



The 47th Annual South African Symposium on Numerical & Applied Mathematics (SANUM)

30 March to 01 April 2026

Department of Mathematical Sciences
Stellenbosch University, South Africa



Stellenbosch
UNIVERSITY
IYUNIVESITHI
UNIVERSITEIT

forward together
sonke siya phambili
saam vorentoe

Local organising committee: Nick Hale, Emma Nel, & André Weideman

The organising committee acknowledges generous sponsorship from the DSI-NRF Centre of Excellence - Mathematical and Statistical Sciences and the London Mathematical Society.



Programme:

		Monday 30 March			Tuesday 31 March			Wednesday 1 April		
Start	To	Room 1005	Room 2002	Room 3001	Room 1005	Room 2002	Room 3001	Room 1005	Room 2002	Room 3001
8:30	9:00	Workshop registration @ GenEng foyer								
9:00	9:50	Dedalus workshop @ MechEng2002			Milewski			Vasil		
9:50	10:40				Sibanda			Nel	Jafari	Fabris-Rotelli
10:40	11:10				Tea and coffee @ 1006			Tea and coffee @ 1006		
11:10	11:35				Landi	Nefale	Alexeeva	van Rensburg	Spoerer	Jooste
11:35	12:00				Ruaro	Mkhatshwa	Smuts	Labuschagne	Marime	Venter
12:00	12:25	Jansma	Nkwana	Barashenkov	Close					
12:25	13:45	Lunch @ Neelsie			Lunch @ Neelsie					
		Registration @ 13:00-13:30								
		Welcoming @ 13:30-13:45								
13:45	14:35	Fornberg			Mathebula-Periola					Session key:
14:35	15:25	Bargmann			Hansraj					RBF methods
15:25	15:55	Tea and coffee @ 1006			Tea and coffee @ 1006				Mins	Flow
15:55	16:20	Nordstrom	Mbokoma	Kara	Mnisi	C Hansraj	Olivier	Plenary talk	50	Math bio
16:20	16:45	Harley	Nyathi	Cloete	Nkomo	Nandi	Weideman	Contributed	25	Nonlinear waves
16:45	17:10	Nchalbeleng			Maregere	Joshi	Rundora	Tea/coffee	30	Spectral methods
								Lunch time	80	Gravity
17:15		Conference photograph @ 17:15								Gen. Numerics
17:20		Reception / posters @ 17:20								Geospatial
18:00					Conference dinner @ 18:00					Beams
										Misc

For changes to the timetable, see <https://sanum.github.io/programme.html>

Presentation details:

- Please load your presentations on the available machines **before** the session in which you are speaking.
- If you are using your own laptop for the presentation, please check the connections in **advance**.
- A laser pointer and a clicker will be available in each room.
- Contributed and special session talks will be kept **strictly** to 25 (20+5) minutes.
- The first speaker in each contributed session will chair, with the second speaker acting as chair for the first talk.
- Student speakers should identify themselves as such to the session chair. The chair should arrange for senior academics in the audience to complete one of the available scoring rubrics for awarding student prizes.

Welcome to SANUM! The SANUM meeting came into existence in 1975 when two eminent mathematicians, Lothar Collatz (Hamburg) and Fritz John (New York), happened to visit South Africa at the same time. A number of local researchers used the opportunity to organise a meeting at the (then) University of Natal, at which a total of ten papers were presented. From here the meeting grew in strength, with numerous famous numerical and applied mathematicians passing through. With a only few exceptions, SANUM has taken place every year since, and the 2026 marks the 47th anniversary of the meeting.

Conference Venue: The 2026 conference is held on the first two floors of the Mathematical Sciences building (number 67 on the campus map – next page). On-site registration and name badge collection takes place on Monday 12:30–13:30 in the foyer on the ground floor. (Delegates attending the workshop may register at the workshop venue on Monday morning from 8:30AM.)

Wifi details: eduroam is available throughout the venue. Logins can be obtained at registration.

Tea & Coffee: Tea & Coffee will be available in room 1006 at the times indicated on the conference programme. (There will be no coffee at registration.)

Lunch Venue: In your registration pack, you will find vouchers for use in the Neelsie Student centre, located on the Rooiplein approximately 100m from the conference venue. Each voucher is loaded with R375 to be used for lunch/coffee. Vouchers may be used at any of the following stalls: Buzz Juice, DCM, Free Bird, Jacks Bagels, Jeff’s Place, Kauai, Maties Coffee Hub, Miriams Kitchen, Moody Blue, My Brew, Nca’Kos, Panda, Romans, Sweetbeet, Tamaties, Toast, Vida e café Neelsie, Water@Neelsie.

Conference Photo: Please assemble in the foyer after the final talk on Monday so that we can take a photo of all the attendees!

Conference Reception: A Canapé welcome reception will be held at 5:15PM on Monday in the foyer of the Mathematical Sciences building.

Conference Dinner: The conference dinner will take place at Neil Ellis wine farm, (approximately a 10 minute drive from the conference venue) at 18:00 on Wednesday. Transportation will be organised during the conference.

Safety: The green dashed line on the map is the “Green Route” which is actively patrolled by campus security in the late evenings and at night. However, although Stellenbosch is a quiet and relatively safe town, it is not entirely free of crime. We therefore advise delegates to take normal safety precautions at night time, such as travelling in groups, avoiding dark alleys, and keeping valuables out of sight.

Emergency contacts: Campus Security, +27 (0)21 808 2333, Emergency services, 112, Police, 10111.

47 SANUM meetings:

1. Durban, 10–11 April 1975
2. Durban, 8–9 April 1976
3. Durban, 6–7 April 1977
4. Durban, 18–20 July 1978
5. Durban, 18–20 July 1979
6. Durban, 21–23 July 1980
7. Durban, 20–22 July 1981
8. Durban, 19–21 July 1982
9. Durban, 18–20 July 1983
10. Ballito, 2–4 July 1984
11. Umhlanga Rocks, 8–10 July 1985
12. Umhlanga Rocks, 14–16 July 1986
13. Umhlanga Rocks, 13–15 July 1987
14. Umhlanga Rocks, 11–13 July 1988
15. Umhlanga Rocks, 17–19 July 1989
16. San Lameer, 9–11 July 1990
17. Umhlanga Rocks, 15–17 July 1991
18. Durban, 13–15 July 1992
19. San Lameer, 12–14 July 1993
20. Umhlanga Rocks, 4–6 July 1994
21. Scottburgh, 10–12 July 1995
22. Cape Town, 15–17 April 1998
23. Stellenbosch, 29–31 March 1999
24. Stellenbosch, 3–5 April 2000
25. Stellenbosch, 9–11 April 2001
26. Stellenbosch, 3–5 April 2002
27. Stellenbosch, 31 March–2 April 2003
28. Stellenbosch, 5–7 April 2004
29. Stellenbosch, 30 March–1 April 2005
30. Stellenbosch, 3–5 April 2006
31. Stellenbosch, 2–4 April 2007
32. Stellenbosch, 2–4 April 2008
33. Stellenbosch, 6–8 April 2009
34. Stellenbosch, 15–17 April 2010
35. Stellenbosch, 23–25 April 2011
36. Wits, 2–4 April 2012
37. Stellenbosch, 3–5 April 2013
38. Wits, 14–16 April 2014
39. Pretoria, 30 March–1 April 2015
40. Stellenbosch, 22–24 March 2016
41. Wits, 28–30 March 2017
42. Stellenbosch, 04–06 April 2018
43. Pretoria, 27–29 March 2019
44. Johannesburg, 3–5 April 2023
45. Stellenbosch, 2–5 April 2024
46. Wits, 2–4 April 2025
47. **Stellenbosch, 30 March–01 April 2026**

STELLENBOSCH CAMPUS MAP INDEX

SUPPORT SERVICES

- 1 Administration, Block A
- 2 Administration, Block B
- 3 Administration, Block C
- 4 JS Geriecke Library
- 5 Neelsie Student Centre
- 6 Campus Security
- 7 Centre for Student Recruitment
- 8 Centre for Student Counselling and Development: Reception (CSCD)
- 9 CSCD: Office for Students with Special Learning Needs (Disabilities); Den Bosch
- 10 CSCD: Unit for Psychotherapeutic and Support Services
- 11 Centre for Teaching and Learning
- 12-14 Division for Student Affairs
- 15 Language Centre: Reading Lab and Language Enrichment Courses
- 16 Campus Health Services
- 17 Matties Sport
- 18 Coetzenburg Centre
- 19 Old Conservatoire
- 20 SU Art Gallery
- 21 University Museum
- 22 Centre for Learning Technology
- 23 Language Centre (Writing Lab)
- 24 WAT, 115 Banghoek Road
- 25 Mattie Community Services, Luckhoff School
- 26 SU Vehicle Fleet
- 27 Legal Aid Clinic
- 28 IT Hub (Help Centre)
- 30 Purchasing and Provision Services
- 31 Facilities Management
- 32 Information Technology

SPORTS FACILITIES

- 33 Danie Craven Stadium
 - 34 Coetzenburg Athletics Stadium
 - 35 Coetzenburg Tennis Courts
 - 36 PSO Club House and Hockey Fields
 - 37 Netball Courts
 - 38 Swimming Pool
 - 39 SU Gymnasium
 - 40 Tennis Courts (Residences)
 - 41 Old Mutual Sports Centre (squash courts)
 - 42 Tennis Courts
 - 43 Heidehof Rugby Fields
- ACADEMIC BUILDINGS**
- 44 Food Science
 - 45 PO Sauer
 - 46 Electrical/Electronic Engineering
 - 47 Civil Engineering
 - 48 Mechanical/Mechatronic/Industrial Engineering

- 49 Process Engineering
- 50 Knowledge Centre
- 51 Engineering, General
- 52 Africa Centre for HIV and AIDS Management
- 53 JC Smuts – Biological Sciences
- 54 De Beers – Chemistry
- 55 Mike de Vries
- 56 Chemistry – first-years
- 57 Inorganic Chemistry
- 58 CGW Schumann
- 59 Van der Sterr
- 60 Accounting and Statistics
- 61 JS Marais
- 62 Polymer Science
- 63 Visual Arts
- 64 JH Neethling
- 65 AI Perold
- 66 Merensky
- 67 Mathematical Sciences and Industrial Psychology
- 68 Nursery
- 69 Natural Sciences
- 70 Chamber of Mines
- 71 RW Wilcocks
- 72 Old Main Building
- 73 HB Thom Theatre
- 74 CL Marais Library
- 75 Journalism
- 76 GG Cillie
- 77 Arts and Social Sciences
- 78 Lombardi
- 79 Konservatorium (University Choir)
- 80 Theological Seminary
- 81 Paul van der Bijl Laboratories
- 82 Agronomy
- 83 Welgevallen Experimental Farm
- 84 Sport Science

WOMEN'S RESIDENCES

- 85 Monica
 - 86 Harmonie
 - 87 Heemstede
 - 88 Huis ten Bosch
 - 89 Lydia
 - 90 Minerva
 - 91 Nerina
 - 92 Erica
 - 93 Nemesia
 - 94 Serruria
 - 95 Tinie Louw Dining Hall: amaMatties hub
 - 96 Sonop (Huis van Niekerk)
 - 97 Irene
- MEN'S RESIDENCES**
- 98 Feendrag
 - 99 Helshoogte

- 100 Simonsberg
- 101 Huis Visser
- 102 Huis Marais
- 103 Dagbreek
- 104 Majuba
- 105 Wilgenhof
- 106 Helderberg

MEN'S AND WOMEN'S RESIDENCES

- 107 Huis Neethling
- 108 Metanoia
- 109 Russel Botman House: ▲ Wimbledon hub
- 110 Academia
- 111 Concordia
- 112 Goldfields
- 113 Huis McDonald

UNIVERSITY FLATS AND HOUSES

- 114 Lobelia
- 115 Crozierhof
- 116 Huis de Villiers
- 117 Waldenhof

LISTEN, LIVE AND LEARN

- 118 LLL village
- LLL houses

CLUSTER HUBS

- ▲ amaMatties 95
- ▲ Wimbledon 108

FACILITIES (IN COLOUR) AND DEPARTMENTS

- Accounting 59
- African Languages 77
- Afrikaans en Nederlands 77
- Agricultural Economics 61

- AGRICULTURES** 44, 45, 53, 55, 61, 64, 78, 81–83
- Agronomy 82
- Ancient Studies 77
- Animal Sciences 55
- Applied Mathematics 51

ARTS AND SOCIAL SCIENCES

- 63, 70, 71, 73, 75, 77, 79
- Biochemistry 53
- Business Management 58
- Botany and Zoology 69
- Chemistry and Polymer Science 54–57, 62
- Computer Science (Math. Sc.) 51
- Conservation Ecology and Entomology 61
- Drama 73
- Earth Sciences 70

- ECONOMIC AND MANAGEMENT SCIENCES** 52, 58–60, 67

EDUCATION

- 76, 84

ENGINEERING

- 47–51

- English 77
- Food Science 44
- Forest and Wood Sciences 45
- General Linguistics 77
- Genetics 53
- Geography and Environmental Studies 70
- History 71
- Horticulture 78
- Industrial Psychology 67
- Information Science 77
- Institute for Wine Biotechnology 64
- Institute for Plantbiotechnology 69
- Journalism 75

LAW

- 72

- Mathematics (Math. Sc.) 67
- Microbiology 53
- Modern Foreign Languages 77
- Music 79
- Philosophy 77
- Physics 66
- Physiological Sciences 55
- Plant Pathology 78
- Political Science 77
- Psychology 71
- Public Development Management 59

SCIENCE

- 51, 53–57, 59, 62, 65, 66, 67, 70

- Social Work 77
- Sociology and Social Anthropology 77
- Soil Science 78
- Sport Science 84
- Statistics and Actuarial Science 60

THEOLOGY

- 80
- Visual Arts 63
- Viticulture and Oenology 64

P

- Parking

Plenary abstracts

Mathematical and computational modeling of biological and tailor-made high-performance materials

Prof Swantje Bargmann (University of Wuppertal, Germany)

Biological materials (e.g., teeth, wood, turtle shells, skeletal muscles) derive their exceptional mechanical performance from complex structure–property relationships across multiple length scales. These natural systems serve as role models for the development of tailor-made high-performance materials, where geometry, hierarchy, and composition are systematically engineered to achieve specific functionalities. Many of today’s key technological innovations are rooted in such bio-inspired materials design.

This talk focuses on the mathematical and computational modeling of biological and architected materials. Alongside experimental and materials-science efforts, the theoretical community has advanced rigorous mathematical formulations—from continuum models and nonlinear constitutive laws to homogenization and multiscale theories—that provide the foundation for computational analysis. Theoretical and computational strategies naturally extend to tailor-made high performance materials such as nanoporous gold and solids based on triply periodic minimal surfaces (TPMS). Both systems mimic key features of biological structures: nanoporous gold exhibits nanoscale ligament–void networks analogous to trabecular or foam-like biological tissues, while TPMS architectures echo the smooth, continuous, and mechanically efficient geometries found in shells and skeletal frameworks. Their mechanical response can be understood—and optimized—through the same mathematical principles used to analyze biological systems.

By linking natural and artificial hierarchical materials through a shared mathematical language, this talk highlights how mathematical modeling enables the rational design of next-generation high-performance materials.

Radial Basis Functions (RBFs) - A half-century journey from concept to mainstream numerics

Prof Bengt Fornberg (University of Colorado Boulder, USA)

The history of finite differences can be traced back to the 16th century. Just over a century ago, these were for the first time used to solve PDEs. Numerous variations have since then been developed to meet increasing needs for geometric flexibility and for high orders of accuracy. The breakthrough we presently focus on started somewhat unexpectedly around 1972 when radial basis functions (RBFs) proved to be superior for generating elevation contours on maps. About 20 years later, it was realized that this interpolation method was also well suited for approximating derivatives and could be used for solving PDEs. Initial obstacles in terms of computational cost and numerical stability were then gradually overcome. In the form of RBF-FD (RBF-generated Finite Differences), we have now arrived at a very flexible method that can naturally combine very high orders accuracy and complete geometric flexibility with simple implementations and computational effectiveness. While this presentation focuses on the task of solving PDEs, application areas of RBFs are much broader and include neural networks, pattern recognition, computer graphics and general modeling of complex data and systems.

Numerical techniques in the mathematics of gravitation

Prof Sudan Hansraj (University of KwaZulu-Natal)

The gravitational field remains the most mysterious of the four fundamental forces in nature. The general theory of relativity (GR) built from the Einstein-Hilbert lagrangian is the most successful theories of gravity to date. Nevertheless, it has a number of serious shortcomings necessitating modifications. Currently GR relies on the possible existence of exotic matter for which no support has emerged to date. However, a different school of thought works on extending the geometrical sector to accommodate what is actually observed. As a consequence a number of modified theories have emerged in the recent past in an effort to succeed GR. Each of these must also pass basic tests such as diffeomorphism and Lorentz invariance and the Bianchi identities. When GR is applied to construct models of stars, a coupled system of 10 nonlinear partial differential equations arises in general in four dimensions. The number reduces to 3 if spherical symmetry is assumed however in the case of isotropic or perfect fluid matter field there are 4 unknowns. This means that an extra closure condition must be stipulated. The most important physical constraint that could be imposed is an equation of state relating two of the variables. The caveat is that the master differential equation is not solvable exactly for physically relevant cases. Numerical schemes must be implemented to reveal the characteristics of the gravitational field. Ad hoc assumptions on some of the dynamical variables may also be made for mathematical expediency however the likelihood of the solution satisfying an equation of state is slim. Some 120 exact solutions have been reported over the past century. Of these only fewer than 10 tick all the boxes for physical acceptability. The trade-off going down the numerical route, on the other hand, is that while the dynamics may be understood to high degrees of accuracy, the geometry in the form of the metric remains unknown. In this talk we shall discuss the numerical schemes employed to solve the Tolman-Oppenheimer-Volkoff equation which emanates from the conservation laws also known as the condition for hydrodynamical stability or continuity equation. This equation follows from the 3 field equations however it may substitute any one. The system has precisely 3 independent equations. We will examine how the mass-radius relations can be found using numerical techniques. Finally we shall comment on how the latest information coming from the Nobel-prize winning work on gravitational waves can be used in the study of stellar structure.

Dynamical systems modelling of infectious diseases in rural and resource-limited settings: From theory to impact

Prof Dephney Mathebula-Periola (University of Fort Hare)

This talk focuses on how mathematical biology and dynamical systems theory contribute to understanding infectious disease systems in rural and resource-limited settings. Building on biomathematical, multiscale, and optimal control modelling approaches, the talk highlights how context-informed frameworks address key challenges such as limited data availability, population heterogeneity, and constrained intervention options. Selected case studies illustrate how mathematical insights can inform sustainable and cost-effective disease control strategies, bridging the gap between theory and real-world public health practice. The talk concludes by discussing key lessons learned and future directions for impactful mathematical modelling of infectious diseases in rural and limited-resource settings.

Resonance of surface water waves in cylindrical containers

Prof Paul Milewski (Penn State University, USA)

Nonlinear waves sloshing in a container of rectangular cross-section can behave very differently than those with other cross sections. Nonlinear resonance is a mechanism by which energy is continuously exchanged between a small number of wave modes and is common to many nonlinear dispersive wave systems. In the context of free-surface gravity waves such as ocean surface waves, nonlinear resonances have been studied extensively over the past 60-years, almost always on domains that are large (or infinite) compared to the characteristic wavelength. In this case, the dispersion relation dictates that only quartic (4-wave) resonances can occur. In contrast, nonlinear resonances in confined three-dimensional geometries have received relatively little attention, where, perhaps surprisingly, stronger 3-wave resonances can occur. We will present the results characterizing the configuration and dynamics of resonant triads in cylindrical basins of arbitrary cross sections, demonstrating that these triads are ubiquitous, with (the commonly studied) rectangular cross section being an exception where they do not occur. I will also mention environmental and engineering applications.

A spectral optimized multi-derivative hybrid block method for FitzHugh-Nagumo equations

Prof Precious Sibanda (University of KwaZulu-Natal)

The FitzHugh-Nagumo equations provide a canonical model for excitable systems in biology and neuroscience and pose significant challenges due to their nonlinear and stiff character. This talk presents a spectral optimized multi-derivative hybrid block method for their efficient numerical solution. The time integration scheme is derived using a multistep collocation and interpolation approach based on an approximated power series and incorporates two optimally selected intra-step points to enhance accuracy and stability. The method is shown to be consistent and convergent, with its absolute stability properties rigorously established.

For the spatial discretization of the associated partial differential equations, the time integrator is coupled with a spectral collocation method, following a linear partitioning of the governing equations. Numerical experiments demonstrate that the resulting fully discrete scheme achieves high accuracy and efficiency compared with existing methods, highlighting its effectiveness for nonlinear evolution problems requiring strong stability and spectral-level resolution.

Sphere we go again: lessons on solving equations in curved spaces with polynomials and tensors

Prof Geoff Vasil (University of Edinburgh, UK)

Numerical analysis is full of perennial tradeoffs: speed versus flexibility, realism versus tractability, and (more subtly) freedom versus commitment. Make too few choices and expect inefficiency; make too many, and lock yourself into a brittle, over-specialised pipeline. This talk is about a sweet spot that appears repeatedly in mathematics and computation: choosing just enough structure.

Spheres, balls, and disks sit right in that sweet spot. They are not the geometry of a race car, but they are excellent for stars, planets, and many laboratory experiments, simple enough to admit powerful representations and rich enough for serious physics. I'll begin with spherical harmonics as the canonical basis for scalar fields on the sphere, then explain why vector and tensor PDEs force you into the broader (and often underused) world of spin-weighted spherical harmonics, with their clean calculus and beautifully sharp identities.

Moving from surfaces to volumes brings in a parallel cast: Zernike polynomials. I'll describe my efforts to build a hierarchy of generalised Zernike-type polynomial bases that support vector and tensor fields across dimensions. The story becomes even more interesting once curvilinear basis elements enter the game, naturally leading to discrete structures that expose hidden algebra and yield non-numerical insights. Along the way, I'll present many motivating applications, for example, elastically driven turbulence in pipe flow and fully compressible convection in a spherical star, with turbulence extending all the way to the centre.

Contributed abstracts

Variational and numerical description of Φ^4 oscillons

Nora Alexeeva, Igor Barashenkov, Alain Dika, & Raphael De Sousa (UCT)

We use two different methods to study the existence and stability of the Φ^4 oscillon. The variational technique approximates it by a periodic function with a set of adiabatically changing parameters. An alternative approach treats oscillons as standing waves in a finite-size box; these are sought as solutions of a boundary-value problem on a two-dimensional domain. The numerical analysis reveals that the standing wave's energy-frequency diagram is fragmented into disjoint segments with $\omega_{n+1} < \omega < \omega_n$, where $\omega_n = \omega_0/(n+1)$, $n = 0, 1, 2, \dots$, and ω_0 is the endpoint of the continuous spectrum (mass threshold of the model). The variational approximation involving the first, zeroth and second harmonic components provides an accurate description of the oscillon with the frequency in (ω_1, ω_0) , but breaks down as ω falls out of that interval.

Swinging Waves in the Ablowitz-Ladik Equation

Igor Barashenkov & Frank S Smuts (UCT)

We construct a novel family of exact cnoidal wave and soliton solutions of the focusing and defocusing Ablowitz-Ladik equations. Unlike cnoidal waves that were obtained by earlier authors, the phase variable of the new solutions exhibits a nonlinear dependence on time and site number; the wave “swings”. Our approach hinges on the existence of a two-point map governing the absolute value of the complex field; this map gives rise to standing waves centred arbitrarily relative to the lattice sites. Having derived stationary solutions, we use these as a basis for constructing waves with nonzero velocity. The localised members of the new family comprise dark solitons with the nontrivial asymptotic behaviour. We identify periodic and quasiperiodic patterns and establish an explicit quantisation rule for the velocity of the wave circulating in a closed loop of N sites.

Stochastic optimization method for large deviations of Markov diffusion processes

Daniël Cloete & Hugo Touchette (Stellenbosch University)

Rare fluctuations of trajectory-dependent observables in stochastic dynamical systems are difficult to study using direct simulation because the events of interest occur with very low probability. Large deviation theory provides a framework for quantifying these fluctuations through the rate function and associated driven process, but computing them is often computationally demanding. We present a stochastic optimization method for estimating large deviation properties of time-additive observables of Markov diffusion processes. The approach is based on a variational characterization of the process that samples rare trajectories most efficiently; by parameterizing this process and optimizing a corresponding objective, the method uses stochastic optimization techniques inspired by machine learning to approximate the relevant large deviation quantities. The method and its numerical implementation are illustrated by considering nonequilibrium work fluctuations in a two-dimensional transverse diffusion.

The Discrete Pulse Transform - a nonlinear filter for image and spatial analysis

Inger Fabris-Rotelli (University of Pretoria)

The LULU operators L_n and U_n operate on neighbourhoods of size n . The Discrete Pulse Transform (DPT) of images is obtained via recursive peeling of the local maximum and minimum sets with the LULU operators as n increases from 1 to the maximum number of elements in the array. The DPT then provides a nonlinear decomposition of a multidimensional array. This nonlinear decomposition will be unpacked and its use cases in image processing and spatial analysis presented.

Advection, diffusion, dispersion, and all that jazz

Nick Hale & JAC Weideman (Stellenbosch University)

In an example in their complex variables textbook, Ablowitz and Fokas apply the method of steepest descent to derive an asymptotic estimate for the solution of the linear Korteweg-de Vries equation for large times. Here the analysis is extended to an equation with a fractional dispersion relation, which contains this KdV equation (which models dispersion), the heat equation (diffusion), and the one-way wave equation (advection) as special cases.

Classifying shear-free perfect fluids: a master equation

Chevarra Hansraj (Stellenbosch University), Jonathan Hakata, Rituparno Goswami, & Sunil D Maharaj (UKZN)

Shear-free perfect fluids in general relativity have an intriguing property: despite the absence of shear, they can still exhibit inhomogeneity and tidal effects. Using a 1+1+2 covariant spacetime decomposition, we derive a complete classification of all spherically symmetric shear-free perfect fluid spacetimes based on the fluid acceleration and expansion. The most interesting class, "dynamical, inhomogeneous, and shear-free", is shown to exist only for very specific equations of state. These are governed by a highly nonlinear, fourth-order autonomous ordinary differential equation that we derive from first principles and analyse both numerically and from a dynamical systems perspective. Our results demonstrate how geometric analysis in general relativity can reduce to concrete, numerically tractable problems in ODEs and dynamical systems, opening new avenues for exploring the interplay between matter content and spacetime geometry.

Stabilizing the nonlinear initial boundary value problem governing thin film flow

Charis Harley (UJ), Jan Nordström (Linköping University & UJ), & Ebrahim Momoniat (UJ)

We solve the nonlinear initial boundary value problem that arises in the study of thin film flows. The generally accepted version is shown not to lead to an energy bound and stability. To deal with that, we firstly consider modifications of the original equation. Secondly, we develop new splitting techniques for the discretisation of the nonlinear terms. Finally, we provide boundary conditions and an implementation procedure that leads to provably conservative and stable nonlinear schemes. Our numerical calculations corroborate the theoretical developments, exemplify the influence of the modifications and reveal new parameter dependencies.

A Comparison Between Some Iterative Methods

Hossein Jafari (UNISA)

In the past centuries, there was no problem or fault in misleading names for same definitions or similar methods due to lack of access to information such as journals and internet. One of the serious challenges in mathematics is the originality of the new definitions and methods which are presented in the last decades. With a little research and accuracy in some of them, it can be shown that some of these definitions and methods are not new but they have new names! I will illustrate in this talk that there are connections between these methods with the classical methods or some other existing methods.

Bayesian inference captures metabolite-bacteria interactions in a microbial community

Jack Jansma, Pietro Landi, & Cang Hui (Stellenbosch University)

Macro-ecosystems, including the human gut, host a vast and diverse set of microbes that indirectly interact with each other through consuming and producing metabolites. Disruptions in this microbial network can affect macro-ecosystem functioning and, in the human gut, contribute to the onset and progression of various disorders, including diabetes, rheumatoid arthritis and Parkinson's disease. A theoretical foundation for understanding the intricate and dynamic interactions between microbes and metabolites is essential for developing microbiota-targeted interventions to improve macro-ecosystem functioning and health. To this end a precise mathematical framework is crucial to capture and quantify the complex dynamics of the microbial system. Here, we develop a dynamic network model of coupled ordinary differential equations and present a computational workflow that integrates a generative model with Bayesian inference for model identification. Our approach infers interaction rates, quantifying metabolite consumption and production from simulated time-series data within a Bayesian framework, incorporating prior knowledge and uncertainty quantification. We show that our approach is accurate and reliable in communities of various sizes, sparsity and with different levels of observational noise. This workflow enables in-silico predictions of system behaviour under perturbations and offers a robust method to integrate high-dimensional biological data with dynamic network models. By refining our understanding of microbial dynamics, this framework is capable of assessing microbiota-targeted interventions and their potential to improve the health of the macro-ecosystem.

Detector Characterization & Signal Processing in CCD Systems

Bhushan Joshi (Southern African Large Telescope, Stellenbosch University),
& Nicolas Erasmus (South African Astronomical Observatory)

Understanding how well a CCD performs is essential if we want to trust the scientific results we derive from it. In this talk, I will focus on simple and practical ways to estimate two key parameters of a CCD system: gain and noise.

Using standard calibration data, I will demonstrate how the photon transfer method can be used to estimate the system gain from flat-field images and how read noise can be determined from bias frames. Along the way, I will briefly discuss the main sources of noise in CCDs such as photon noise, dark current, and readout noise and how they affect the final image quality.

I will also touch on how signal processing techniques, like correlated double sampling and careful readout design, help in reducing noise and improving performance. The goal of this talk is to present a clear, hands-on approach to detector characterization, supported by simple numerical examples that can be easily applied in practice.

On the zeros of discrete orthogonal polynomials

Alta Jooste (University of Pretoria)

The Hahn, Meixner, Krawtchouk and Charlier polynomials are orthogonal with respect to a discrete weight function, on the finite (or infinite) set of points $\{0, 1, \dots, N\}$. We will discuss results on the spacing of the zeros, and (outer) bounds for the extreme zeros of these polynomials, given by Krasikov and Zarkh [1] in 2009. New results on the zeros of Charlier polynomials from different sequences will be given. The equations used to prove our results are obtained from different contiguous relations satisfied by the Charlier polynomials. We will also discuss the location of the zeros of the associated Charlier polynomials.

[1] I. Krasikov, A Zarkh. On the zeros of discrete orthogonal polynomials. *Journal of Approximation Theory* 156 (2009), 121-141.

On first integrals and reduction of classes of Emden and Lienard Equations

Abdul Hamid Kara (University of the Witwatersrand)

We present a general method to construct first integrals for some classes of the well known second-order ordinary differential equations, viz., the Emden and Lienard classes of equations.

Numerical Floquet Theory: The Dynamics of Numerical Methods

Melusi Khumalo (University of South Africa)

Nonautonomous differential equations possessing periodic solutions arise naturally in a wide range of scientific and engineering applications, including mechanics, electrical circuits, control theory, and biological models subject to periodic forcing. Of particular interest are systems for which the period of the exact solution is known a priori, as this structural information can be exploited in both analysis and numerical approximation.

In this presentation, we focus on a class of simple nonautonomous systems whose solutions are periodic with a prescribed period. We investigate how such systems are transformed when discretized by linearized collocation methods. To analyze the qualitative behavior of the resulting numerical solutions, we employ stroboscopic sampling, which can be viewed as a discrete analogue of the classical Poincaré map obtained by sampling the continuous solution once per period. This approach reduces the study of periodic orbits to the analysis of fixed points of an associated discrete dynamical system.

Using this framework, we examine the stability properties of the numerical schemes and their ability to reproduce the correct long-term behavior of the underlying continuous system. In particular, we demonstrate that there is an inextricable connection between AN-stability (or BN-stability) of the collocation methods and the faithful qualitative reproduction of periodic solutions. Methods lacking the appropriate stability properties may generate spurious growth or decay in the stroboscopic map, leading to incorrect conclusions about the stability of the true periodic orbit. Our results highlight the crucial role of numerical stability theory in the reliable simulation of nonautonomous periodic systems and provide guidance for the selection of collocation schemes in practical computations.

Finite Element analysis of non-linear rod models for small vibrations

Madelein Labuschagne & Kirstin Hohls (University of Pretoria)

Following the research presented previously on the Local Linear Timoshenko (LLT) model for the planar motion of rods, this talk investigates an alternative non-linear simplification: a semi-linear Timoshenko (Sapir-Reiss) model, or SLT-SR.

We compare the properties of these models, specifically focusing on how they handle small vibrations and buckling under axial loads. Using a mixed finite element method, we demonstrate that while linear models predict instability past this threshold, the SLT-SR model accurately captures the transition to stable oscillations around non-trivial buckled equilibria.

Roof Material Classification in South Africa from Aerial RGB Using Discrete Pulse Transform Features

Mark de Lancey & Inger Fabris-Rotelli (University of Pretoria)

This work investigates whether Discrete Pulse Transform (DPT) features can discriminate roof covering material classes in high-resolution aerial RGB imagery. Using standardised per-roof image crops (and footprint-derived roof masks where available), we compare a raw-pixel baseline with compact DPT features computed from the pulse decomposition, using matched, controlled preprocessing and the same lightweight classification pipeline across representations. Performance is assessed using macro-averaged F1 and related metrics to quantify class separability. We find that DPT-derived statistics capture material-relevant texture cues and provide a strong, low-dimensional alternative to raw pixels for roof material recognition. In South African where roof covering material and roof slope are practical indicators for settlement mapping, infrastructure planning, and solar panel feasibility screening—this low-cost, image-based approach could support scalable analysis when LiDAR/DSM coverage or ground survey data are limited.

Eco-evolutionary modelling with Adaptive Dynamics

Pietro Landi (Stellenbosch University)

Adaptive dynamics is a mathematical modelling framework based on ordinary differential equations to describe evolutionary dynamics driven by the underlying ecological interactions. When such interactions are mediated by phenotypes (traits), adaptive dynamics allows the definition of invasion fitness and selection gradient, that drive the evolution of the traits. Furthermore, when selection vanishes, higher order derivatives of the fitness determine the possibility of evolutionary branching and phenotypic diversification. This framework is applied to a variety of ecological models.

FOFD-PINN for Singularly Perturbed Turning Point Problems with Interior Layers

William Manamela (UWC), Justin Munyakazi (UWC), & George Buzuzi (CPUT)

This paper introduces a Fitted Operator Finite difference Physics-Informed Neural Network (FOFD-PINN) method designed to effectively handle singularly perturbed turning point problems characterized by an interior layer. Unlike conventional PINNs that rely on automatic differentiation, the FOFD-PINN method uses finite difference discretization to approximate derivatives in the loss function. Moreover, to ensure the convergence of the FOFD, PINN scheme, an a priori error bound theorem is also included.

Finally, to show the accuracy of the FOFD, PINN approach, some test problems were solved at various values of the perturbation parameter, ε , and the L^2 , errors were calculated and compared with those obtained using the conventional PINN and hard constraint PINN methods. The outcomes confirm that FOFD-PINN is consistently more accurate and requires less training time than the other two methods.

A Multi-scale Modelling of Lymphatic Filariasis Disease System

Bothwell Maregere, Winston Garira, & Dimpho Mothibi (Sol Plaatje University)

Multi-scale modelling of infectious disease dynamics is emerging as a critical area of research, although it remains less developed than traditional single-scale approaches (e.g., transmission mechanism theory). Its importance lies in its ability to inform strategies for the control, elimination, and potential eradication of infectious diseases by capturing interactions across multiple biological and ecological levels (micro-scale and macro-scale). In this study, we introduce a coupled multi-scale model for lymphatic filariasis (LF), a mosquito-borne parasitic disease, formulated at the level of the whole organism. The model integrates replication and transmission processes of the filarial parasite, capturing key mechanistic elements across its full life cycle and addressing important knowledge gaps. The well-posedness of the coupled multi-scale for LF disease system is discussed. The influence of the within-host scale sub-model parameters on between-host scale variables and vice-versa is numerically illustrated. We also evaluate the comparative effectiveness of two intervention strategies: long-lasting insecticide-treated nets (LLINs) and mass drug administration (MDA), using the reproductive number as an indicator of the effectiveness of these interventions. Results suggest that combining interventions at high efficacy offers the most effective strategy for reducing community transmission. Among single interventions, two LLIN-based mechanisms: (i) mosquito-repelling effects and (ii) prevention of mosquito bites on human's effects, emerge as the most effective. By integrating within-host and between-host scales, our framework establishes a foundation for mathematical infrastructure capable of generating insights critical for designing novel tools to monitor, prevent, and treat LF.

A reflection on two NSFD schemes for a malaria model with control

Calisto Blessmore Marime (Stellenbosch University) & Justin B Munyakazi (University of the Western Cape)

In the quest to extract quantitative information on mathematical models, many researchers have used standard numerical methods such as the explicit Euler and the Runge-Kutta schemes. However, these methods suffer from time step restriction and may give rise to spurious and non-physical solutions. In my talk, I will propose nonstandard finite difference (NSFD) methods to solve malaria models. We proved that the proposed NSFD schemes are dynamically consistent with the continuous malaria model regardless of the step size. Dynamic consistency refers to the fact that the NSFD schemes preserve positivity, boundedness, and stability of the solutions and that their equilibrium points and basic reproduction number coincide with those of the continuous model, regardless of the step size. Both theoretical analysis and comprehensive numerical experiments validate the superiority of the developed scheme.

RBF-FD Approach to the Burgers Equation using the Medusa C++ Library

Gabriel Mbokoma (University of South Africa, Namibia University of Science Technology), Marcia Moremedi (University of South Africa), Nguetchue Neossi (Namibia University of Science and Technology), & Celestin Wafo-Soh (Jackson State University)

The Burgers equation is a fundamental nonlinear partial differential equation (PDE) that models transport-diffusion processes in areas such as fluid mechanics and traffic flow. Its combination of nonlinear advection and viscous diffusion presents numerical challenges, particularly in capturing steep gradients and handling shock formation, which complicates stability and convergence. In this work, we present a meshless approach to solve the one- and two-dimensional Burgers equation using the Radial Basis Function–Finite Difference (RBF-FD) method implemented in the Medusa C++ library. The RBF-FD method offers the flexibility of scattered node discretization while retaining the computational efficiency of finite differences. The library provides a streamlined framework for constructing differentiation operators, managing node distributions, and implementing explicit and implicit time-stepping schemes. Our results demonstrate that the RBF-FD approach achieves high accuracy with relatively few nodes, maintains stability across a range of viscosity parameters, and is easily extendable to higher-order problems.

Non-similar solutions for Darcy-Forchheimer flow with multiple dissipative mechanisms: A spectral collocation approach

Musawenkosi Mkhathshwa (University of Limpopo), Muhammad Afridi (Hanjiang Normal University, China), Muhammad Qasim (COMSATS University Islamabad, Pakistan), & Ali Chamkha (Kuwait College of Science and Technology, Kuwait)

The Darcy-Forchheimer flow of an electrically conducting, dissipative fluid over a stretching flat sheet is investigated, considering three distinct dissipative mechanisms: viscous, ohmic, and porous dissipation. The governing equations are modelled to analyze the flow and thermal characteristics within the boundary layer region using suitable assumptions. A novel transformation is introduced wherein the Forchheimer number varies along the streamwise direction. This transformation yields a set of non-similar partial differential equations from the governing boundary layer equations. The numerical solutions of these nonlinear equations are obtained using an efficient computational approach, namely the overlapping multi-domain spectral collocation scheme. The influence of various flow parameters on different flow characteristics and relevant quantities is presented graphically and discussed in detail. Important findings include that the velocity profile decreases with increasing magnetic parameter and Forchheimer number but increases with a higher porosity parameter. Temperature gradients near the wall intensify at higher Eckert numbers and magnetic parameters, while thermal boundary layer thickens with a higher Forchheimer number. These findings are significant for optimizing engineering systems involving porous materials, heat exchangers, and systems influenced by magnetic fields and inertial effects.

Maximizing Malaria Elimination: A Mathematical Model for Optimizing IRS, LLINs, and ATSB Interventions

Siyamthanda Gift Mnisi (University of KwaZulu-Natal), Hloniphile Sithole Mthethwa (University of KwaZulu-Natal), & Vusi Mpendulo Mgagula (University of Eswatini)

Malaria is a public health problem for more than 2 billion people globally. About 219 million cases of malaria occur worldwide and 660,000 people die, mostly (91%) in the African Region despite decades of efforts to control the disease. Although the disease is preventable, it is life-threatening and parasitically transmitted by the bite of the female *Anopheles* mosquito. A deterministic mathematical model with intervention strategies is developed in order to investigate the effectiveness and optimal control strategies of indoor residual spraying (IRS), long lasting insecticide nets (LLINs) and attractive toxic sugar bait (ATSB). This model executed two equilibrium points, namely the disease free and epidemic equilibrium points. The DFE was proved to be stable when $\mathcal{R}_0 < 1$ and the EE was proved to be stable when $\mathcal{R}_0 > 1$. The sensitivity analysis was computed using the PRCC technique. MATLAB ode45 solver is used to compute a numerical analysis of the population dynamics. Optimal control analysis was introduced and solved using Pontryagin's maximum principle. The dynamics of the optimal control showed that the infections decrease with an increase in the control to reduce the disease.

Overlapping grid block hybrid method for unsteady second grade tetra hybrid nanofluid flow through a stretching Riga surface with microgravity conditions

Juliet Molepo & Musawenkosi Mkhathshwa (University of Limpopo)

The suspension of non sized nanoparticles with base fluid has gained attention to researchers. This study examine the unsteady second grade tetra hybrid nanofluid flow through a stretching Riga surface with microgravity environment. The governing nonlinear partial differential equations are transformed into non dimension system of ordinary differential equations using suitable transformation. The dimensionless equations are solved using overlapping grid block hybrid method. The flow and thermal performance for the tetra hybrid case is greater than trihybrid cases subject to microgravity environment. The impact of several parameters on the velocity, temperature, skin friction and Nusselt number are discussed in graphics.

Quantum Structure of Spacetime and Its Imprint on Hawking Radiation

Partha Nandi (Stellenbosch University)

Hawking radiation reveals a deep connection between gravity, quantum theory, and thermodynamics. In the standard semiclassical description, quantum fields propagate on a classical spacetime background, leading to the well-known Bekenstein–Hawking entropy–area relation. However, if spacetime itself possesses an underlying quantum structure, this picture may be modified. In this talk we explore how such quantum spacetime effects can influence Hawking radiation. Using the quantum Hamilton–Jacobi formulation of the tunneling method, we analyze quantum corrections to the particle action and show how they generate logarithmic corrections to black hole entropy. We then demonstrate that the coefficient of this correction is controlled by the trace anomaly of the quantum stress tensor. Motivated by the possibility that spacetime coordinates may become non-commutative at short distances, we investigate Hawking radiation in a non-commutative black hole geometry. We show that the non-commutative parameter modifies the curvature invariants entering the anomaly and consequently alters the coefficient of the logarithmic entropy correction. Our results suggest that the quantum structure of spacetime can leave imprints on Hawking radiation, providing a possible window into quantum gravity effects.

A pre-existing fluid-driven permeable fracture with Darcy flow

Mathibele Nchabeleng (University of Pretoria) & Adewunmi Fareo (University of the Witwatersrand)

This study examines the propagation of a pre-existing fluid-driven fracture in a permeable rock. Incompressible laminar Newtonian fluid drives the fracture which experiences fluid loss through the fracture interface into the surrounding rock matrix. Because the Carter’s model derived from Darcy law has its many flaws, a new model for the fluid loss relating the leakoff depth to the net fluid pressure in the fracture is employed in this work. The elasticity of the rock is modelled using the Khristianovic-Geertsma-de Klerk (KGD) model. Starting out with lubrication equations, a system of partial integro-differential equations relating the width of the fracture to the net pressure and the leak-off depth is derived. Similarity solutions derived for the fracture half-width, net pressure, and depth of leak-off are used to reduce the system of partial integrodifferential equations to a system of ordinary integro-differential equations. Numerical results are obtained for the fracture length, fracture half-width, leak-off depth and the net fluid pressure.

Block Hybrid Methods for Second-Order Boundary Value Problems

Mpho Mendy Nefale (University of the Witwatersrand), Olumuyiwa Otegbeye (University of the Witwatersrand), & Shina Daniel Oloniju (Rhodes University)

This study introduces a block hybrid method (BHM) in obtaining approximate solutions to second-order nonlinear boundary value problems (BVPs). The approach involves partitioning the domain into discrete blocks, where a set of collocation points is introduced within each block to approximate the unknown solution. By integrating the collocation equations across these points, the BVP is reformulated into a structured system of algebraic equations. The performance of the BHM is evaluated by benchmarking it against spectral methods on single-domain and overlapping grids. The results obtained from the preliminary analysis indicate that the BHM achieves higher accuracy and enhanced numerical stability. Specifically, it is apparent from the results that the BHM maintains a significantly lower matrix condition number compared to established spectral techniques, which often suffer from ill-conditioning as the number of grid points increases. This study confirms that the BHM provides a highly reliable alternative for solving complex second-order BVPs.

Stable Laguerre differentiation matrices for pseudospectral methods

Emma Nel & Nick Hale (Stellenbosch University)

Laguerre spectral and pseudospectral methods provide a natural framework for solving differential equations on semi-infinite domains, but their practical implementation is often limited by numerical instability. In particular, classical constructions of Laguerre pseudospectral differentiation matrices involve intermediate quantities that grow or decay rapidly, leading to overflow, underflow, and loss of significance in floating-point arithmetic. As a result, widely used implementations may fail for a relatively modest number of collocation points. In this paper we present a stable and efficient framework for constructing Laguerre pseudospectral differentiation matrices. The approach provides a reformulation of the off-diagonal entries and computes all required quantities simultaneously using an existing fast algorithm that also produces the collocation nodes. For the diagonal entries, a direct closed-form expression is employed to improve numerical accuracy. The resulting method avoids the catastrophic cancellation and yields an all-in-one procedure for generating differentiation matrices. Numerical experiments demonstrate that the proposed construction improves robustness and maintains high accuracy for substantially larger numbers of collocation points than standard implementations.

Multiscale Modelling to Support Evidence-Based Control and Elimination of Environmentally Transmitted Infectious Disease Systems

Rendani Netshikweta (University of Limpopo)

Environmentally transmitted infectious diseases remain a major public health challenge in resource-limited settings, where transmission is sustained by environmental contamination and inadequate sanitation. This study develops a multiscale mathematical model to support evidence-based control and elimination strategies, using human ascariasis as a case study. The model links parasite life-cycle dynamics at the host level with environmental pathogen stages and population-level transmission dynamics. We investigated key mathematical properties of the model, including positivity, the disease-free equilibrium, and the basic reproduction number. Sensitivity analysis is conducted to identify parameters that most strongly influence transmission potential and disease persistence. The model is further extended to evaluate combined intervention strategies, including treatment, sanitation, environmental decontamination, and behavioural change. Results from numerical simulations and comparative effectiveness analyses indicate that combined control strategies consistently outperform single-control approaches, leading to substantial reductions in transmission potential and endemic infection burden. These findings highlight the importance of coordinated medical and public health interventions and demonstrate how multiscale models can inform strategic planning for sustainable disease control. The proposed framework provides practical insights for policymakers and public health practitioners seeking to design effective long-term intervention programs for environmentally transmitted infectious diseases.

A Numerical Approach to a Fractional Model of Cassava Mosaic Disease

Nolwazi Nkomo, Hossein Jafari, & Maluti Kgarose (University of South Africa)

In this paper, we analyze a fractional-order cassava mosaic disease model derived using the Caputo–Fabrizio derivative, since classical integer-order models often fail to capture memory effects in plant–vector disease dynamics. The model describes interactions between healthy and infected cassava plants, and susceptible and infected whitefly vectors. We establish the existence and uniqueness of solutions to the model. For the numerical analysis, we employ a three-step Adams–Bashforth scheme adapted to the Caputo–Fabrizio operator. Numerical simulations show that the fractional order significantly influences transient dynamics and disease persistence, with convergence to the classical model as the fractional operator reduces to its integer-order counterpart.

Solving higher order Korteweg–de Vries equations using multi-domain spectral collocation method

Mokiri Nkwana, Dr Musawenkosi Mkhathshwa, & Dr Kgomotshwana Thosago (University of Limpopo)

Solving the Korteweg–de Vries (KdV) equations has long been a challenging task, particularly over large computational domains. This study presents a multi-domain spectral collocation method (MDSCM) for efficiently solving higher-order KdV equations defined on extended time intervals. The KdV equations are first reformulated into a linearized iterative form using the quasi-linearization method (QLM). The time domain is partitioned into non-overlapping subintervals, while the spatial domain is divided into overlapping subintervals. Solutions within each time subinterval are computed independently, with continuity conditions imposed to generate initial values for subsequent intervals. Conversely, the spatial solutions are determined simultaneously across overlapping subdomains.

The obtained numerical solutions are compared with exact analytical solutions to assess accuracy. In addition, the efficiency, stability, and accuracy of the proposed method are verified through analysis of computational error analysis, condition number evaluation, and computation time assessment. The results confirm that the proposed approach is a robust and reliable numerical tool capable of accurately capturing the complex structures typical of KdV-type models—such as solitons and intricate wave interactions. The method achieves high precision, with numerical results closely matching exact solutions and presenting very small absolute errors.

A canonical evaporation model problem for phase-change in multi-phase flows

Jan Nordström (Linköping University and UJ), Hanifa Hanif (UCT), & Arnaud G. Malan (UCT)

We formulate a well posed formulation of the canonical one-dimensional nonlinear evaporation multi-phase model problem (the Stefan and sucking problem). This extremely stiff model problem is commonly used to validate existing production codes such as e.g. AlphaFlow developed at UCT-Elemental Numerics and FLOW-3D developed by Flow Science Inc. After transformation to a steady frame, we derive interface conditions at the unsteady moving evaporation interface and boundary conditions at the steady outer boundaries. Next, we discretize using provably stable high order accurate continuous Galerkin finite element methods and perform numerical experiments. Finally, we speculate on which level/order of accuracy that is necessary for the production codes in order to achieve reasonable run times.

Stability and Positivity of RBF Interpolation in Compressible Flows

Freeman Nyathi (UNISA), Marcia Moremedi (UNISA), & Neossi Nguetchue (Namibia University of Technology)

Radial Basis Function (RBF) interpolation has become a powerful meshless technique for spatial approximation in computational fluid dynamics, particularly for complex geometries and scattered data. However, when applied to compressible flow simulations, numerical instabilities and violations of physical constraints such as positivity of density and pressure may occur. In this study, we investigate the numerical stability and positivity-preserving properties of RBF interpolants when reconstructing compressible flow variables. Several commonly used RBF kernels, including Gaussian, Multiquadric, and Polyharmonic splines, are examined. The analysis focuses on their interpolation behaviour under steep gradients and shock-like profiles typically encountered in compressible flows. Numerical experiments are performed on representative one-dimensional and two-dimensional flow problems. The results demonstrate that while smooth RBF kernels offer high accuracy, they may introduce non-physical oscillations near discontinuities. In contrast, locally supported or polynomially augmented RBF interpolants exhibit improved stability and better preservation of positivity. These findings provide useful guidelines for selecting appropriate RBF interpolants for compressible flow simulations and contribute toward the development of robust meshless CFD solvers.

Cnoidal wave solutions of the KdV equation

Carel Olivier (Akademia and North-West University)

Cnoidal wave solutions are nonlinear periodic solutions expressed in terms of Jacobi elliptic functions that arise in many nonlinear partial differential equations. In this presentation, we return to these solutions that was reported by Korteweg and De Vries in 1895. These solutions are characterized by the elliptic integral modulus parameter. Interestingly, two different limits of this parameter tend to either linear wave solutions or soliton solutions. To unravel this, a subset of “mass-conserving” cnoidal wave solutions are considered. These solutions are used to demonstrate both limits.

Variational Analysis and Integration of the $(2 + 1)$ Fourth-Order Time-Dependent Biharmonic Equation via Energy and Momentum Conservation and Applications

Ali Raza (Stellenbosch University) & Abdul Hamid Kara (University of the Witwatersrand)

We study the integrability via conservation laws and discuss the non-linearity of the fourth-order $(2 + 1)$ biharmonic equations corresponding to some well-known potential functions $h(u)$. Variational analysis has been performed based on the various potential functions $h(u)$. Conservation laws are investigated by Noether's theorem and presented in the form of conserved vectors. The obtained conserved flows define energy, momentum and flow dynamics supporting the system integrability.

Theory of the transverse vibration of an axially loaded beam

Nic van Rensburg & Kirstin Hohls (University of Pretoria)

We consider the simplest nonlinear model problem posed in Sapir and Reiss 1987. In this model, stretching, bending and shear of the beam is included. The authors derived some properties of a solution but did not consider existence theory. Tucsna 1992 presented a local existence result with proof. In Ammari 2002 we find a theorem stating the existence of a global weak solution. The author proves the existence of a weak global solution using a fixed point approach and results for an associated linear problem. In her doctoral thesis Kirstin Hohls expressed reservations regarding the proofs in these articles and proved the existence of a mild solution.

In this presentation we elaborate on the results above. We discuss classical, weak and mild solutions in general and the relevance of these results for applications.

Assessing the Effect of Spatial Partitioning on Model Predictions of White Rhino (*Ceratotherium simum simum*) Population Dynamics in Kruger National Park, South Africa

Lorenzo Ruaro, Cang Hui, Pietro Landi, & Sandra MacFadyen (Stellenbosch University)

Reliable forecasting of wildlife populations is critical for conservation, particularly for long-lived, low-fecundity species such as the southern white rhino (*Ceratotherium simum simum*) in Kruger National Park (KNP), South Africa. A major limitation stems from the spatially explicit nature of species distributions. This often manifests as discrete habitat patches or intrinsic metapopulation structures, particularly in large populations occupying enclosed landscapes. We therefore examined whether spatial partitioning and temporal calibration could enhance the predictive accuracy of population forecasts based on annual census data (1988–2012). To do so, the park was divided into discrete cells, and local relative growth rates were estimated using KNP census data while accounting for temporal autocorrelation and spatial synchrony. Forecasts were calibrated to different Final Calibration Years (FCYs) and assessed using Root Mean Squared Percentage Error (RMSPE). Models incorporating temporal autocorrelation generally improved predictive performance, especially during periods of consistent population trends. Predictive accuracy varied non-linearly with spatial partitioning: row-based divisions minimized RMSPE with three rows, whereas column-based partitions captured north–south spatial correlations more effectively and yielded lower overall RMSPE. Our analyses suggest potential metapopulation structure within southern KNP and demonstrate that combining spatial and temporal considerations enhances population trend forecasting. This framework provides a robust tool for identifying early demographic shifts, optimizing monitoring efforts, and informing adaptive management strategies. By linking spatially explicit growth rates with temporal dynamics, the approach supports proactive, evidence-based conservation of white rhino's populations under changing environmental and anthropogenic pressures.

Comparative Lie Symmetry Analysis of Four Fundamental One-dimensional Wave Equations

Lazarus Rundora & Dimakatso Raesibe Seroka (University of Limpopo)

A systematic comparative Lie symmetry analysis of four paradigmatic one-dimensional wave equations: the linear wave equation, Sine-Gordon equation, Klein-Gordon equation, and Boussinesq equation is presented. We investigate their symmetry algebras, group-invariant solutions, conservation laws, and integrability properties through unified Lie-theoretic methods. The study reveals how different physical mechanisms such as dispersion, nonlinearity, and higher-order effects modify symmetry structures and solution properties.

An Integrable Parity-Time Symmetric Lattice with Gain and Loss

Frank Smuts & Igor Barashenkov (University of Cape Town)

A parity-time-symmetric oligomer is a finite-size discrete nonlinear Schrödinger equation with balanced gain and loss of energy. These equations may model networks of optic fibres, or Bose-Einstein condensates confined to multiple potential wells. One may intuitively assume that these systems are nonintegrable, owing to the presence of gain and loss terms. To the contrary - at least in the two-site case (or dimer case) there exists a four-parameter class of Hamiltonian integrable oligomers. Two questions remain unanswered:

1) Do these integrable dimers possess a Lax pair? 2) Are there any integrable oligomers with more than two interacting sites?

To gain insight into both questions, we construct a prototypical integrable PT-symmetric oligomer, as a nonlocal reduction of the Ablowitz-Ladik equation. The oligomer admits a Lax pair and remains Hamiltonian integrable for arbitrarily many interacting sites.

The linearized oligomer exhibits symmetry breaking when the gain loss coefficient is raised past a critical value. The fully nonlinear system is, of course, much more complicated. We solve for the two-site case exactly, in terms of elementary functions, and show that the system conspires to completely suppress any symmetry-broken orbits.

Symbol-Based Analysis of a Multigrid Method for a PDE-Constrained Optimal Control Problem

René Spoerer (Bergische Universität Wuppertal)

Multigrid methods are a promising approach for solving certain classes of sparse linear systems. When employed successfully, they offer optimal convergence: computational cost grows linearly with the number of unknowns.

If the system additionally displays a block-Toeplitz or block-circulant structure (which occurs when differential equations are discretised on a regular grid), symbol-based techniques provide an elegant analytical framework for describing the behaviour of multigrid methods. Indeed, the symbol characterises the spectral behaviour of the system matrix across all levels of the multigrid method and enables a convergence analysis irrespective of the number of levels employed.

As a specific application, this contribution examines an optimal control problem constrained by the heat equation. Special attention is given to the boundary conditions, ensuring that the structural properties required for the symbol analysis are preserved across all levels of the multigrid cycle.

Greedy vertex colouring algorithms in a dynamic vertex-deleting network model

Gerhard Venter (Norwegian University of Life Sciences), Leonardo Rydin Gorjão (Norwegian University of Life Sciences, Open University of the Netherlands), & Jacques M. Maritz (University of the Free State)

In a simple model for a complex network, a node can lose its functionality based on the functional states of its neighbours. In an abstraction, vertices are sequentially removed from the underlying graph when they lose functionality in the network. The original degree of a vertex v is denoted by $\deg(v)_0$ and its current degree, after i steps in the removal sequence, is denoted by $\deg(v)_i$. Starting with a graph G , a vertex v is removed if, at any moment i in the sequence, $(\deg(v)_i)/(\deg(v)_0) < \theta$, for a fixed threshold $\theta \in (0, 1)$. The sequence begins with the removal of a single vertex. By artificially fixing a relatively small number of vertices, so that they are not removed when the removal criterion is satisfied, it is possible to terminate the removal sequence while the graph is still mostly intact. The challenge lies in correctly choosing a minimal subset of vertices to keep fixed. Two recent studies, [Smolyak *et al.*, *Sci. Rep.* **10**, 16124 (2020), Singh *et al.*, *Commun. Phys.* **8**, 170 (2025)], aim to fix a small fraction of the vertices in G that satisfy $(\deg(v)_0 - 1)/(\deg(v)_0) < \theta$. One proposed strategy relies on a proper vertex colouring, determined by the largest-first greedy algorithm. We illustrate that the effectiveness of the strategy is limited by the application of different greedy algorithms and the density of G . Additionally, we show that the proposed algorithms do not yield a unique solution for colouring the graphs, nor for ensuring a sufficient selection of vertices to achieve minimal removal.

Depth-dependent singularities in electrohydrodynamic waves

Zhan Wang (Institute of Mechanics, Chinese Academy of Sciences),
Demetrios Papageorgiou (Imperial College London), & Tao Gao (University of Essex)

Interfacial instabilities in dielectric fluids can be driven to collapse by a strong normal electric field. This study investigates finite-time singularities in electrocapillary-gravity waves on a fluid of finite depth. We examine two distinct physical regimes to characterize the topological evolution of the interface. In the shallow-water limit, a quasi-linear hyperbolic model is derived to describe the evolution of unstable periodic disturbances. Both theoretical analysis and fully nonlinear computations reveal a touchdown singularity, where the interface ruptures upon contact with the bottom boundary. In the deep-water regime, a nonlinear dispersive model predicts a blowup singularity, with amplitudes tending to infinity in finite time. However, full Euler simulations indicate that as depth increases, the system transitions toward a $2/3$ power cusp singularity or non-physical self-intersecting structures. These results elucidate how the depth-dependent competition between nonlinearity, dispersion, and electric forcing dictates the specific mechanism of interfacial collapse. This work provides a rigorous comparison between reduced-order asymptotic models and high-fidelity numerical simulations in electrohydrodynamics.

The nonlinear Schrödinger equation: Modulational instability and rogue waves

André Weideman (Stellenbosch University)

Uniform wave train solutions of the periodic nonlinear Schrödinger equation can be unstable to perturbations. This so-called modulational instability has been proposed as a possible mechanism for rogue wave formation on the ocean (and in other applications). In this talk we discuss numerical and asymptotic approximations to this instability as well as its pole dynamics.