach. The adaptive multidomain pseudospectral method offers a robust solution for handling the challenges posed by singularly perturbed differential equations common in fields such as wave propagation, astrophysics and quantum mechanics.

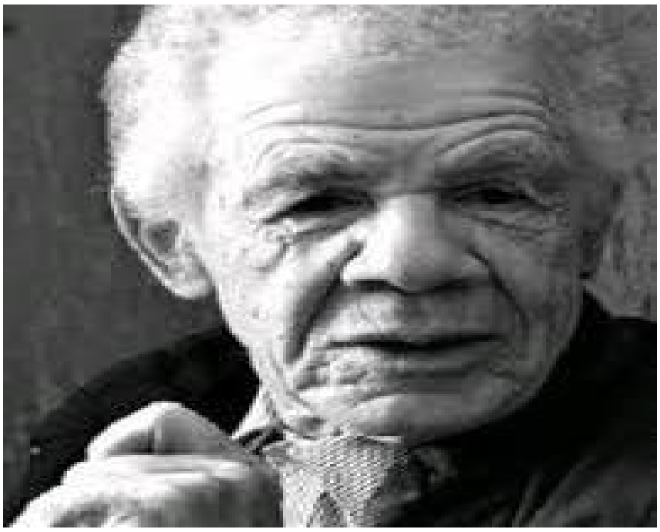
Sensitivity and numerical simulation of unsteady reactive hydromagnetic Eyring-Powell flow in a non-horizontal channel

Yusuf Tijani (Rhodes University)

This study investigates the unsteady reactive magnetohydrodynamic Eyring-Powell fluid in a microchannel, incorporating the effects of suction/injection and heat source. The governing nonlinear deterministic two-variables differential equations, derived from the principles of conservation of mass, momentum, and species concentration, are transformed into a non-dimensional system using appropriate similarity variables. The coupled equations are solved numerically using the implicit finite difference method (IFDM), which ensures stability and accuracy for stiff systems. Sensitivity analysis is perform to check the impact of the parameters at the walls of the channel. The influence of key parameters such as the Brinkman number, Grashof number, suction/injection Reynolds number parameter, chemical reaction rate, and Eyring-Powell fluid parameters on the concentration, temperature, and velocity profiles is discussed. This work provides valuable insights into the behaviour of non-Newtonian fluids in a microchannel under the combined effects of some thermophysical properties, with potential applications in microfluidics, biomedical devices, and chemical engineering processes.

TW KAMBULE MATHEMATICAL SCIENCES BUILDING

(a) TW Kambule Mathematical Sciences Building



(b) Dr Thamsanqa W. Kambule

In September 2017, in its continuing initiative to name and rename several University places and spaces, the University's Naming Committee approved the renaming of the Mathematical Sciences Building to the TW Kambule Mathematical Sciences Building after Dr Thamsanqa Wilkinson "Wilkie" Kambule.

Dr Kambule is widely acknowledged as an inspirational teacher and leader who fought for high quality black education in apartheid South Africa. He was a leading mathematics educator who joined Wits in 1978 as a Senior Tutor in the then Department of Mathematics, serving as a role-model for about a decade. He was elected to the Council of the University in 1989 and awarded an honorary doctorate from Wits in 1997. In 2002, Dr Kambule received the Order of the Baobab in Gold for his exceptional contribution to mathematics education, human development and community service. This was followed by an honorary doctorate from the University of Pretoria and honorary membership of the Actuarial Society of South Africa.

Dr Kambule held a Teachers Diploma from Adams College (1946) and a BA degree from UNISA (1954) and also founded the Pace College in Soweto. Dr Kambule is an example of a mathematician who served the field of mathematics and mathematics education in the country with distinction. It is hoped that by having renamed the former Mathematical Sciences Building after a leading African mathematician, it will inspire future generations of mathematical scientists to excel in the discipline.

* *The above extract was taken from a communication from Wits VCO News sent on 11th October 2017.*